Improved Numerical Calculation of the Airfoil Transonic Drag Applied Within a Zonal Flowfield Modeling Concept

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Very high cost efficiency of the flight is a crucial requirement specially in the contemporary commercial airplane design. Beside the low engine fuel consumption, advanced aerodynamics is another dominant factor which must be satisfied to fulfill this request. Many of these aircraft cruise at speeds slightly lower than the speed of sound, so their lifting surfaces and corresponding airfoils must be optimized primarily for this domain. One of the first steps in that process is selection or even design of the customized airfoils for the particular wing and other lifting surfaces that will produce the least possible shock wave drag in cruising flight. Nowadays the numerical airfoil optimization is very important part in that process. Algorithm presented in this paper enables the numerical calculation of wave drag both for the existing and the airfoils designed specially for a certain aircraft, and it is primarily aimed for use in the operational aircraft design. This algorithm is fairly simple and very reliable, which has been proven by comparing it's results, obtained through the computer program Tranpro, with the experimental results for airfoils tested at several most competent aeronautical institutions throughout the world.

Keywords: *wave drag, aviation airfoils, turbulent boundary layer, numerical modeling, zonal flowfield analysis.*

1. INTRODUCTION

Calculations presented in this paper are the results of the computer program named Tranpro [4], an upgraded version of the Trandes [3] computer program. The Trandes was developed by Prof. L.A.Carlson of the Texas A&M Univ. USA, under the contract for the NASA agency, in the late seventies. Very soon the Trandes has also been accepted by many leading aircraft corporations (such as Boeing Aircraft Corp. USA, for example) and the universities throughout the world. This program is used for the aviation airfoil analysis and design, primarily for the transonic speed domain. Although the state of the art software of the time due to many qualities it possesses, it has also been the subject to some critics in scientific papers ever since the time of it's issue, and many of it's users have developed their own upgraded versions of this program. Nowadays, the Trandes has practically become a public domain software, while the upgrades, still well in operational use, are more or less classified. The author of this paper has had a chance to use Trandes for many years, and from these experiences, both good and bad, the Tranpro computer program has been developed. One of the most important problems of Trandes, the wave drag calculation, mostly based on rather flexible algorithm and "the user's experience" (many of the solutions can fail for that reason), has been solved and unified by an original approach in Tranpro, and every calculation has a stable and unique solution, regardless of how experienced the user is. This paper presents the achievements in wave drag calculation based on this calculation model, which is still more than useful and successful, due to the accuracy of the results for practical aircraft design applications, and the moderate time and computer resources that it requires.

Globally speaking, in this calculation the zonal approach is applied. The airflow outside the turbulent boundary layer is treated as potential, and solved by the method of finite differences (in aircraft operational use this type of boundary layer is totally dominant at higher subsonic Mach numbers and the corresponding relatively large Reynolds numbers). In the local supersonic domains on the airfoils, if and when they exist, the Jameson's [3] "rotated differences" model is used. Flow calculation inside the boundary layer is done by the application of an improved [5, 6] algorithm, derived by modifying and spreading the Nash-Macdonald model used originally in Trandes and by adding some completely new features for calculations at higher angles of attack. Wave drag coefficient evaluated by calculation of the differences between the corresponding pressure drag coefficients from supercritical and subcritical pressure coefficient distributions for a same lift coefficient. This particular approach was originally used

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in Trandes as well, but in a way that largely depended on the user's experience. On the other hand, here presented method is applied in such a way that it's solutions do not depend on this influence factor, and thus it is much more reliable and enables very wide scope of applications.

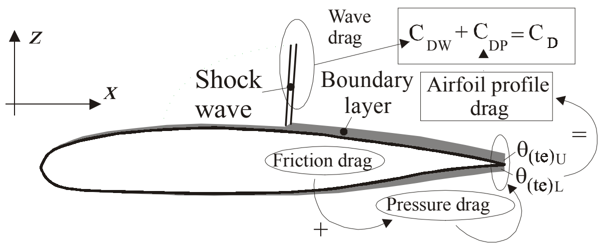

Figure 1. Total airfoil drag coefficient C_D at transonic speeds.

Numerical calculation of the wave drag coefficient can not be isolated and verified separately. It can be checked through the verification of the total airfoil drag coefficient at transonic speeds (Fig. 1), which consists of the profile drag coefficient C_{DP} (including pressure and friction drag coefficients) and the wave drag coefficient C_{DW}. In transonic domain only total C_D can be obtained experimentally, so numerical value of the C_{DW} can be checked only if the numerical calculation of C_{DP} is sufficiently accurate. For here presented algorithm, average error in the improved C_{DP} calculation for lower subsonic speeds is only about 1,5% [1, 2, 5] compared with the relevant experimental data (mostly NACA & NASA sources). Applied with proper compressibility corrections, this level of accuracy is fully retained in lower transonic domain as well.

2. AIRFLOW CALCULATION

Both in the Trandes and the Tranpro computer programs, the zonal approach in airflow calculation is applied. This approach is still very successful in many operational engineering applications, especially for strictly defined narrow domains of application such as aviation airfoil calculations are. The advantage of this approach over more complex methods lies mostly in extremely high time and computer resource efficiency, while at the same time the accuracy of the results can be brought to more than satisfactory level.

2.1. Calculation of the inviscid part of the flow

The inviscid part of the flow is calculated over the displacement surface of the airfoil, i.e. the airfoil contour increased by the numerically smoothed local distribution of the δ^*. In the Tranpro, this calculation is done by the same general algorithm as the one applied in Trandes. It is based on the solution of the full nondimensional perturbation potential $\bar{\phi}$ nonlinear partial differential equation, which in the physical x - z space for the unit airfoil chord length takes the form:

$$(a^2-u^2)\bar{\phi}_{xx}+(a^2-w^2)\bar{\phi}_{zz}-2uw\bar{\phi}_{xz}=0 \qquad (1)$$

while, applied in the calculation space ξ - η , changes to:

$$(a^2-u^2)f(\bar{\phi}_\xi)_\xi+(a^2-w^2)g(\bar{\phi}_\eta)_\eta-2uwfg\bar{\phi}_{\xi\eta}=0 \qquad (2)$$

where f=dξ/dx and g=dη/dz. Specially, in the local supersonic domain, where Jameson's rotated finite difference s - n scheme is used, the governing equation takes the form:

$$(1-\mathrm{M}^2)\bar{\phi}_{ss}+\bar{\phi}_{nn}=0 \qquad (3)$$

in which:

$$\bar{\phi}_{ss}=\frac{1}{V^2}\left[u^2f(f\bar{\phi}_\xi)_\xi+2uwfg\bar{\phi}_{\xi\eta}+w^2g(g\bar{\phi}_\eta)_\eta\right] \qquad (4)$$

$$\bar{\phi}_{nn}=\frac{1}{V^2}\left[w^2f(f\bar{\phi}_\xi)_\xi-2uwfg\bar{\phi}_{\xi\eta}+u^2g(g\bar{\phi}_\eta)_\eta\right] \qquad (5)$$

Very quick convergence of the solution is obtained by calculating the flow on the series of rectangular grids, starting with 13×7, then 25×13, 49×25 and 97×49. Very often the final solution is obtained on the 49×25 grid, so the finest grid need not be applied, which reduces the computation time.

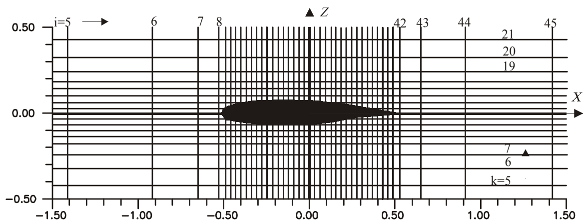

Figure 2. Grid 49×25 in the vicinity of the airfoil

2.2. Boundary layer and profile drag calculations

In this paper only turbulent boundary layer case with transition point fixed close to the leading edge will be discussed. In Trandes, the Nash-Macdonald integral turbulent boundary layer calculation is used, while in Tranpro, the modified [1, 2, 5] version of this model is applied. The momentum integral equation [7, 8]:

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^{[n]}=-\frac{(\theta^+)^{[n-1]}}{u_e^+}\frac{\mathrm{d}u_e}{\mathrm{d}x}\left(\mathrm{H}+2-\mathrm{M}_\mathrm{e}^2\right)+\frac{1}{\left(\zeta^{[n]}\right)^2} \qquad (6)$$

is solved for the momentum thickness θ. In (6), "e" denotes the values on the outer edge of the turbulent boundary layer, while $[n]$ denotes a certain iteration cycle value. Parameter ζ is defined by:

$$\zeta^{[n]}=\mathrm{F_C}\left[2.4711\cdot\ln\left(\mathrm{F_R}\,\mathrm{R}_\theta^{[n-1]}\right)+4.75\right]+$$
$$+1.5\,\mathrm{G}^{[n-1]}+\frac{1724}{\left(\mathrm{G}^{[n-1]}\right)^2+200}-16.87 \qquad (7)$$

in which:

$$\mathrm{F_C}=1+0.066\left(\mathrm{M_e}\right)^2-0.008\left(\mathrm{M_e}\right)^3 \qquad (8)$$

$$\mathrm{F_R}=1-0.134\left(\mathrm{M_e}\right)^2+0.027\left(\mathrm{M_e}\right)^3 \qquad (9)$$

where G is the Clauser parameter. Shape factor $\mathrm{H}=\delta^*/\theta$ is calculated by:

$$\bar{H}^{[n]} = \frac{1}{1 - G^{[n-1]}(1/\zeta)} \tag{10}$$

and

$$H^{[n]} = \left(\bar{H}^{[n]} + 1\right)\left[1 + 0.178\left(M_e^{+}\right)^2\right] - 1 \ . \tag{11}$$

In the modified Tranpro's model [1,2,5], the Clauser parameters G and β_p are related by :

$$G^{[n]} = 6.1\sqrt{\beta_p^{[n]} + 1.81} - 4.1 \ , \tag{12}$$

while in the original Carlson/Nash-Macdonald model used in Trandes, the usual equation of Nash [1, 10] is applied, giving quite inaccurate results. Once θ is determined, turbulent boundary layer displacement thickness is calculated by $\delta^* = H \cdot \theta$. Finally, the distribution of δ* is smoothed [3, 4] over the airfoil, and so the airfoil displacement surface is obtained.

For the complete profile (joined pressure and friction) drag coefficient calculations, the spreadded Squire-Young formula, with the separate trailing edge values for upper and lower surface, is used:

$$C_{DP} = 2 \cdot \left[\theta_{(te)_U} \left(\frac{u_{e(te)}}{u_\infty} \right)_U^{\frac{\bar{H}_{(te)U}+5}{2}} + \theta_{(te)_L} \left(\frac{u_{e(te)}}{u_\infty} \right)_L^{\frac{\bar{H}_{(te)L}+5}{2}} \right] . \tag{13}$$

In case of the transonic (supercritical) flow, it is necessary to calculate and add the wave drag coefficient to this value.

2.3. Wave drag coefficient calculation

As already mentioned, in the case of supercritical free-stream (M_∞) Mach numbers, total drag coefficient is obtained as:

$$C_D = C_{DP} + C_{DW} \tag{14}$$

where for C_{DP} calculations equation (13) in combination with modified, i.e. Tranpro's turbulent boundary layer model must be used.

Results presented in this paper will be confined to smaller transonic values for two reasons. As first, at these Mach numbers shock wave that appears on an aviation airfoil is weak and it does not induce massive boundary layer separation at here considered angles of attack that are close or equal to $\alpha = 0^o$, which is usual α domain for this kind of numerical calculations. Terms "weak, moderate and massive separation" [3, 4] are formally used and come out from the specific approaches in the numerical treatment of this phenomenon. So far, the Tranpro deals very well with weak and moderate turbulent boundary layer separation, while Trandes had some inherent errors problems even in case of very weak separation. Secondly, generally speaking the transonic flows are not potential, but in the lower transonic domain which is characterized by weak shock waves, the potential approximation of the flow outside the boundary layer can be readily accepted without affecting the relevancy of the results.

The C_{DW} is obtained as a difference of form (pressure) drag coefficients - C_{Df} at supercritical and subcritical Mach numbers for the same C_L, where these coefficients are obtained by integrating the Cp distributions along the direction perpendicular to the airflow. The problem of C_{Df} calculations on locally uniformly spaced rectangular grids (such as the grid in Fig. 2) exists especially in the leading and sometimes in trailing edge domains (Fig. 3), which may lead to incorrect C_{DW} results if not treated properly (also one of the remarkable problems in the Trandes). In the Tranpro, the following approach in C_{DW} calculation is applied:

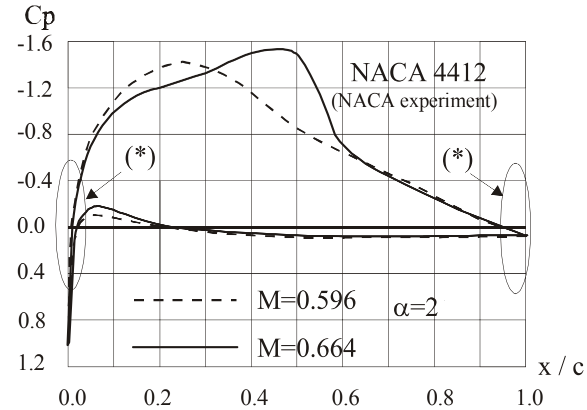

Figure 3. (*) - Critical domains of the numerical calculations of pressure drag coefficient; example for constant angle of attack

(A) For several *subcritical* M_∞ values and preselected constant α, by integrating the calculated Cp distributions with respect to axes normal and tangential to the airfoil chord, normal and tangential aerodynamic force coefficients $C_{N(-)}$ and $C_{T(-)}$ should be obtained (here "(-)" and "(+)" subscripts will denote subcritical and supercritical conditions respectively). Then for each of those Mach numbers coefficient C_A is calculated as:

$$C_{A(-)} = C_{T(-)} + C_{N(-)} \tan\alpha \tag{15}$$

(B) For a desired *supercritical* M_∞ and keeping the same α as in (A), values $C_{N(+)}$, $C_{T(+)}$ and $C_{A(+)}$ are calculated in the same manner as in subcritical cases.

(C) Lift coefficient C_L initially increases as transonic domain is encountered and entered while keeping the same α. We can assume that coefficient $C_{A(-)}$ would keep on behaving in the same manner with respect to the C_L at M_∞ from (B) if we suppose that no shock wave exists (purely fictive case) so the character of their mutual dependence established in subcritical domain could also be mapped here. For the higher C_L obtained in (B) at supercritical M_∞, and from the known previously derived function C_L- $C_{A(-)}$ obtained in (A), by extrapolation a fictive "subcritical" value of $C_{A(-)fic}$ is calculated for that new higher supercritical C_L.

(D) The difference, denoted as C_{A,C_L} for that C_L is then calculated as:

$$C_{A,C_L} = C_{A(+)} - C_{A(-)fic} \tag{16}$$

(E) The wave drag coefficient is then:

$$C_{DW} = C_{N(+)} \sin\alpha + C_{A,C_L} \cos\alpha \tag{17}$$

This new algorithm is completely free from any problems associated with C_{Df} calculations on rectangular grids that exist in original Trandes algorithm [3]. Four sample cases of total drag C_D calculations by new method for airfoils NACA 2312, NACA 2315, NACA GA(W)-2 and NACA 0012-34 at nominal angle of attack of $\alpha = 0^o$ are given in Tables 1 ÷ 4 (only cases using improved turbulent boundary layer calculation are presented; coordinates for the three classical airfoils derived according to [9], GA(W)-2 derived by thickness scaling from [11]).

Table 1. Example of the calculation of the wave drag coefficient for NACA2312 airfoil

NACA 2312 ; calculation for nominal $\alpha = 0^o$; improved TBL model used in *Tranpro*; α correction applied [3]							
M	C_L	C_{DP}	$C_{A(-)}$	$C_{A(+)}$	$C_{A(-)fic}$	C_{DW}	C_D
0.40	0.2260	0.0148	0.00129	-	-	-	0.0148
0.45	0.2348	0.0143	0.00140	-	-	-	0.0143
0.50	0.2428	0.0139	0.00167	-	-	-	0.0139
0.55	0.2529	0.0136	0.00200	-	-	-	0.0136
0.60	0.2664	0.0134	0.00244	-	-	-	0.0134
0.65	0.2853	0.0132	0.00295	-	-	-	0.0132
0.70	0.3037	0.0133	-	0.00436	0.00353	0.0008	0.0141
0.75	0.3250	0.0158	-	0.01253	0.00415	0.0084	0.0242
0.78	0.3494	0.0162	-	0.02298	0.00487	0.0181	0.0343
Extrapolation function: $C_{A(-)} = 0.0294\ C_L - 0.0054$; Re = 750.000 ÷ 1.400.000; Transition fixed at 6% chord (simulated standard roughness)							

Table 2. Example of the calculation of the wave drag coefficient for NACA2315 airfoil

NACA 2315 ; calculation for nominal $\alpha = 0^o$; improved TBL model used in *Tranpro*; α correction applied [3]							
M	C_L	C_{DP}	$C_{A(-)}$	$C_{A(+)}$	$C_{A(-)fic}$	C_{DW}	C_D
0.40	0.2346	0.0157	-0.00504	-	-	-	0.0157
0.45	0.2415	0.0153	-0.00472	-	-	-	0.0153
0.50	0.2503	0.0149	-0.00430	-	-	-	0.0149
0.55	0.2616	0.0146	-0.00379	-	-	-	0.0146
0.60	0.2703	0.0144	-0.00277	-	-	-	0.0144
0.65	0.2948	0.0148	-0.00269	-	-	-	0.0148
0.70	0.3109	0.0155	-	0.00259	-0.00058	0.0032	0.0187
0.72	0.3242	0.0159	-	0.00826	0.00022	0.0080	0.0239
0.75	0.3463	0.0163	-	0.01965	0.00154	0.0181	0.0344
Extrapolation function: $C_{A(-)} = 0.0599\ C_L - 0.0192$; Re = 750.000 ÷ 1.400.000; Transition fixed at 6% chord (simulated standard roughness)							

Table 3. Example of the calculation of the wave drag coefficient for GA(W)-2 airfoil

NACA GA(W)-2 ; calculation for nominal $\alpha = 0^o$; improved TBL model used in *Tranpro*							
M	C_L	C_{DP}	$C_{A(-)}$	$C_{A(+)}$	$C_{A(-)fic}$	C_{DW}	C_D
0.45	0.5334	0.0106	0.00235	-	-	-	0.0106
0.50	0.5499	0.0104	0.00283	-	-	-	0.0104
0.55	0.5705	0.0103	0.00343	-	-	-	0.0103
0.60	0.5972	0.0103	0.00423	-	-	-	0.0103
0.65	0.6341	0.0103	0.00541	-	-	-	0.0103
0.70	0.6916	0.0106	-	0.00782	0.00715	0.0007	0.0113
0.72	0.7131	0.0112	-	0.01231	0.00781	0.0045	0.0157
0.74	0.7432	0.0124	-	0.02280	0.00872	0.0141	0.0265
0.75	0.7350	0.0158	-	0.02734	0.00847	0.0189	0.0347
Extrapolation function: $C_{A(-)} = 0.0303\ C_L - 0.0138$; Re = 4.000.000 ÷ 6.670.000; Transition fixed at 6% chord (simulated standard roughness)							

Table 4. Example of the calculation of the wave drag coefficient for NACA2315 airfoil

NACA 0012-34 ; calculation for nominal* $\alpha = 0^o$; improved TBL model used in *Tranpro*							
M	C_L *	C_{DP}	$C_{A(-)}$	$C_{A(+)}$	$C_{A(-)fic}$	C_{DW}	C_D
0.40	-0.0211	0.0121	0.00488	-	-	-	0.0121
0.45	-0.0214	0.0118	0.00499	-	-	-	0.0118
0.50	-0.0219	0.0115	0.00512	-	-	-	0.0115
0.55	-0.0225	0.0113	0.00527	-	-	-	0.0113
0.60	-0.0233	0.0111	0.00545	-	-	-	0.0111
0.65	-0.0256	0.0110	0.00553	-	-	-	0.0110
0.70	-0.0348	0.0110	0.00647	-	-	-	0.0110
0.75	-0.0336	0.0110	0.00653	-	-	-	0.0110
0.80	-0.0347	0.0113	-	0.00979	0.00653	0.0033	0.0146
0.85	-0.0468	0.0129	-	0.03500	0.00790	0.0271	0.0400
Extrapolation function: $C_{A(-)} = -0.1132\ C_L + 0.0026$; Re = 2.000.000 ÷ 4.000.000; Transition fixed at 6% chord (simulated standard roughness)							

* - rectangular grids generally have small inherent error in α settings; in this example actual α is just slightly different from 0^o and so $C_L \neq 0$ for the given symmetrical airfoil. Otherwise, the angle of attack for such airfoils should be set a little bit above or below zero to enable calculations using this particular model, as the first member in extrapolation function for $C_{A(-)}$ should not be constant and equal to 0.

3. RESULTS AND COMMENTS

Results presented in Fig's 4 ÷ 7 show that modified turbulent boundary model combined with here presented new algorithm for transonic drag calculations generally gives much better agreements with experimental results than the original Nash-Macdonald model combined with the same new transonic drag model. It should be emphasized that around and above M_{cr} sudden increase in drag coefficient in very narrow domain of Mach numbers may cause proportionally larger scattering of the measured experimental results than in subsonic domain, and the presentation of these results may also be affected by the way the test points are fitted afterwards (this could become very obvious when experimental results for the same airfoils obtained in different wind tunnels are compared). That can slightly affect the level of agreement of numerical results with experimental curves in this particular domain. The author's opinion is that results obtained by here presented calculation model even in that domain can be considered more than satisfactory for engineering purposes.

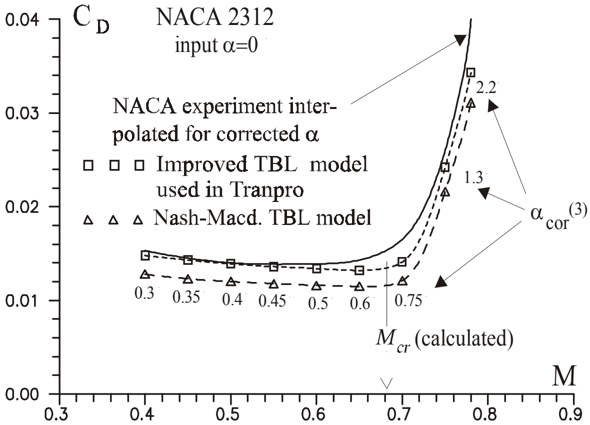

Figure 4. Re=750.000÷1.400.000; standard roughness. (α_{cor}[3] - in some cases angle of attack correction is necessary for calculations on rectangular grids).

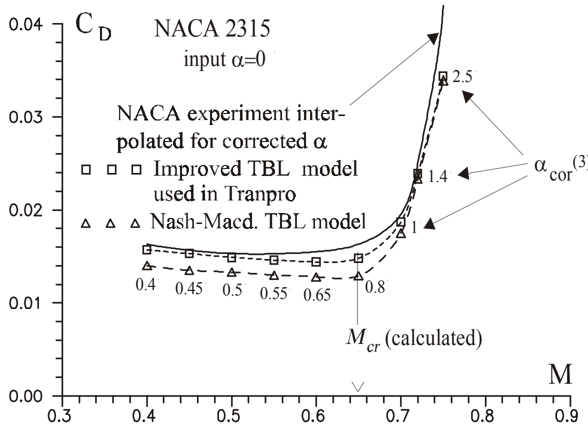

Figure 5. Re=750.000÷1.400.000; standard roughness. (α_{cor}[3] - in some cases angle of attack correction is necessary for calculations on rectangular grids)

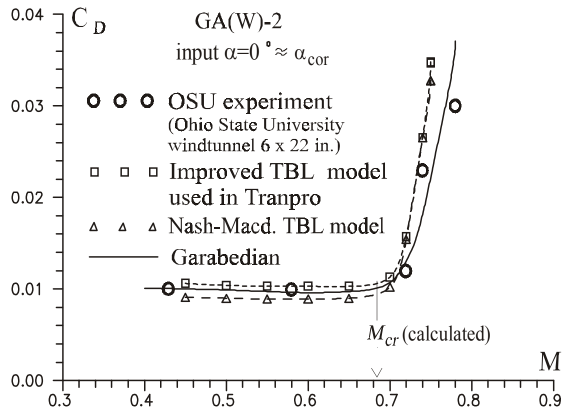

Figure 6. Re=4.000.000÷6.670.000; standard rough.

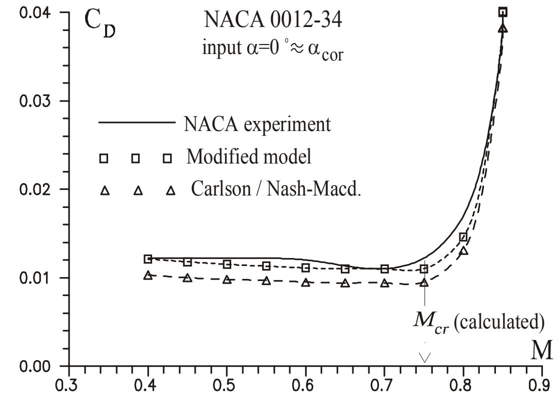

Figure 7. Re = 2.000.000 ÷ 4.000.000; standard rough.

Comparison of numerical calculations using new transonic drag model with the experiment in Figures 4 ÷ 7. Original transonic drag model could not give stable solutions and these results are not presented. Calculations were done using original Nash-Macdonald [3,10] and improved turbulent boundary layer (TBL) model used in Tranpro, also briefly described in this paper.

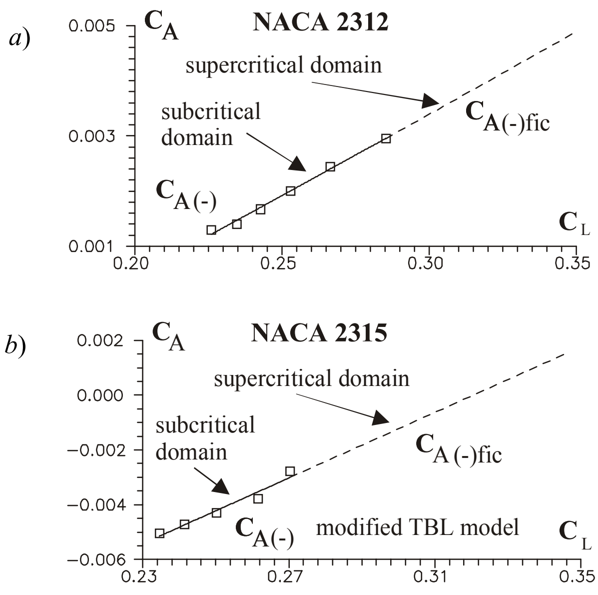

Figure 8.

Examples of the derivation of extrapolation functions for the estimation of $C_{A(-)fic}$ from subcritical values of $C_{A(-)}$; this is the most important new step in here presented algorithm for wave drag calculations, which leads to stable solutions.

Rather high values of profile drag (both experimental and numerical – Fig's 4÷7) come out from the fact that transition to turbulent boundary layer in all cases was forced very close to the leading edge. Also, in cases of airfoils NACA 2312 and 2315, very low test Re numbers contribute to additional increase of profile drag. On a vast number of test cases (detailed presentation is out of the scope of this paper), the author has shown that linear fittings for the extrapolation functions $C_{A(-)} – C_L$ give very good final results – Fig. 8.

The earlier works of this author contained more or less empirically based proofs of his approach to this particular problem. After gaining the additional experience by improving the quality level of problem treatment applied in successive Tranpro versions [4] and refining some steps of the algorithm, the author believes that at this moment he is able draw the conclusions which lead to full understanding of the problem background, giving clear explanations of all the advantages of here presented model.

The key advantage of here presented model for the calculation of wave drag, compared to the Carlson's original approach used in Trandes, is contained in step (C). The Carlson's idea was the following: by subtracting the values of supercritical and subcritical C_A for the same C_L, inevitable numerical errors in the domains of the leading and the trailing edge will cancel because Cp distribution in these problematic areas will not change relevantly at the same C_L, and only the "pure" difference which leads to the wave drag coefficient will remain. But, since the C_L for the constant α in this speed domain increases with Mach number, it is obvious that supercritical lift coefficient will be larger than the subcritical at given α. At that point, Carlson suggested that the user should select some subcritical M_∞ and simply slightly increase the angle of attack until the same C_L as in supercritical case is reached, and then the subcritical C_A should be determined for the rest of the calculation. Unfortunately, this step practically canceled his initially very good approach (and he was aware of that himself), because with altering α the character of Cp at the leading and the trailing edges will change and in most of the cases new type of numerical error in subcritical case will appear compared to that in supercritical calculation. These errors will not cancel; they would be superimposed, and the final result will be incorrect. To overcome that, Carlson suggested that the user should apply his own experience and select such subcritical M_∞ at which, for the increased α, the numerical error will be as close as possible to the one in the supercritical case. As a consequence, in case of a not too experienced user, the C_D curve in supercritical domain can become widely scattered, as some of the values of C_{DW} due to the numerical error can even be obtained as negative (shock wave acting as a thrust device, completely opposite from the real life).

On the other hand, in here presented algorithm (A) – (E), the subcritical C_A is first determined for the real subcritical M_∞ -s and the selected α as a function of the progressively increasing C_L, until the critical Mach number is reached. For some supercritical M_∞ and the corresponding consequently higher lift coefficient C_L, the $C_{A(-)} – C_L$ function is extrapolated and for that supercritical value of C_L the C_A is determined. By that approach, even if due to the mentioned problems of numerical integration $C_{A(-)}$ appears as negative (see example in table 2), the appropriate difference which should finally give the wave drag is always positive and gives numerical results that match the experiment very well.

It should also be mentioned that turbulent boundary layer separation due to the shock wave (Fig's 9 & 10) was modeled in all shown examples as the weak separation, which is quite acceptable if angles of attack are kept close to zero.

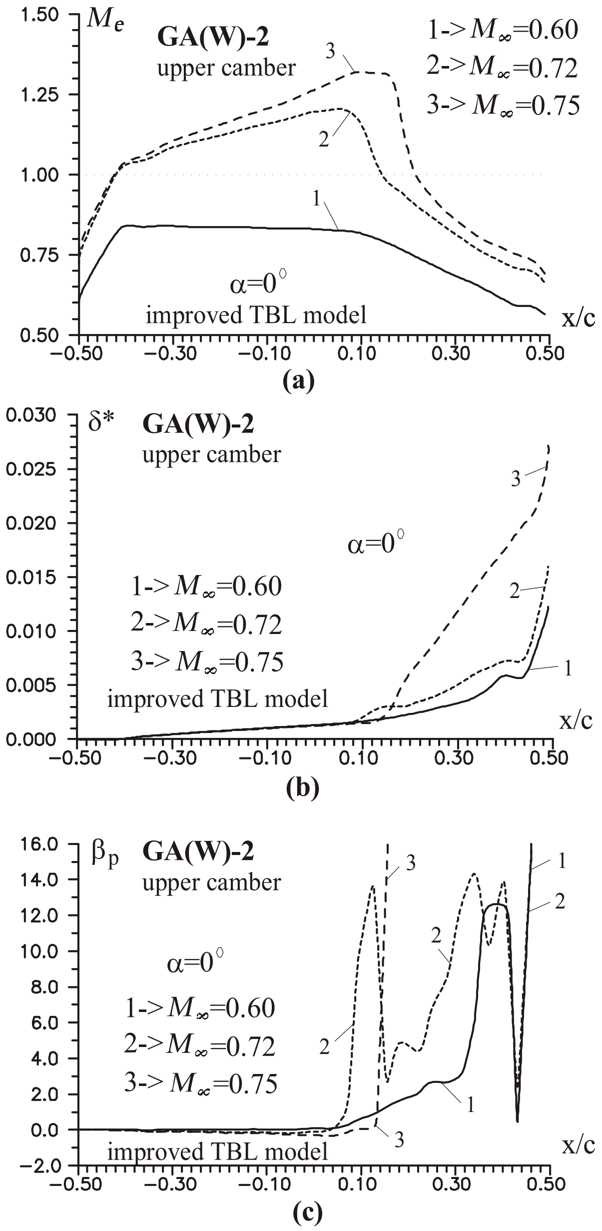

Figure 9.

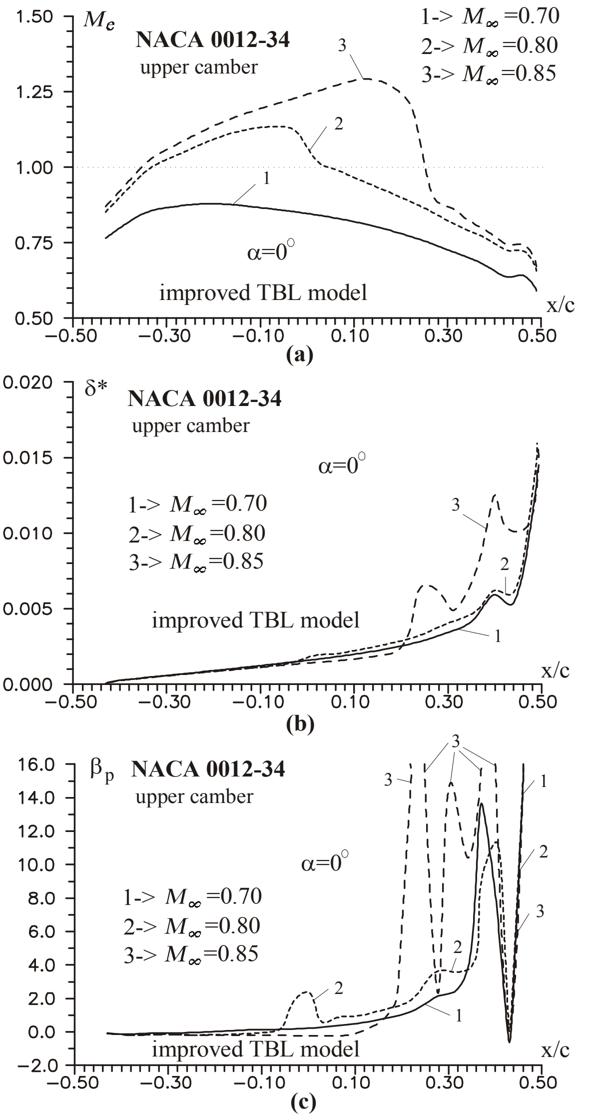

Figure 10.

Figures 9 and 10 show the chordwise distributions of local Mach number M_e (a), displacement thickness δ^* (b), and Clauser parameter β_p (c), along the upper cambers of the two airfoils. For the highest analyzed Mach numbers, β_p shows irrecoverable turbulent boundary layer separation at the shock wave position on GA(W)-2, while on 0012-34 there is turbulent boundary layer reattachment behind the shock wave and new separation close to the trailing edge.

4. CONCLUSION

The presented algorithm for wave drag calculations is the latest version of a numerical model the author has been using in airfoil analyses. The first steps in this work were initialized by real-life problems, and the first fully successful solutions that were reached were covered practically only by empirical proofs. After years of gaining experience in this area, the author believes that now he is able to give simple and logical, but very sound scientific proofs of the algorithm he has introduced. They have been presented in this paper together with the brief theoretical background, algorithm itself and a certain number of experimental verifications.

It has been shown that the results obtained by here presented model produce very smooth drag coefficient curves in transonic domain and always give the unique solutions. Also, they do not depend on the user's experience, and what is the most important, they coincide very well with the appropriate experimental results. For the reasons explained in detail in this paper, the author could not compare his results with the Carlson's C_{DW} algorithm, since it was producing instabilities in solutions. Also, the modified turbulent boundary layer model applied for profile drag calculations (originally derived by the author for smaller speed analyses) has proven its advantages in transonic domain as well over the original Nash-Macdonald turbulent boundary layer model.

NOMENCLATURE

a - local speed of sound,
c - airfoil chord length (unit),
C_L - airfoil lift coefficien,
C_{DP} - airfoil profile drag coefficient,
C_{DW} - airfoil wave drag coefficient,
C_D - airfoil total drag coefficient,
Cp - local pressure coefficient,
f - $d\xi / dx$,
g - $d\eta / a dz$,
H - boundary layer shape factor,
$\overline{H}$ - compressibility corrected boundary layer shape factor,
M - local Mach number,
M_e - local Mach number at the outer edge of the boundary layer,
M_∞ - free stream Mach number,
Re - Reynolds number,
R_θ - Reynolds number defined by boundary layer momentum thickness as characteristic length,
u - local velocity component in x direction
u_e - local velocity component at the outer edge of the boundary layer,
V - local velocity,
w - local velocity component in z direction,
x, z - physical space coordinates,
x/c - relative chordwise coordinate,

α - angle of attack,
δ^* - boundary layer displacement thickness,
$\overline{\phi}$ - nondimensional velocity perturbation potential,
θ - boundary layer momentum thickness,
ρ_e - local density at the outer edge of the boundary layer,
ξ, η - calculation space coordinates.

Subscript / superscript

L -subscript denoting a parameter value on the lower airfoil camber,
$[n]$ - superscript denoting the "n"-th iteration cycle value,
TBL - abb. for "turbulent boundary layer",
(te) - subscript denoting a parameter value on the trailing edge,
U - subscript denoting a parameter value on the upper airfoil camber.

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ПОБОЉШАНИ АЕРОДИНАМИЧКИ ПРОРАЧУН ТРАНСОНИЧНОГ ОТПОРА АЕРОПРОФИЛА ПРИ ЗОНАЛНОМ МОДЕЛИРАЊУ СТРУЈНОГ ПОЉА

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При пројектовању савремених комерцијалних ваздухоплова висока економичност лета један је од најбитнијих захтева које треба испунити. Поред избора економичних мотора, врло битна ставка за задовољење овог услова је и примена савремених техника при аеродинамичком пројектовању. Велики број ових ваздухоплова крстари брзинама које су нешто мање од брзине звука, па је узгонске површине и њихове аеропрофиле потребно оптимизирати превасходно за овај домен. Један од првих корака у том процесу је избор или наменско пројектовање аеропрофила за крило и остале узгонске површине конкретне летелице који ће производити што мањи таласни отпор у крстарећем лету. Нумеричка оптимизација аеропрофила данас представља изузетно важан део тог поступка. Алгоритам приказан у овом раду омогућава нумерички прорачун таласног отпора како на постојећим тако и на аеропрофилима који се наменски парве за одређену летелицу и превасходно је намењен оперативном аеродинамичком пројектовању ваздухоплова. Алгоритам је релативно једноставан и врло поуздан, што је показано поређењем резултата које он даје у оквиру програма названог *Транпро* са експерименталним резултатима из неколико најкомпетентнијих светских ваздухопловних центара који се баве овом проблематиком.