Asymptotic Solution of Nonlinear Vibrations of Antisymmetric Laminated Angle-Ply Plate

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In this paper single frequency vibration of laminated angle-ply rectangular plate which is freely supported on its own edges are analyzed. The classical Kirhhoff theory is used and the vibration equations of Karman type are analyzed using the Airy function. Asymptotic solution in the first approximation is given. Numerical example includes analyses of single frequency plate vibration in stationary and instationary conditions under the activity of time-dependence outer impulse. Amplitude-frequency and phase-frequency characteristics of plate in stationary and instationary conditions for different laminate characteristics are presented graphically.

Keywords: *Nonlinear vibration, laminated, plate, amplitude, phase, frequency*

1. INTRODUCTION

The problem of laminated composite vibrations has been the object of consideration during the past five decades. The equations of laminated plate vibrations are essentially identical to those for a single-layer orthotropic plate. Jones (1975) gave the fundamental basis for tension-deformation state of laminated plates and differential equations of linear plates vibrations. Khdeir and Reddy (1999) consider the free vibrations of laminated composite plates, for different boundary conditions, comparing the Kirhhoff theory with applied one. Tylikovski (1993) considers stability of nonlinear symmetrical laminated cross-ply plates. The equation of vibration of cross-ply laminated plate is derived by introduction of Airy function. Ghazarian and Locke (1995) with the invoking of Galerkin method determine equations of laminated plate vibrations, which are simple for analysis.

Very applicable asymptotic method of Krilova-Bogoloubova-Mitropolski 1964) for solving of nonlinear vibrations continuum problems is applied in papers of Pavlović (1984) and K. Hedrih and others (1974), (1978), (1986). Pavlović (1984) published a study about analysis for resonant regime two-frequent vibrations of shallow shells; K.Hedrih and the others (1986) analyze four-frequent vibrations of thin shells with an initial irregularity. Janevski (2001) analyze single frequency vibration symmetrically laminated cross-ply plate, consider influence of mechanical and others characteristics to amplitude and phase of asymptotic solution.

In the present paper single frequency vibrations of laminated plate under the time dependent external force effect are considered. Also, influence of mechanical and others characteristics to amplitude and phase of asymptotic solution is given in the first approximation.

2. PROBLEM FORMULATION

Components of deformation tensor and components of curvature of the plate middle surface are defined as follows:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}(\dfrac{\partial w}{\partial x})^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}(\dfrac{\partial w}{\partial y})^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix},$$

$$\{\kappa\} = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w}{\partial x^2} \\ -\dfrac{\partial^2 w}{\partial y^2} \\ -2\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix}, \tag{1}$$

where $u(x,y,t)$, $v(x,y,t)$ are in-plane displacements whether $w(x,y,t)$ is displacement normal to the middle surface of plate..

Membrane force, moments of bending and torsion moment in the cross section along axes can be presented as:

$$\{N\} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}, \ \{M\} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}. \tag{2}$$

The relationship of forces and moments in the middle surface of plate is expressed by the equation

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = [C]\begin{Bmatrix} \{\varepsilon\} \\ \{\kappa\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix}\begin{Bmatrix} \{\varepsilon\} \\ \{\kappa\} \end{Bmatrix}, \tag{3}$$

Received: June 2002, accepted: November 2002.
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Matrix of stiffness $[C]$ for antisymmetric angle-ply laminates has the form:

$$[C]=\begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & B_{16} \\ A_{12} & A_{22} & 0 & 0 & 0 & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & 0 \\ 0 & 0 & B_{16} & D_{11} & D_{12} & 0 \\ 0 & 0 & B_{26} & D_{12} & D_{22} & 0 \\ B_{16} & B_{26} & 0 & 0 & 0 & D_{66} \end{bmatrix}, \tag{4}$$

and matrices of extensional stiffness $[A]$, coupling stiffness $[B]$ and bending stiffness $[D]$ are defined as:

$$[A]=\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix},\ [B]=\begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{16} & B_{26} & 0 \end{bmatrix},$$
$$[D]=\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}. \tag{5}$$

Elements of matrix of stiffness are defined as

$$\left(A_{ij}, B_{ij}, D_{ij}\right)=\int_{-h/2}^{h/2}(1, z, z^2)\, Q_{ij}\, \mathrm{d}z,$$

where Q_{ij} are the reduced in-plane stiffness of an individual lamina, and h is thickness of plate.

Differential equations of plate vibration are obtained from condition that forces and moments in coordinate direction are balanced dynamically

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\ ,\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0,$$
$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_y}{\partial y^2}+N_x\frac{\partial^2 w}{\partial x^2}+ \tag{6}$$
$$+2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}+N_y\frac{\partial^2 w}{\partial y^2}+q=\rho h\frac{\partial^2 w}{\partial t^2}+2\beta\rho h\frac{\partial w}{\partial t}$$

where ρ is the density material of plate, β is damping coefficient and $q(x, y, t)$ is external disturbing force.

From equation (3) moment of bending as well as moment of torsion components can be expressed in terms of transverse displacement of middle surface plate:

$$M_x=B_{16}\gamma_{xy}-D_{11}\frac{\partial^2 w}{\partial x^2}-D_{12}\frac{\partial^2 w}{\partial y^2},$$
$$M_y=B_{26}\gamma_{xy}-D_{12}\frac{\partial^2 w}{\partial x^2}-D_{22}\frac{\partial^2 w}{\partial y^2}, \tag{7}$$
$$M_{xy}=B_{16}\varepsilon_x+B_{26}\varepsilon_y-2D_{66}\frac{\partial^2 w}{\partial x\,\partial y}.$$

Introduce the function of tension $\psi=\psi(x, y, t)$ so that

$$N_x=\frac{\partial^2\psi}{\partial y^2},\ N_y=\frac{\partial^2\psi}{\partial x^2},\ N_{xy}=-\frac{\partial^2\psi}{\partial x\,\partial y}, \tag{8}$$

the first and the second equation of system Eq. (6) are satisfied. The condition of deformation compatibility can be expressed as

$$\frac{\partial^2\varepsilon_x}{\partial y^2}+\frac{\partial^2\varepsilon_y}{\partial x^2}=\frac{\partial^2\gamma_{xy}}{\partial x\,\partial y}. \tag{9}$$

and according to Eq. (1) can be rewritten in the next form:

$$\frac{\partial^2\varepsilon_x}{\partial y^2}+\frac{\partial^2\varepsilon_y}{\partial x^2}-\frac{\partial^2\gamma_{xy}}{\partial x\,\partial y}=\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}. \tag{10}$$

From equation (3) it follows that

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \kappa_x\\ \kappa_y\\ \kappa_{xy}\end{Bmatrix}=[C]^{-1}\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{Bmatrix}, \tag{11}$$

where $[C]^{-1}$ is the inverse of the matrix of stiffness $[C]$:

$$[C]^{-1}=\begin{bmatrix} A_{11}^* & A_{12}^* & 0 & 0 & 0 & B_{16}^* \\ A_{12}^* & A_{22}^* & 0 & 0 & 0 & B_{26}^* \\ 0 & 0 & A_{66}^* & B_{16}^* & B_{26}^* & 0 \\ 0 & 0 & B_{16}^* & D_{11}^* & D_{12}^* & 0 \\ 0 & 0 & B_{26}^* & D_{12}^* & D_{22}^* & 0 \\ B_{16}^* & B_{26}^* & 0 & 0 & 0 & D_{66}^* \end{bmatrix}. \tag{12}$$

From Eqs. (8), (11) and (12) components of tensor of deformation can be expressed in terms of function of tension

$$\varepsilon_x=A_{11}^*\frac{\partial^2\psi}{\partial y^2}+A_{12}^*\frac{\partial^2\psi}{\partial x^2}+B_{16}^* M_{xy},$$
$$\varepsilon_y=A_{12}^*\frac{\partial^2\psi}{\partial y^2}+A_{22}^*\frac{\partial^2\psi}{\partial x^2}+B_{26}^* M_{xy}, \tag{13}$$
$$\gamma_{xy}=-A_{66}^*\frac{\partial^2\psi}{\partial x\,\partial y}+B_{16}^* M_x+B_{26}^* M_y.$$

Substituting Eqs. (7) and (8) into third equation of the system Eq. (6) and including of Eq. (13), after its differentiation, into left side of Eq. (10) gives:

$$\rho h\frac{\partial^2 w}{\partial t^2}+2\beta\rho h\frac{\partial w}{\partial t}+L_{AU}(w)+$$
$$+e_1\frac{\partial^4\psi}{\partial x^3\partial y}+e_2\frac{\partial^4\psi}{\partial x\,\partial y^3}-L(w,\psi)=q(x,y,t), \tag{14}$$

$$\Theta_{AU}\psi=-\frac{1}{2}L(w,w)-k_1\frac{\partial^4 w}{\partial x^3\partial y}-k_2\frac{\partial^4 w}{\partial x\,\partial y^3}, \tag{15}$$

where following denotations are:

$$L_{AU}(w)=g_{11}\frac{\partial^4 w}{\partial x^4}+g_{12}\frac{\partial^4 w}{\partial x^2\partial y^2}+g_{22}\frac{\partial^4 w}{\partial y^4}, \tag{16}$$

$$g_{11}=b_6 B_{16}(B_{16}^* D_{11}+B_{26}^* D_{12})+D_{11},$$
$$g_{12}=b_6 B_{16}(B_{16}^* D_{12}+B_{26}^* D_{22})+b_6 B_{26}(B_{16}^* D_{11}+$$
$$+B_{26}^* D_{12})+2(D_{12}+2b_6 D_{66}),$$
$$g_{22}=b_6 B_{26}(B_{16}^* D_{12}+B_{26}^* D_{22})+D_{22},$$
$$e_1=b_6\left[B_{16}A_{66}^*-2(B_{16}A_{12}^*+B_{26}A_{22}^*)\right],$$
$$e_2=b_6\left[B_{26}A_{66}^*-2(B_{16}A_{11}^*+B_{26}A_{12}^*)\right],$$

$$\Theta_{AU}\psi = h_{11}\frac{\partial^4\psi}{\partial x^4} + h_{12}\frac{\partial^4\psi}{\partial x^2\partial y^2} + h_{22}\frac{\partial^4\psi}{\partial y^4}, \quad (17)$$

$$h_{11} = b_6 B_{26}^*(B_{16}A_{12}^* + B_{26}A_{22}^*) + A_{22}^*,$$

$$h_{12} = b_6 B_{16}^*(B_{16}A_{12}^* + B_{26}A_{22}^*) + b_6 B_{26}^*(B_{16}A_{11}^* + B_{26}A_{12}^*) + b_6 A_{66}^* + 2A_{12}^*,$$

$$h_{22} = b_6 B_{16}^*(B_{16}A_{11}^* + B_{26}A_{12}^*) + A_{11}^*,$$

$$k_1 = b_6(B_{16}^* D_{11} + B_{26}^* D_{12} - 2B_{26}^* D_{66}),$$

$$k_2 = b_6(B_{16}^* D_{12} + B_{26}^* D_{22} - 2B_{16}^* D_{66}),$$

Eqs. (14) and (15) are differential equations of plate vibration. By solving of Eqs. (14) and (15), knowing the boundary and initially conditions, one can determine transverse displacements of middle surface $w(x,y,t)$ of laminated plate, as well as function of tension $\psi(x,y,t)$. Also according to equations (7), (8) and (13) all necessary tensor of deformation components, force components and moments are determined.

3. SINGLE FREQUENCY VIBRATIONS OF ANTISYMMETRIC LAMINATED ANGLE-PLY PLATES

Let us consider plate vibration described by the system of differential equations (14)- (15). Suppose that disturbing force $q(x,y,t)$ is acting on the system. The force is 2π-periodical in $\theta_1(t)$ with the constant amplitude P_1^* in the form

$$q(x,y,t) = \varepsilon P_1^* \sin\theta_1\, w_1(x,y), \quad (18)$$

where $d\theta_1/dt = \nu_1(t)$ is momentary frequency and ε is a small positive parameter. For the laminated plate, freely supported along edges, boundary conditions are (Tylikovski (1993))

$$\left.\begin{matrix} x=0 \\ x=a \end{matrix}\right\} w=0;\ M_x=0,\ N_x=0,\ N_{xy}=0;$$
$$\left.\begin{matrix} y=0 \\ y=b \end{matrix}\right\} w=0;\ M_y=0,\ N_y=0,\ N_{xy}=0; \quad (19)$$

Let initially conditions are

$$w(x,y,t)\big|_{t=0} = \sum_{i=1}^{N}\sum_{j=1}^{M} p_{ij} w_{ij}(x,y),$$
$$\left.\frac{\partial w(x,y,t)}{\partial t}\right|_{t=0} = \sum_{i=1}^{N}\sum_{j=1}^{M} q_{ij} w_{ij}(x,y), \quad (20)$$

where $w_{ij}(x,y) = \sin(i\pi x/a)\sin(j\pi y/b)$ are either normal functions and p_{ij} and q_{ij} are real numbers. According to boundary and initially conditions (Eqs. (19), (20)), in the single frequency regime of plate vibration transverse displacement $w(x,y,t)$ as solution of system Eqs. (14)-(15) is supposed in the form

$$w(x,y,t) = f_1(t)w_{11}(x,y) = f_1(t)\sin(\frac{\pi x}{a})\sin(\frac{\pi y}{b}), \quad (21)$$

where $f_1(t)$ is unknown function of time, which will be determined from the equation of vibration.

Taking Eq. (16) into consideration, function $L(w,w)$ is evaluated in the form:

$$L(w,w) = 2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 =$$
$$= -\frac{\pi^4}{a^2 b^2} f_1^2(t)\left[\cos(\frac{2\pi x}{a}) + \cos(\frac{2\pi y}{b})\right],$$

and included in the Eq. (15). Solving of partial differential equation one determines function of tension in the form:

$$\Psi = \frac{\lambda^2 f_1^2}{32 h_{11}}\cos(\frac{2\pi x}{a}) + \frac{f_1^2}{32\lambda^2 h_{22}}\cos(\frac{2\pi y}{b}) + \frac{\lambda(k_1 + \lambda^2 k_2)}{h_{11} + \lambda^2 h_{12} + \lambda^4 h_{22}} f_1 \cos(\frac{\pi x}{a})\cos(\frac{\pi y}{b}). \quad (22)$$

where $\lambda = a/b$ is ratio of plate sides.

Multiply Eq. (14) by $w_{11}(x,y)\mathrm{d}x\mathrm{d}y$, after substitution of disturbing force equation (18) and expressions Eqs. (21)-(22) in its right and left side respectively, to integrate by plate surface ($x \in (0,a)$, $y \in (0,b)$). If we introduce substitution

$$\xi_1(t) = f_1(t)/h, \quad (23)$$

after the integration, we will obtain differential equation in unknown function $\xi_1(t)$

$$\ddot{\xi}_1 + \omega_1^2\xi_1 = -2\beta\dot{\xi}_1 + \alpha_1\xi_1^3 + \varepsilon P_1 \sin\theta_1, \quad (24)$$

where:

$$\omega_1^2 = \frac{1}{\rho h}\frac{\pi^4}{a^4}\left(g_{11} + \lambda^2 g_{12} + \lambda^4 g_{22}\right) - \frac{1}{\rho h}\frac{\pi^4}{a^4}\left[\lambda^2\frac{(k_1 + \lambda^2 k_2)(e_1 + \lambda^2 e_2)}{h_{11} + \lambda^2 h_{12} + \lambda^4 h_{22}}\right], \quad (25)$$

$$\alpha_1 = -\frac{1}{16}\frac{h}{\rho}\frac{\pi^4}{a^2 b^2}(\frac{\lambda^2}{h_{11}} + \frac{1}{\lambda^2 h_{22}}), \quad (26)$$

$$P_1 = P_1^*/\rho h^2. \quad (27)$$

The Eq. (24) represents differential equation of compulsory vibration of plate in the single frequency regime with the frequency given by the Eq. (25).

For the composition of the asymptotic approximations of the solution of the perturbed vibration equation (3.24), which corresponds to the single frequency vibrations, it is indispensable that $\nu_1(t) \approx \omega_{sr}$, where ω_{sr} is circular frequency "unperturbed" vibration. Also, the following conditions should be fulfilled (Mitropolski and Mosenkov (1976), Hedrih (1978)):

a) The possible harmonic vibrations with proper circular frequency ω_{sr}, only depending on two arbitrary constants,

b) By an unique solution of the equation of "unperturbed vibrations" which to the balance of the plate, a trivial solution $w(x,y,t)=0$ appears,

c) In the perturbed system are absent the internal resonant states, that is, $\omega_{sr} \neq (p/q)\ \omega_{mn}$ where m,n=2,3, 4,... and p and q are reciprocally simple numbers.

With these assumptions, the asymptotic solution of the equation of perturbed vibrations is (Mitropolski (1964)):

$$\xi_1 = a_1 \cos(\theta_1 + \varphi_1) + \varepsilon u^{(1)} + \varepsilon^2 u_j^{(2)} + ... \tag{28}$$

where $\tau = \varepsilon t$ is "slowly-changed time" and $u_j^{(1)}(\tau, \theta_1, a_1, \varphi_1)$, $u_j^{(2)}(\tau, \theta_1, a_1, \varphi_1), ...$ are periodical functions whose arguments are: θ_1 and φ_1 with the period 2π; amplitude and phase of solution (Eq. (28)) which can be found from differential equations

$$\begin{aligned} \frac{\mathrm{d}a_1}{\mathrm{d}t} &= \varepsilon A_1 + \varepsilon^2 A_2 + ... \\ \frac{\mathrm{d}\varphi_1}{\mathrm{d}t} &= \omega_1 - \nu_1 + \varepsilon B_1 + \varepsilon^2 B_2 + ... \end{aligned} \tag{29}$$

where $A_1(\tau, a_1, \varphi_1)$, $A_2(\tau, a_1, \varphi_1)$, $B_1(\tau, a_1, \varphi_1)$, $B_2(\tau, a_1, \varphi_1)$, ... are unknown functions in "slowly-changed time" and amplitude and phase. These functions can be determined of supposed solution (Eq. (28)) in the equation (24) and equalizing of coefficient along by the same harmonics. Staying on the first approximation, the solution of equation (24) will have the form

$$\xi_1 = a_1 \cos(\nu_1 t + \varphi_1), \tag{30}$$

where differential equations in the first approximation will be

$$\begin{aligned} \frac{da_1}{dt} &= -\beta a_1 - \frac{P_1}{\omega_1 + \nu_1} \cos\varphi_1 , \\ \frac{d\varphi_1}{dt} &= \omega_1 - \nu_1 - \frac{3}{8}\frac{\alpha_1}{\omega_1} a_1^2 + \frac{P_1}{a_1(\omega_1 + \nu_1)} \sin\varphi_1 . \end{aligned} \tag{31}$$

4. NUMERICAL ANALYSIS OF COMPULSIVE VIBRATION OF LAMINATED PLATE IN STATIONARY CONDITIONS

If expressions (31), which the first approximation differential equations of solution (30), are equal to zero, i.e.

$$\begin{aligned} -\beta a_1 - \frac{P_1}{\omega_1 + \nu_1} \cos\varphi_1 &= 0, \\ \omega_1 - \nu_1 - \frac{3}{8}\frac{\alpha_1}{\omega_1} a_1^2 + \frac{P_1}{a_1(\omega_1 + \nu_1)} \sin\varphi_1 &= 0, \end{aligned}$$

the equations, which define relationship of amplitude and phase of asymptotic solution (30), will be obtained.

Solving these equations in amplitude $a_1 = f_1(\nu)$ and phase $\varphi_1 = f_2(\nu)$ we obtain amplitude-frequency and phase-frequency characteristics of laminated plates vibration for stationary conditions. For all numerical examples the following characteristics of laminated plate are taken: dimension of plate a=2 m, b=1 m; thickness of plate h=1 cm; specific density of plate material ρ=2600 kg/m^3; damping coefficient β=6 s^{-1} and amplitude of disturbance P_1^*=1300 N/m^2. Changes of amplitude-frequency and phase-frequency characteristics due to changes of some laminates characteristics are given in the next examples.

Amplitude-frequency characteristics of four-layered antisymmetric laminate with lamina orientation 0^0/ 90^0/ 0^0/ 90^0 and thickness 0.2h / 0.3h / 0.3h / 0.2h while changing the relation of longitudinal and transverse modulus of elasticity (E_1/E_2=5;10;40) are shown in Fig.1a. With the increasing of the relation E_1/E_2 amplitude-frequency curves are getting displaced to higher amplitudes and lower frequencies. Frequency region in which for some frequencies of external force there is possibility of three stationary states (two are stable, one is unstable), that is frequency region between resonant jumps, is getting displaced to lower frequencies. Fig.1b shows phase frequency characteristics whose examination gives the same conclusions.

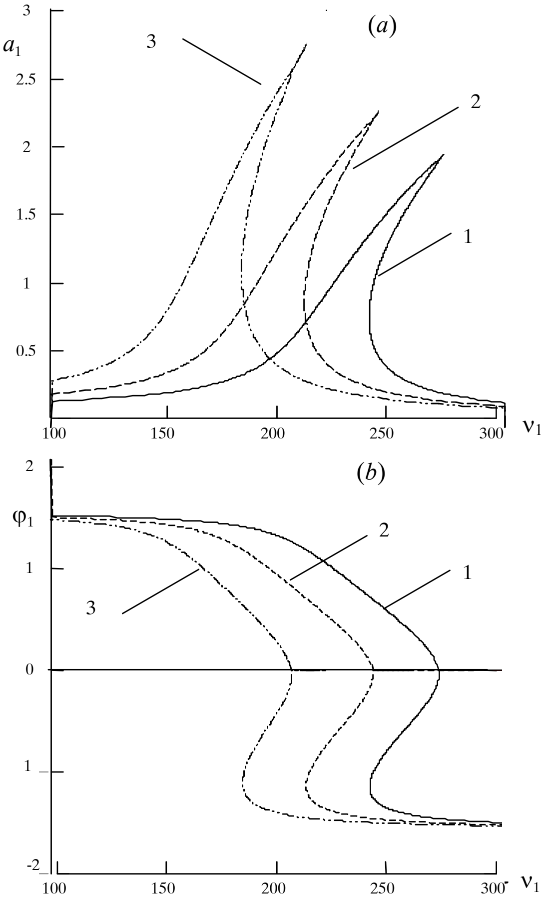

Figure 1. Amplitude-frequency (a) and phase-frequency (b) characteristic for different ratio E_1/E_2 (1-E_1/E_2=5, 2 - E_1/E_2=10, 3 - E_1/E_2=40)

In the next example the laminate analysis for the ratio E_1/E_2=10, and for the same orientation of individual lamina $0^0/90^0/0^0/90^0$ are performed. While analyzing the thickness of lamina is changed. Amplitude-frequency and phase-frequency characteristics for different thickness are shown in Fig.2. It is obvious that with the decreasing of external lamina thickness and with the increasing of internal lamina thickness, amplitudes have lower values on the higher frequencies of external disturbing force.

Amplitude - frequency and phase - frequency characteristics for different number of lamina are shown in Fig. 3. At the ratio E_1/E_2=10 it is taken that total thickness of longitudinal lamina is the same as the thickness of transversal laminae (by h/2), i.e. for two layered laminate (30^0/-30^0, 0.5h/0.5h), for four-layered laminate (30^0/ -30^0/ 30^0/ -30^0, 0.25h/0.25h/0.25h/0.25h) and for six layered laminate (30^0/-30^0/30^0/-30^0/30^0/-30^0,

$0.1h/0.15h/0.25h/0.25h/0.15h/0.1h$). With the increasing of number of lamina amplitude maximums are decreasing, and they are occurring on the higher frequencies. Amplitude-frequency and phase-frequency characteristics for different angle of lamina are shown in Fig. 4. With the increasing of angle of lamina amplitude maximums are decreasing, and they are occurring on the higher frequencies.

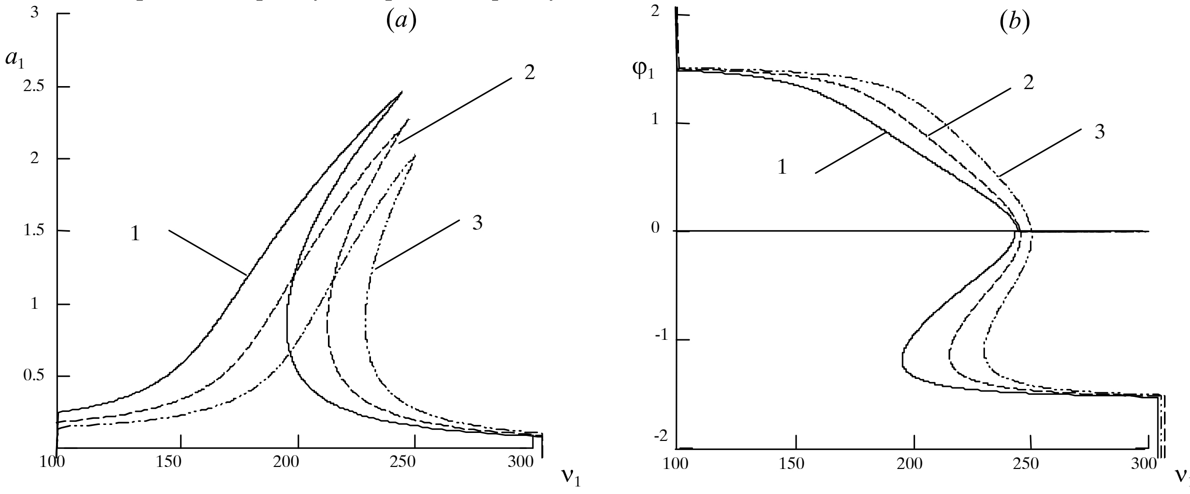

Figure 2. Amplitude-frequency (a) and phase-frequency (b) characteristics for different thickness lamina (1-0.4h/ 0.1h/ 0.1h/0.4h, 2 - 0.3h/0.2h/0.2h/0.3h, 3 - 0.2h/0.3h/0.3h/0.2h)

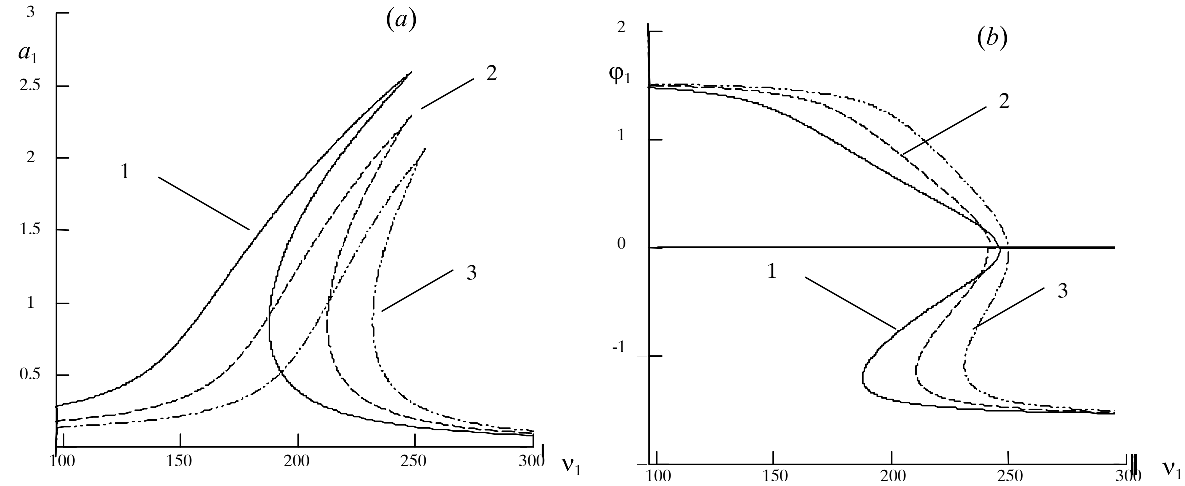

Figure 3. Amplitude-frequency (a) and phase-frequency (b) characteristics for different number of laminae (1-II laminae +φ / -φ/, 2-IV laminae +φ/-φ/+φ/-φ/, 3-VI laminae +φ/-φ/+φ/-φ/+φ/-φ)

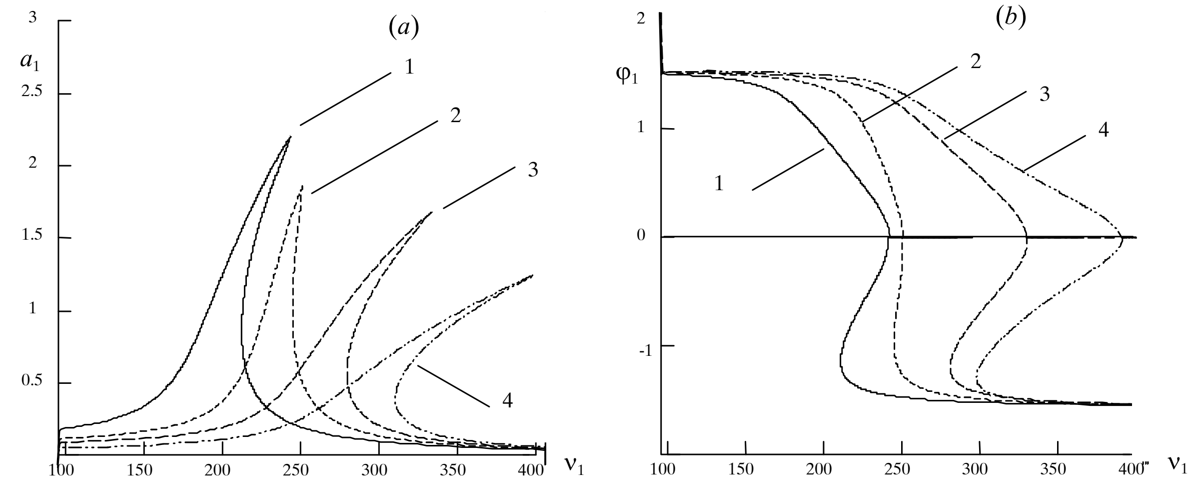

Figure 4. Amplitude-frequency (a) and phase-frequency (b) characteristics for different angle of lamina (1-$\varphi=30^0$, 2-$\varphi=45^0$, 3-$\varphi=60^0$, 4-$\varphi=75^0$)

5. NUMERICAL ANALYSIS OF COMPULSIVE VIBRATION OF LAMINATED PLATES IN INSTATIONARY CONDITIONS

The equations (31) are the first approximation differential equations of asymptotical solution of differential equation (24). Numerical solving of these equations by means of Runge-Kutta method (the fourth order), gives amplitude frequency characteristics of single frequency regime of laminated plate vibration in instationary conditions. The dependence of these curves on changing of same laminate characteristics is given in the next examples.

Amplitude-frequency characteristics of four-layered laminate (0^0/ 90^0/ 0^0/ 90^0, 0.2h/ 0.3h/ 0.3h/ 0.2h) for different ratios of longitudinal and transverse modulus of elasticity are shown in Fig. 5. Amplitude-frequency characteristics at linear increasing of external force frequency are shown in Fig. 5*a*. Passing by resonant state is realized by decreasing of external force frequency as in Fig. 5*b*. It can be concluded, on the basis of both diagrams, that due to increasing of ratio E_1/E_2, maximums of amplitude are increasing too, and are displaced toward lower frequencies.

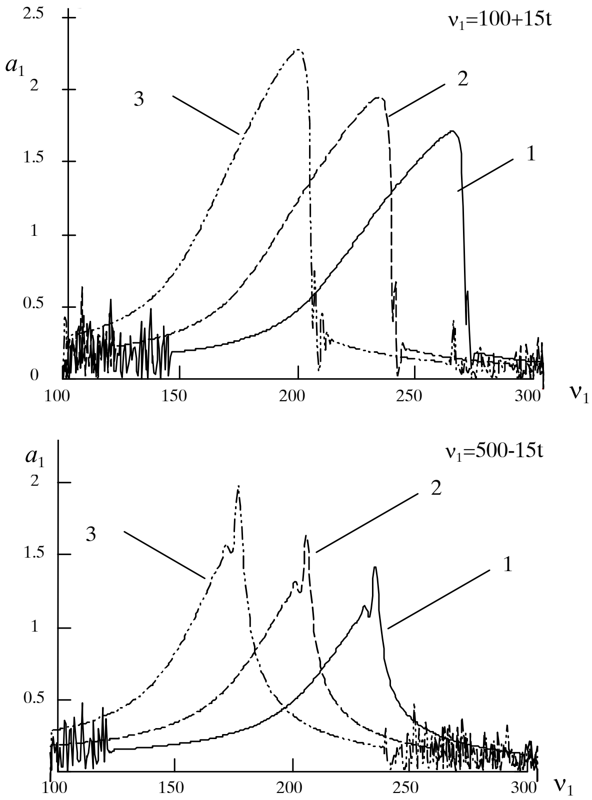

Figure 5. Amplitude-frequency characteristics for different ratio E_1/E_2 (1 - E_1/E_2=5, 2 - E_1/E_2=10, 3 - E_1/E_2=40)

Amplitude-frequency characteristics for different thickness of longitudinal and transverse lamina's orientation while E_1/E_2=10 are shown in Fig. 6. It is obvious that with increasing of internal lamina thickness, maximums of amplitude are decreasing.

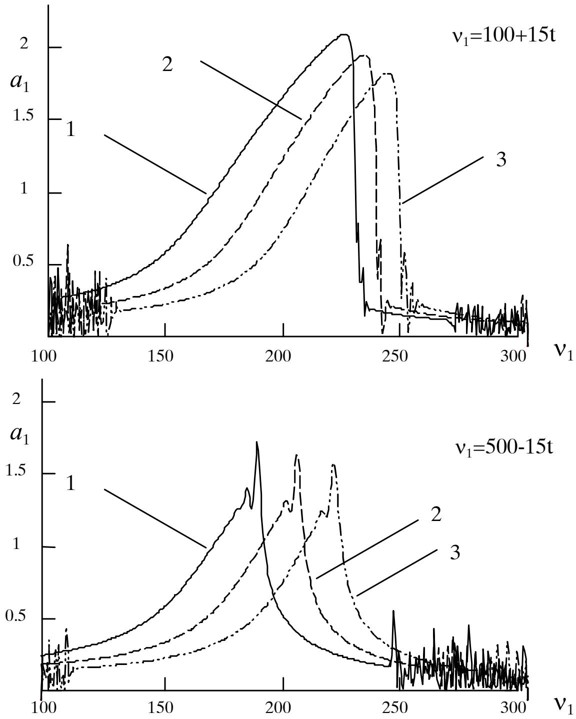

Figure 6. Amplitude-frequency characteristics for different thickness of laminae (1-0.4h/0.1h/0.1h/0.4h, 2-0.3h/ 0.2h/ 0.2h/ 0.3h, 3-0.2h/0.3h/0.3h/0.2h)

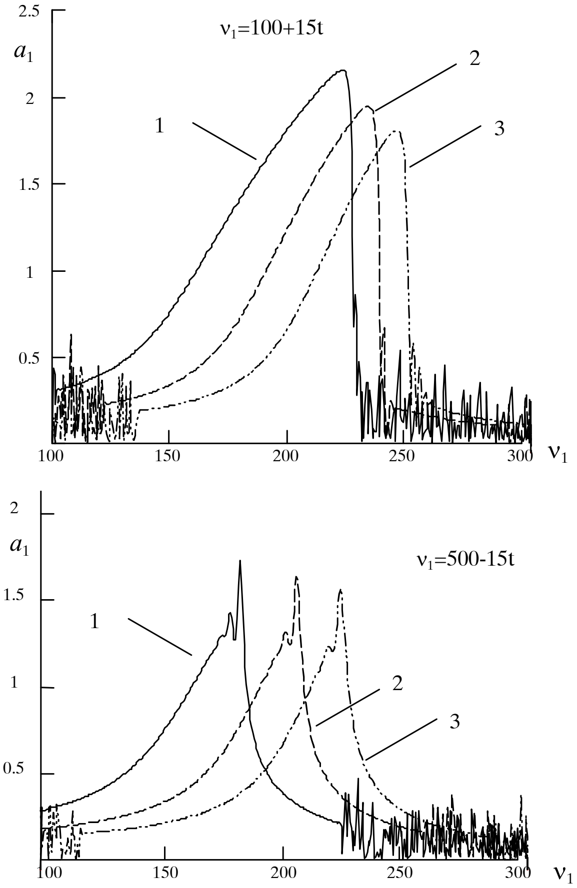

Figure 7. Amplitude-frequency characteristics for different number of laminae (1-II laminae +φ/-φ/, 2-IV laminae +φ/-φ/+φ/-φ/, 3-VI laminae +φ/-φ/+φ/-φ/+φ/-φ)

The influence of number of laminae on the amplitude-frequency characteristics in instationary state is shown in Fig. 7. At the ratio E_1/E_2=10 it is taken that total thickness of longitudinal lamina is the same as thickness of transverse laminae (by h/2), that is for two layered laminate ($0^0/90^0$, 0.5h/0.5h), for four-layered laminate ($0^0/ 90^0/ 0^0/ 90^0$, $0.25h/0.25h/0.25h/0.25h$) and for six layered laminate ($0^0/90^0/0^0/90^0/0^0/90^0$, $0.1h/0.15h/0.25h/0.25h/0.15h/0.1h$). It is evident that with increasing of lamina number, maximums of amplitude are decreasing.

The influence of angle of laminae on the amplitude-frequency characteristics in instationary state is shown in Fig. 8. It is evident that with increasing of lamina angle, maximums of amplitude are decreasing.

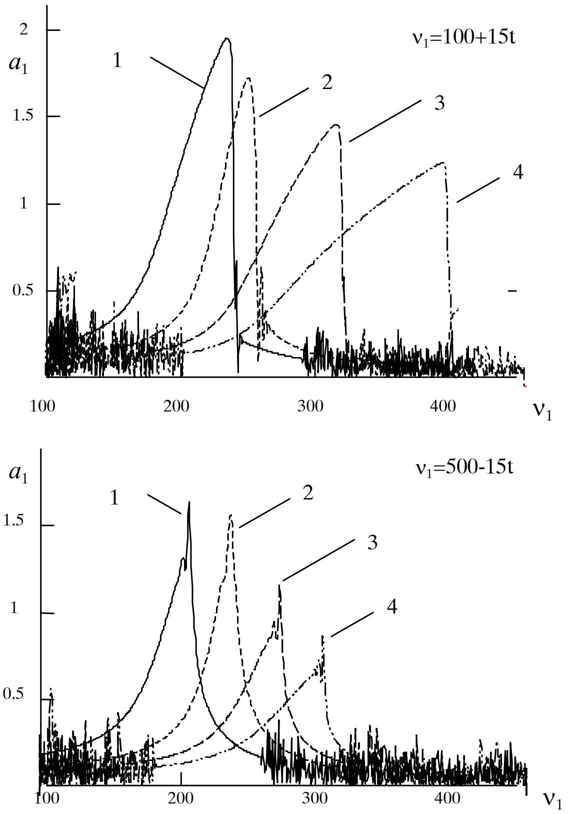

Figure 8. Amplitude-frequency characteristics for different angle of lamina (1-φ=30⁰, 2-φ=45⁰, 3-φ=60⁰, 4-φ=75⁰)

6. CONCLUSIONS

On the basis of analysis of amplitude-frequency characteristics for single frequency regime of laminated plate vibrations in stationary and instationary we can conclude:

- while increasing of the ratio of longitudinal and transverse modulus of elasticity E_1/E_2, absolute values of vibration amplitudes increase,

- in the four-layered laminates where laminae orientation is $0^0/90^0/0^0/90^0$, amplitudes of vibration are larger on the lower frequencies with increasing of thickness internal laminae, and

- while increasing of lamina number, by the same thickness of longitudinal and transverse lamina, amplitudes are decreasing,

- while increasing angle of lamina, amplitudes are decreasing.

ACKNOWLEDGEMENT

The research was supported by Ministry of Science, Technology and Development of Republic Serbia, through the project N^0 1409.

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Асимптотско решење нелинеарних осцилација антисиметричних угаоних ламеластих плоча

Горан Јаневски

У раду су анализиране једнофреквентне осцилације антисиметричне угаоне ламеласте плоче, правоугаоног облика, слободно ослоњене на својим крајевима. Коришћена је класична теорија Кирхоф-а, увођењем Аириј-еве напонске функције анализирана је једначина осциловања ламеласте плоче Карман-овог типа. Дата су асимптотска решења у првој апроксимацији. Нумерички пример обухвата анализу осциловања плоче у стационарним и нестационарним условима под дејством временски зависне спољашње побуде. Графички су приказане амплитудно-фреквентне и фазно-фреквентне карактеристике плоче у стационарним и нестационарним условима за различите карактеристике ламелата.