Determination of the Acceleration of the Characteristic Mechanism by Introducing a Fictitious Bar

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This paper treats the problem of determination of the acceleration of a group of specific mechanisms differing only by the number of attributed dyads. The solution is simple in the case where the first term of such a mechanism receives the motive power. When it is the last term of the mechanism that the motive power is conveyed to, then the problem arises. The problem is to solve by the method of characteristical instantaneous teamed centres of rotation of the terms.

Keywords: *Mechanism, fictitious bar, acceleration, coupled centres.*

1. INTRODUCTION

It is known that a four-bar linkage mechanism can be simply extended with a dyad with pin constraints in such a way that one bar of the dyad is connected to the connecting rod and the second one is pin supported. The mechanism thus achieved can be extended with a new dyad by connecting it with the preceding one in the same way the preceding is connected to the basic mechanism. This adding of dyads to the mechanism can repeatedly continue. The determination of the kinematics values (of the acceleration above all) in such a mechanism is simple. The problem arises when, instead of the first moving bar in such kinematics chain, the last one becomes the motive bar. Classical methods in theories of mechanisms solve such cases only if the number of bars of these mechanisms is reduced to a minimum.

In this paper the procedure of the determination of the acceleration by introducing a fictitious bar will be exposed for the group of mechanisms mentioned above. It has been noticed that certain groups of bars of these mechanisms may be temporarily replaced by an equivalent fictitious bar in order to enable us to determine the characteristic accelerations by means of the Method of coupled centers.

2. THESIS

A kinematics chain given in Fig. 1 receives its motive power through the bar 2. The kinematics chain can not have less than six bars and their number must be always even. For solving such problems using the Method of coupled centers a fictitious bar is to be introduced, which is equivalent to the teamed kinematics three-bar-group in a given moment (Fig. 2). The equivalent bar thus achieved forms together with two other bars in the chain a new teamed kinematics three-bars-group that is subject to a new replacement by a new equivalent bar... and so on until we come to the motive bar. This replacement process goes from the end to the beginning of the kinematics chain.

Having finished the procedure of introducing the fictitious equivalent bars, accelerations of bars of the mechanism are to be determined starting from the motive bar, whose law of motion is known, until the end of the kinematics chain. The determination of the acceleration of a teamed group's bars makes "disappear" the fictitious bar of that group.

The equivalent bar represents the radius of curvature of the trajectory of the point of the middle bar of the teamed three-bar-group, the point being receiver of the motive power for the group. The determination of the radius of curvature is explained in the paper [2].

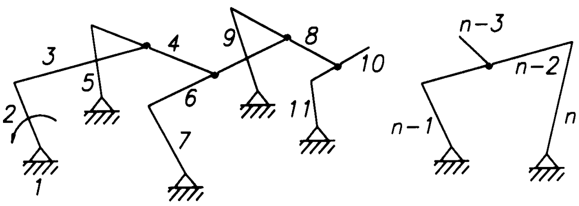

Figure 1. The kinematics chain.

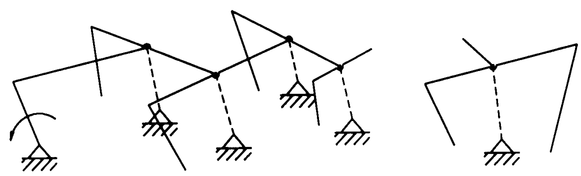

Figure 2. The mechanism with fictitous bas.

In order to prove this assertion we will proceed as follows. In the six-bar mechanism OABECDH, illustrated in Fig. 3, we determine the equivalent bar f(BG) that replaces the three-bar kinematics group 5-4-6 and forms a temporary mechanism OABG. Apart from the mechanism OABG there are also the mechanisms GBCE, GBDH and ECDH in the new kinematics chain. By means of the motive bar 2 we determine the acceleration of the fictitious bar f of the mechanism OABG. The same procedure applies for determination

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of the acceleration of the bar 6 of the mechanism GBCE by using the motive bar f. The acceleration of the bar 5 of the mechanism ECDH is to be determined by using the motive bar 6. Finally, the determination of the acceleration of the bar 5 of the mechanism GBDH follows by using the motive bar f.

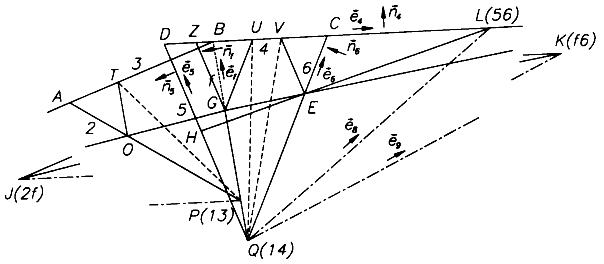

Figure 3. The six-bar meshanism OABECDH.

If the acceleration values of the bar 5 of the mechanism GBDH, that are obtained through the acceleration of the fictitious bar f, are the same as the acceleration values of the bar 5 of the mechanism ECDH, that are obtained though the acceleration of the bar 6, then it is true that the acceleration of the mentioned kinematics chain can be determined by means of a fictitious bar.

3. PROOF

Suppose the acceleration of the point A of the mechanism OABG to be known; on the acceleration plane (Fig. 4) it is represented as segment SM

$$SM = \left|\vec{a}_{An}\right| = AO\omega_2^2 \ . \tag{1}$$

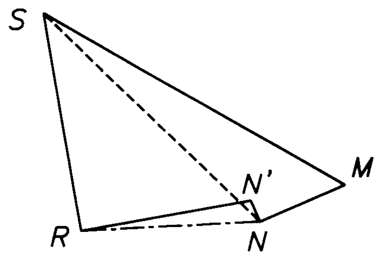

Figure 4. The acceleration (plane) of the mechanism OABG

Other accelerations in the mechanism OABG are determined by the Method of coupled centers [1]: a straight line through O, parallel to GB, is cutting the bar AB in the point T (OT||GB). A straight line through S, parallel to TP, as well as a straight line through M, parallel to BA, determine the point N (SN||TP; MN||BA) i.e. the magnitude of the acceleration $\vec{a}_{Bn}^{A}$

$$MN = \left|\vec{a}_{Bn}^{A}\right| . \tag{2}$$

A straight line through S, parallel to BG (Fig. 4) and the one through N, parallel to PJ, determine the point R (SR||BG; NR||PJ) i.e. the magnitude of the acceleration $\vec{a}_{Bn}$ of the equivalent bar f

$$SR = \left|\vec{a}_{Bn}\right| \ . \tag{3}$$

A perpendicular to SR through R and a perpendicular to MN through N determine the point N' (RN'...MN) i.e. the magnitudes of the acceleration $\vec{a}_{Bt}^{A}$ and $\vec{a}_{Bt}$

$$NN' = \left|\vec{a}_{Bt}^{A}\right| \ , \tag{4}$$

$$RN' = \left|\vec{a}_{Bt}\right| \ . \tag{5}$$

Let us now take the mechanism GBCE (Fig. 3). The bar f plays the role of a motive bar with known acceleration components $\vec{a}_{Bn}$ and $\vec{a}_{Bt}$. By the Method of coupled centers we find out the characteristic accelerations of the bars 4 and 6:

$$RR' = \left|\vec{a}_{Cn}^{B}\right| \ , \tag{6}$$

$$SF' = \left|\vec{a}_{Cn}\right| \ . \tag{7}$$

The acceleration of the point C is to be determined by means of well-known mechanical relations (Fig. 5):

$$\vec{a}_C = \vec{a}_{Cn} + \vec{a}_{Ct} \ , \tag{8}$$

$$\vec{a}_C = \vec{a}_B + \vec{a}_{Cn}^{B} + \vec{a}_{Ct}^{B} \ , \tag{9}$$

where N'T'=RR' and $SN' = \left|\vec{a}_B\right| = \left|\vec{a}_{Bn} + \vec{a}_{Bt}\right|$ which defines the magnitudes of tangential components $\vec{a}_{Ct}$ and $\vec{a}_{Ct}^{B}$ a_{Ct}

$$T'K' = \left|\vec{a}_{Ct}^{B}\right| \ , \tag{10}$$

$$F'K' = \left|\vec{a}_{Ct}\right| \ . \tag{11}$$

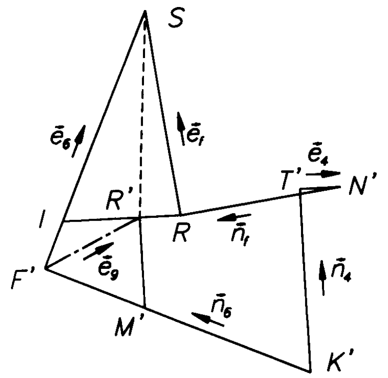

Figure 5. The acceleration of the mechanism GBCE.

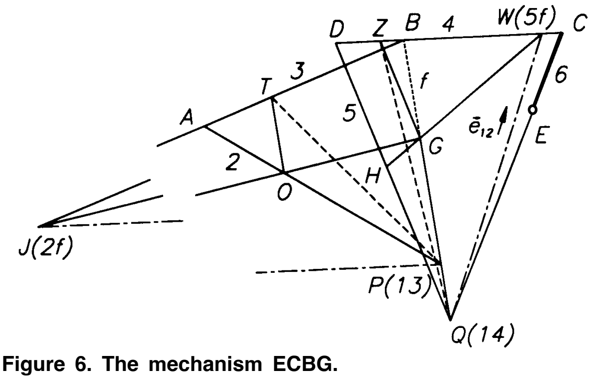

Figure 6. The mechanism ECBG.

The magnitudes of the tangential accelerations can be calculated according to [3] from these relations:

$$\frac{F'K'-F'M'}{CQ}=\frac{RN'}{BQ}\ , \tag{12}$$

$$\frac{T'K'-R'M'}{BC}=\frac{RN'}{BQ}\ . \tag{13}$$

Together with the bars 1, 5 and, partially, 4, the fictitious bar f forms the mechanism GBDH (Fig..6). By the Method of coupled centers it is possible to define the acceleration of the point D of the bar 5 (Fig. 7):

$$SX=\left|\vec{a}_{Dn}\right|\ , \tag{14}$$

$$XY=\left|\vec{a}_{Dt}\right|\ . \tag{15}$$

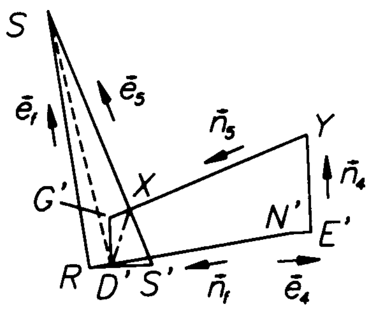

Figure 7. The acceleration of the mechanism GBDH.

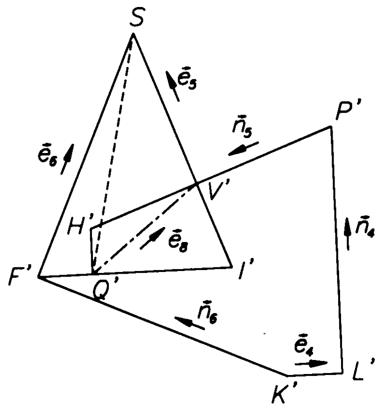

Figure 8. The acceleration of the mechanism HDCE.

Now we can consider the mechanism ECDH. By the Method of coupled centers we determine the acceleration of the point D of the bar 5 through the acceleration of the point C (Fig. 8)

$$SV'=\left|\vec{a}_{Dn}\right|\ , \tag{16}$$

$$V'P'=\left|\vec{a}_{Dt}\right|\ . \tag{17}$$

If the acceleration values of the point D thus obtained are the same as those obtained through the point B of the fictitious bar, then it is true that the problem is solvable by introducing the fictitious bar.

Let us orientate the bars f, 4, 5 and 6 by unit vectors $\vec{e}_f, \vec{e}_4, \vec{e}_5$ and $\vec{e}_6$ respectively. Then we orientate the perpendiculars to these bars by unit vectors $\vec{n}_f, \vec{n}_4, \vec{n}_5$ and $\vec{n}_6$ respectively, as well as the radius vectors QL, QK and QW by unit vectors $\vec{e}_8, \vec{e}_9$ and $\vec{e}_{12}$ respectively. The normal and total acceleration of the point C of the mechanism GBCE (Fig. 3), taking into account the point B (Fig. 5), can be expressed through normal and total accelerations:

$$-SF'\vec{e}_6=-SR\vec{e}_f-RR'\vec{e}_4-R'F'\vec{e}_9\ , \tag{18}$$

$$\begin{aligned}&-SF'\vec{e}_6-F'K'\vec{n}_6=\\&=-SR\vec{e}_f-RN'\vec{n}_f-N'T'\vec{e}_4-T'K'\vec{n}_4\end{aligned} \tag{19}$$

Similar procedure can be prepared for the determination of the acceleration of the point D if we take as pole the point C in the mechanism ECBG (Fig. 8):

$$-SV'\vec{e}_5=-SF'\vec{e}_6+F'Q'\vec{e}_4+Q'V'\vec{e}_8\ , \tag{20}$$

$$\begin{aligned}&-SV'\vec{e}_5-V'P'\vec{n}_5=\\&=-SF'\vec{e}_6-F'K'\vec{n}_6+K'L'\vec{e}_4+L'P'\vec{n}_4\ ,\end{aligned} \tag{21}$$

i.e., for the determination of the acceleration of the point D if we take as pole the point B in the mechanism GBDH (Fig. 7):

$$-SX\vec{e}_5=-SR\vec{e}_f+RD'\vec{e}_4+D'X\vec{e}_{12}\ , \tag{22}$$

$$\begin{aligned}&-SX\vec{e}_5-XY\vec{n}_5=\\&-SR\vec{e}_f-RN'\vec{n}_f+N'E'\vec{e}_4+E'Y\vec{n}_4\end{aligned} \tag{23}$$

In order to prove the above assertion it should be demonstrated that:

$$-SV'\vec{e}_5=-SX\vec{e}_5\ ,$$

and

$$-SV'\vec{e}_5-V'P'\vec{n}_5=-SX\vec{e}_5-XY\vec{n}_5\quad \text{i.e. } SV'=SX$$

and

$$V'P'=XY\ .$$

Substituting (18) into (20) and equalizing it with (22) we obtain the expression:

$$\begin{aligned}&-SR\vec{e}_f+RD'\vec{e}_4+D'X\vec{e}_{12}=\\&=-SR\vec{e}_f-RR'\vec{e}_4-F'R'\vec{e}_9+F'Q'\vec{e}_4+Q'V'\vec{e}_8\ ,\end{aligned} \tag{24}$$

whose final form is:

$$\left(RD'+RR'-F'Q'\right)\vec{e}_4=Q'V'\vec{e}_8-F'R'\vec{e}_9-D'X\vec{e}_{12}\ . \tag{25}$$

From the similarity of the triangles: $\Delta SRR'\cong\Delta QBU$, $\Delta SF'Q'\cong\Delta QCV$ and $\Delta SRD'\cong\Delta QBZ$ follow the relations:

$$RR'=\frac{BU}{BQ}SR\ , \tag{26}$$

$$F'Q'=\frac{VC}{CQ}SF'\ , \tag{27}$$

$$RD'=\frac{ZB}{BQ}SR\ . \tag{28}$$

From the similarity of the triangles: $\Delta R'IF'\cong\Delta KCQ$, $\Delta Q'I'V'\cong\Delta LDQ$ and $\Delta S'XD'\cong\Delta DQW$ follow the relations:

$$F'R'=IR'\frac{QK}{CK}\ , \tag{29}$$

$$Q'V' = Q'I'\frac{QL}{DL}\ , \tag{30}$$

$$D'X = D'S'\frac{QW}{DW}\ . \tag{31}$$

Substituting (26), (27), (28), (29), (30) and (31) into (25) we obtain:

$$\left(\frac{ZB}{BQ}SR + \frac{BU}{BQ}SR - \frac{VC}{CQ}SF'\right)\vec{e}_4 =$$
$$= \frac{QL}{DL}Q'I'\vec{e}_8 - \frac{QK}{CK}IR'\vec{e}_9 - \frac{QW}{DW}D'S'\vec{e}_{12}\ . \tag{32}$$

By introducing the expressions:

$$\vec{e}_8 = \frac{QD}{QL}\vec{e}_5 + \frac{DL}{QL}\vec{e}_4\ , \tag{33}$$

$$\vec{e}_9 = \frac{QD}{QK}\vec{e}_5 + \frac{DK}{QK}\vec{e}_4\ , \tag{34}$$

$$\vec{e}_{12} = \frac{QD}{QW}\vec{e}_5 + \frac{DW}{QW}\vec{e}_4\ , \tag{35}$$

into the expression (32), we obtain as its final form:

$$\left(\frac{ZB}{BQ}SR + \frac{BU}{BQ}SR - \frac{VC}{CQ}SF' - Q'I' + \frac{DK}{CK}IR' + D'S'\right)\vec{e}_4 =$$
$$= \left(\frac{QD}{DL}Q'I' - \frac{QD}{CK}IR' - \frac{QD}{DW}D'S'\right)\vec{e}_5\ . \tag{36}$$

For the equality (36) to be valid, it is necessary end sufficient that following conditions are satisfied:

$$\frac{ZB}{BQ}SR + \frac{BU}{BQ}SR - \frac{VC}{CQ}SF' - Q'I' + \frac{DK}{CK}IR' + D'S' = 0\ , \tag{37}$$

i.e.

$$\frac{QD}{DL}Q'I' - \frac{QD}{CK}IR' - \frac{QD}{DW}D'S' = 0\ . \tag{38}$$

For the reasons that are easily understandable, we will analyze only one of them; let it be the expression (38). By introducing the relations:

$$Q'I' = F'I' - F'Q'\ , \tag{39}$$

$$IR' = IR - RR'\ , \tag{40}$$

$$D'S' = RS' - RD'\ , \tag{41}$$

into (38) and by using also the relations (26), (27) and (28), we obtain:

$$\frac{QD}{DL}\left(\frac{DC}{CQ}SF' - \frac{VC}{CQ}SF'\right) - \frac{QK}{CK}\left(\frac{BC}{BQ}SR - \frac{BU}{BQ}SR\right) -$$
$$- \frac{QD}{DW}\left(\frac{DB}{BQ}SR - \frac{ZB}{BQ}SR\right) = 0\ , \tag{42}$$

whose final form, after appropriate reductions, becomes:

$$\frac{QD(DC - VC)}{DL \cdot CQ}SF' - \frac{QD(BC - BU)}{CK \cdot BQ}SR -$$
$$- \frac{QD(BD - BZ)}{DW \cdot BQ}SR = 0\ . \tag{43}$$

Using well-known relations in the mechanisms:

$$SR = |\vec{a}_{Bn}| = |\vec{v}_B|^2 / BG\ , \tag{44}$$

$$SF' = |\vec{a}_{Cn}| = |\vec{v}_C|^2 / CE\ , \tag{45}$$

$$|\vec{v}_B| = |\vec{\omega}_4| BQ\ , \tag{46}$$

$$|\vec{v}_C| = |\vec{\omega}_4| CQ\ , \tag{47}$$

and after appropriate reductions, the expression (43) acquires the following form:

$$\frac{QD\,DV}{DL\,CE} - \frac{QD \cdot BQ\,UC}{CK\,BG\,CQ} - \frac{QD\,BQ\,DZ}{DW\,BG\,CQ} = 0\ . \tag{48}$$

From the similarity of the triangles: $\Delta\,HDL \cong \Delta\,EVL$, $\Delta\,GUK \cong \Delta\,ECK$ and $\Delta\,HDW \cong \Delta\,GZW$, follow the relations:

$$\frac{DV}{DL} = \frac{HL - EL}{HL}\ , \tag{49}$$

$$\frac{UC}{CK} = \frac{GK - EK}{EK}\ , \tag{50}$$

$$\frac{DZ}{DW} = \frac{HW - GW}{HW}\ . \tag{51}$$

By using the instantaneous centers of zero velocity L(67), K(f6), W(5f) and C(55) we obtain the relations:

$$\frac{EL}{HL} = \frac{\omega_5}{\omega_6}\ , \tag{52}$$

$$\frac{GK}{EK} = \frac{\omega_6}{\omega_f}\ , \tag{53}$$

$$\frac{GW}{HW} = \frac{\omega_5}{\omega_f}\ , \tag{54}$$

$$\frac{CE}{CQ} = \frac{\omega_4}{\omega_6}\ , \tag{55}$$

and after substituting (52), (53) and (54) into (49), (50) and (51), and these ones into (48), we obtain:

$$\left(1 - \frac{\omega_5}{\omega_6}\right)\frac{QD}{CE} - \frac{QD \cdot BQ}{BG \cdot CQ}\left[\left(\frac{\omega_6}{\omega_f} - 1\right) + \left(1 - \frac{\omega_5}{\omega_f}\right)\right] = 0\ , \tag{56}$$

and after some reductions it follows:

$$QD(\omega_6 - \omega_5)\left(\frac{1}{\omega_6} - \frac{CE}{CQ\omega_4}\right), \tag{57}$$

and finally, substituting (55) into (57) it follows:

$$QD(\omega_6 - \omega_5)\left(\frac{1}{\omega_6} - \frac{1}{\omega_6}\right) = 0\ , \tag{58}$$

so that the condition (38) is fulfilled and it is proven that SX=SV'.

In order to prove the assertion that the magnitudes of tangential accelerations are equal, i.e. that XY=V'P', we start from total accelerations of the point D, that are expressed by using the point C, i.e. B. Suppose that:

$$-SX\vec{e}_5 - XY\vec{n}_5 = -SV'\vec{e}_5 - V'P'\vec{n}_5\ . \tag{59}$$

Substituting (19) into (21) and equalizing with (23) we obtain:

$$-S R\vec{e}_f - RN'\vec{n}_f + N'E'\vec{e}_4 + E'Y\vec{n}_4 = -S R\vec{e}_f -$$
$$-RN'\vec{n}_f - N'T'\vec{e}_4 - T'K'\vec{n}_4 + K'L'\vec{e}_4 + L'P'\vec{n}_4 \,. \quad (60)$$

By following reduction of the expression, and taking into account that N'E'=RD', N'T'=RR', K'L'=F'Q', we obtain:

$$(RD'+RR'-F'Q')\vec{e}_4 = (L'P'-T'K'-E'Y)\vec{n}_4 \,. \quad (61)$$

For the equality (61) to be valid, it is necessary end sufficient that following conditions are satisfied:

$$RD'+RR'-F'Q'=0 \;, \quad (62)$$

i.e.

$$L'P'-T'K'-E'Y=0 \;. \quad (63)$$

Now we analyze the expression (62). By substituting the expressions (26), (27) and (28) into (62) we obtain:

$$\left(\frac{ZB}{BQ}+\frac{BU}{BQ}\right)SR-\frac{VC}{CQ}SF'=0 \;. \quad (64)$$

By using afterwards the expressions (44), (45), (46), and (47) we obtain:

$$\frac{ZB+BU}{BG}-\frac{VC\cdot CQ}{CE\cdot BQ}=0 \;. \quad (65)$$

From the similarity of the triangles

$$\Delta ZUG \cong \Delta DCQ \cong \Delta VCE \quad \text{i.e.} \quad \Delta BUG \cong \Delta BCQ$$

follow the relations:

$$\frac{ZB+BU}{BG}=\frac{DC}{BQ} \;, \quad (66)$$

$$\frac{VC}{CE}=\frac{DC}{CQ} \;. \quad (67)$$

Substituting (66) and (67) into (65) we obtain:

$$\frac{DC}{BQ}-\frac{DC\cdot CQ}{CQ\cdot BQ}=0 \;, \quad (68)$$

so that the condition (62) is fulfilled and it is proven that V'P'=XY and that three teamed bars may be replaced by an equivalent bar in order to determine the acceleration of specific forms of kinematics chains.

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ОДРЕЂИВАЊЕ УБРЗАЊА КОД КАРАКТЕРИСТИЧНОГ МЕХАНИЗМА УВОЂЕЊЕМ ФИКТИВНОГ ЧЛАНА

С. Ђорђевић

Рад се бави проблемом одређивања убрзања код групе специфичних механизама који се међусобно разликују само по броју придодатих di jada. Kada pogon pr i ma pr vi č~l an mehani zma решење је једноставно. Када погон прима задњи члан механизма настаје проблем. Метода којом се решава проблем базирана је на спрезању kar akt er i st i čni h t r enut них центара ротације чланова.