Gear Vibration in Supercritical Mesh-Frequency Range

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Gear vibrations havv been studied intensively for many years. They are mainly treated as forced damped vibrations caused by stiffness fluctuation of the gear teeth in mesh. Such approach gives good results in the sub-critical and resonant mesh frequency range. In the super-critical frequency range the vibration level increases slightly with the increase of the gear speed of rotation. The starting point of this work is the assumption that gear vibrations are free damped in which excitation is repeated by teeth impact at each entrance of teeth into the meshing – singular process. The new calculation model based on singular systems theory and on measurement results has been worked out. This model supports the measurement results in all three frequency ranges, the sub-critical, resonant and supercritical

Key words: *Vibration, Gear transmission, Singular systems*

1. INTRODUCTION

The gear teeth meshing presents by itself a very complex physical process. From the teeth meshing area arise internal dynamic forces, vibration and noise. Besides that, gear mesh excites dynamic disturbances for wider area of machine systems, which under that excitation generate vibration and noise emission. The main subject of investigation are disturbance processes which follow teeth conjugation. They are caused by teeth in mesh stiffness fluctuation, stiffness non-linearity, teeth in mesh clearance, transmission errors, etc. The first aim of that investigation were internal dynamic forces caused by gear meshing [2]. Higher level of that research was gear vibration and noise [1], [4]. The key excitation in that analytical analysis is the teeth stiffness time function. That function is complex and includes main disturbances in the teeth mesh, non-linearity, etc. [7], [11]. Fourier transformation in the collection of elementary sin-time-functions with different phases makes a possibility for numerical calculations. Such approach allows investigators to use linear differential equations for practical solutions. For more exact representation of gear dynamics some authors [12], [13] include non-linearity. Clearance, backlash, flank friction etc. presents additional effects of gear vibration excitation and analysis.

All mentioned approaches and analyse are carried out to complete the effects of excitation force functions and teeth stiffness in the model of force gear vibration. Majority of investigators took parallel experimental and analytical research. The model of force vibration, gave good results in comparing with measured data. The force fluctuation corresponds to teeth stiffness fluctuation and can include some other influences. Many good results are obtained using comparing between analytical and experimental results. It is possible to observe good correlation between analytical and experimental results, but only in the sub critical and critical teeth mesh frequencies. Supercritical mesh frequency range was not sufficiently investigated. However, using the model of forced vibration with polar coordinates, in that frequency area, analytical vibration values are obtained that are close to measured [5].

Apart from stiffness fluctuation, teeth impacts also excite dynamic forces, vibration and noise. At the moment of teeth entrance into the mesh, teeth impact occurs, and excites free vibration. For impact analysis a simplified model presented at Fig.11a and equivalent model of supported mass (Fig. 1*a*) are sufficient. Measured results, in that frequency region, gradually increase (Fig. 1*b*). For very high mesh frequencies and for high speeds of rotation that vibration levels get very high.

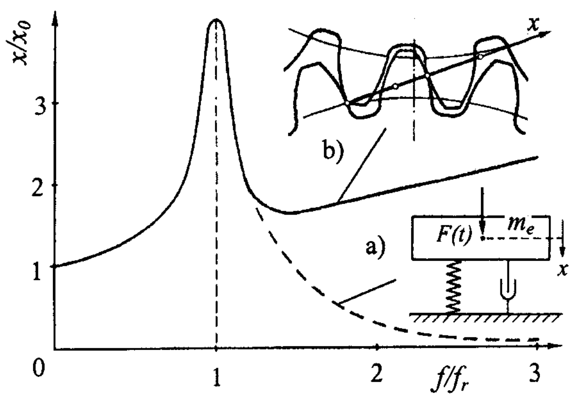

Figure 1. Frequency responce function: a) elastically suported forced mass, b) gear pair in mesh

W. Knabl [1] very clearly presented difference between measured and simulated results in supercritical mesh frequency region. That difference was very high. The aim of his research was to establish correlation between gear noise and vibration, but that difference was not analyzed. Rettig [2], using experimental

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approach, established empirical model for dynamic factor K_v (dynamic forces) calculation. That model is founded on a hypotheses that main direction of dynamic force increase is linear in all mesh frequency range except in the region around of main resonance frequency (Fig. 2*b*). A few years later similar model for K_v calculation (Fig. 2*a*) was standardized by DIN 3990 and ISO 6336.

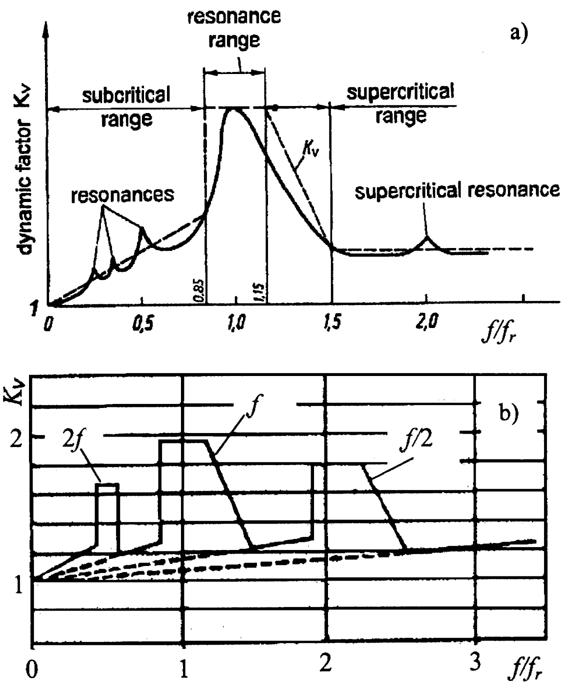


Figure. 2. Dynamic factor caused by gear meshing K_v a) total value [14] b) for separate excitation harmonics [2]

Noticed difference between measured and calculated vibration level in supercritical mesh frequency range (Fig. 1) some investigators tried to solve by some of the next approaches. To create wider critical frequency range using more complex forced vibration model and by including higher number of natural frequencies. It makes possibility to obtain a number of resonant frequencies and wither critical frequency range. For that purpose analysis includes effects of the other parts of machine system, for example effects of bearings, shafts, etc. That approach makes it possible to cover wider mesh frequency range and to hide the problem presented at the Figure 1. Calculation results presented in article [15] show that very clearly. Another possibility contains set of different excitation functions [9]. The harmonics with higher frequencies create sub critical resonances which makes wider critical mesh frequency range too.

In this work experimental results show very clearly differences, presented in Figure 1. It succeeded using experimental installation with extremely high speeds of rotation. The force model approach did not give answers which can explain that phenomena. It is necessary to search for other possibilities. In the following text suggested is a model of repeatable free damped vibration and theory of singularly systems [5].

2. EXPERIMENTAL RESEARCH

Figure 3 presents testing rig based on the back to back system which has been used in this research. The gear center distance was 125 mm, with a continuous rotation speed change from zero to 6000 rpm. The rotational gear vibrations were measured in the direction of the tangent line onto the kinematics circle of the diameter of 125 mm. On the disc, which has been put at the end of an outlet shaft, in the direction of the tangent line onto this circle, an accelerometer has been attached which rotates with the disc. The connection with the coaxial electric cable has been achieved through sliding rings. The output value from the preamplifier is introduced into the A/D card and into a computer, where the processing and frequency analysis of the measured vibrations is materialized. The rotation speed is measured by a tachogenerator, the output of which is also introduced into the computer and conjugated with the measured vibration level.

The test was performed with spur gears of the transmission ratio u=1, with the teeth number z=41, module m_n=3 mm, width 20 mm, using back to back system (Fig. 3*a*). Two loading values were used, one very small of 30 Nm, and a somewhat bigger of 130 Nm, and with speed of rotation in range 0-6000 rpm. Using the disc (Fig. 3*b*) attached to the shaft and piezoelectric accelerometer, rotational vibration measurement was carried out. Accelerometer, including sliding rings for signal transmission, preamplifier and software system for data acquisition was calibrated by a source of known vibrations. Tachogenerator, for shaft speed measurement, was connected with a system for data acquisition and correlated with measured vibrations.

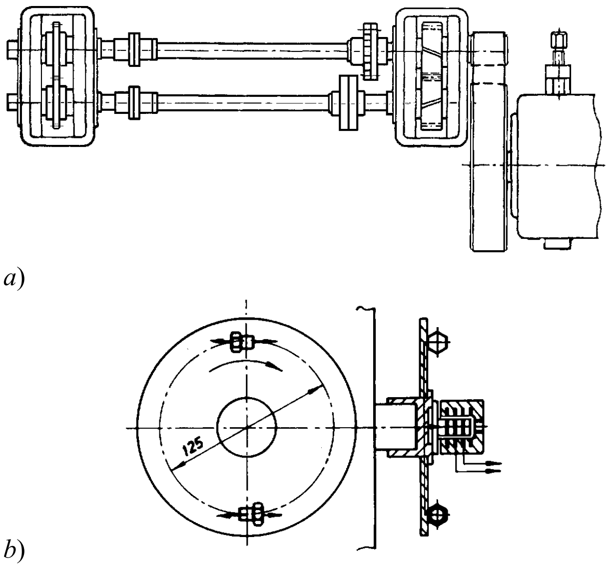


Figure 3. a) Testing rig for low and media speeds, b) Accelerometer on the rotation disc,

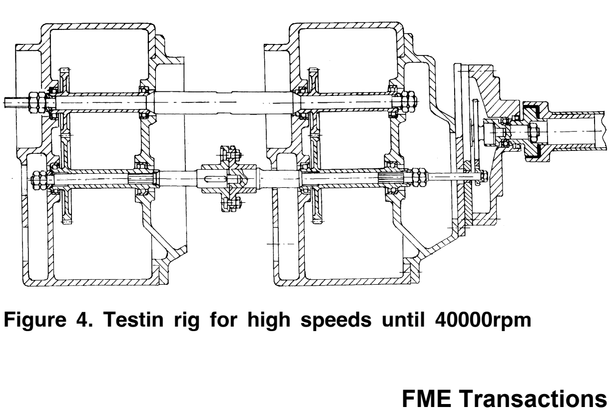


Figure 4. Testin rig for high speeds until 40000rpm

High speeds of gear rotation make it possible to create supercritical mesh frequency range, very wide. It is needed for the identification of more details about gear vibration nature in that mesh frequency range. For that purpose (for speeds from zero to 40000 rpm), the testing rig is presented in Figure 4. That system was constructed using two aircraft gear boxes, driven by a multiplicator and an electric variator. Spur gear pairs with the teeth number 47/32, center distance 85mm, module m_n=2 mm, width 8.5 mm, are tested. Gear and bearing lubrication and sealing is solved and controlled during testing process in a specific way. Vibration measurement system is very similar to the system presented in Fig. 3. Accelerometer was attached to the gearbox wall in direction of gear contact line, and linear vibrations were measured.

The measurement of the total level of rotational vibration correlated with the gear rotation speed has been achieved through the presented installation (Fig. 3), and the obtained measurement results have been presented in Figure 5. With the increase of the angular speed i.e. teeth mesh frequency, there is an increase in the vibration level. Diagram in Figure 5 shows another two resonances in the range after main resonance speed. They are the result of the conjugation of the drive gear pair in the back to back system (Fig. 3), i.e. a consequence of the natural frequency of 2708 Hz. With an increase of the load i.e. torque T which the gears transmit, there is an increase in the vibration level. The torque is increased from 30 Nm to 130 Nm, which has led to an increase in the vibration level in the resonance regions by 400 m/s^2 i.e. by 40g. These acceleration values for gears are usual, since it is a question of extremely small displacements, which change in extremely short time intervals.

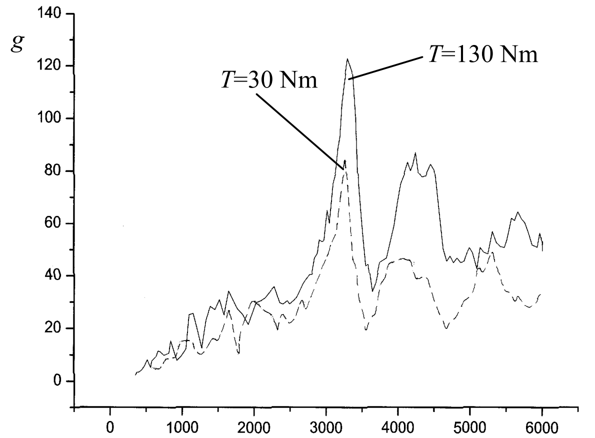
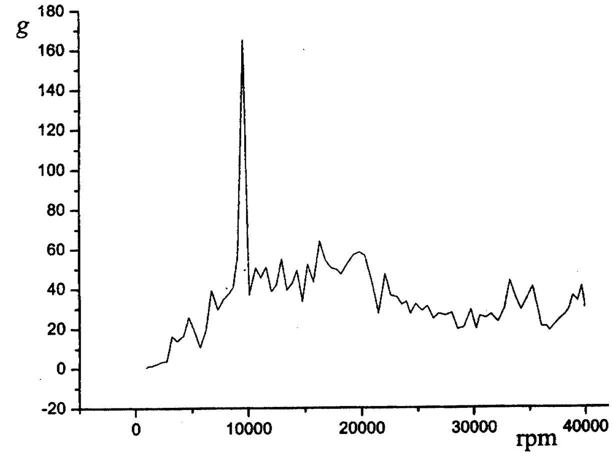


Figure 5. Total vibration level of the spur gears obtained by the system presented in Figure 3

The diagram in Figure 6, obtained by testing rig for high speeds presented at Figure 4, gives much more clear results. Sub-critical, critical and supercritical mesh frequency ranges are separated very clearly. Supercritical range is very wide and shows some interesting trends in fluctuation of gear vibration total level. That level gradually increases until 20000 rpm and then fluctuates. That is a special phenomenon which has to be investigated separately.

Gear vibration frequency analyze are carried out using software FFT - Fast Fourier Transformation. The results in the shape of frequency spectra have been presented in Figures 7, 8 and 9. The amplitude values of the elementary sine functions expressed by the number of gravitation g are on the ordinates, whereas the frequencies of these functions in Hz are on the abscissa. Figure 7 shows the frequency spectra of the gear vibrations in the sub critical range of angular speeds i.e. teeth mesh frequencies. The rotation speeds of 500 pm and 2000rpm have been selected also because the teeth mesh frequencies $f=nz/60$ of 341 Hz and of 1366 Hz are significantly smaller than the natural frequency of the meshed gear pair of 2197 Hz. Although the excitation frequencies are much smaller, the vibration levels are the highest for the natural gear frequency. Free vibrations for n=500 rpm dominate these spectra. With the increase of the teeth mesh frequency f there is an increase in the level of free vibrations. Besides these for



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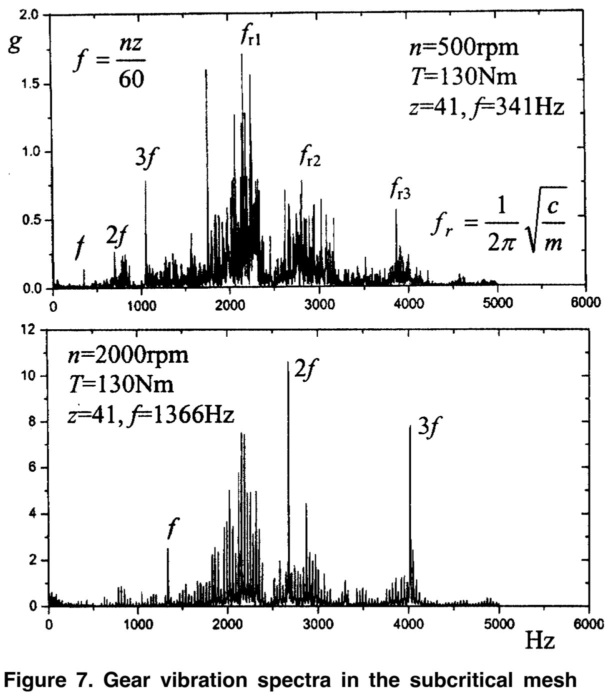

Figure 7. Gear vibration spectra in the subcritical mesh frequency range obtained by testing rig for lower speeds

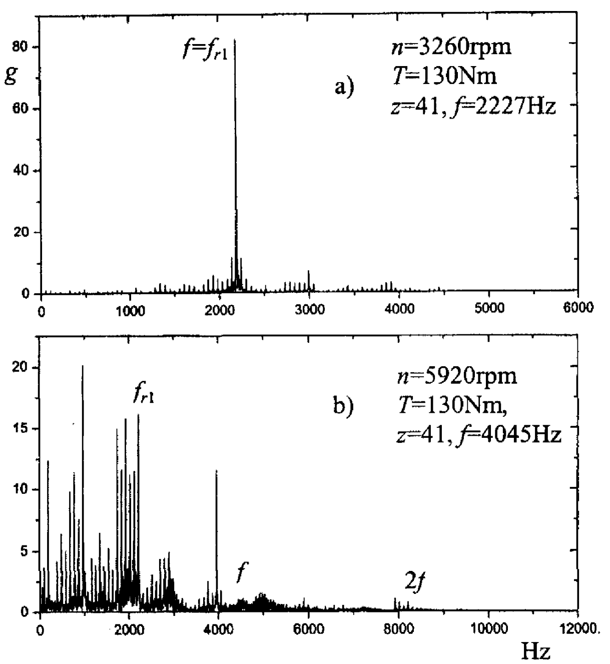

Figure 8. Gear vibration spectra obtained by testing rig for lower speeds: a) in the critical and b) supercritical mesh frequency range

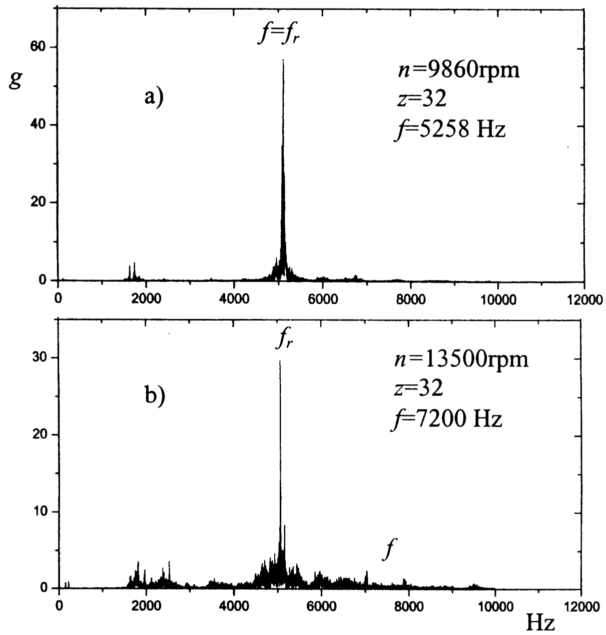

Figure 9. Gear vibration spectra obtained by testing rig for high speeds: a) in the critical and b) supercritical mesh

n=2000 rpm the higher harmonics of excitation function 2f and 3f are in the range of natural frequencies and make possibilities for sub critical resonances.

When teeth mesh frequency gets equal to the natural one (Fig. 8*a* and Fig. 9*a*), the main resonance arises vibration level only for that frequency. All disturbance energy absorbs and emits by the same (by this) frequency. Vibration spectra gets cleaned of vibration with other frequencies. Vibration level for that frequency and total vibration level (Fig. 5 and 6) for that speed of rotation are extremely high. With further rise of teeth mesh frequency by rising speed of rotation, mesh frequency gets higher than the natural one. Vibration level for natural frequencies gets lower (Fig. 8*b* and Fig. 9*b*), but it is possible to notice very clearly the phenomenon described by Figure 1. In mentioned frequency spectra for supercritical mesh frequency f, vibration do not exist. Spectrum at Figure 9*b*, obtained by testing rig for extremely high speeds of rotation, shows that phenomena more clearly in comparison with Figure 8*b*. By mesh frequency f, gear system only absorb disturbance energy. That absorbed energy reduced by different kinds of damping, is released by free vibrations with natural frequencies. That situation exists in whole supercritical mesh frequency range. With further increase of the teeth mesh frequency f, absorbed energy gets higher and free vibration level too.

3. NATURE OF GEAR VIBRATION

At each entrance of a new pair of teeth into the mesh a collision and teeth deformation change occur. After that natural gear vibration with the frequency of f_r occurs. Figure 10 shows the time functions of vibrations for a relatively slow gear rotation i.e. for the state when $f<<f_r$. After each entrance into the mesh, which is repeated with the frequency f (Fig. 10*b*), free damped gear vibrations follows with the frequency f_r (Fig. 10*a*). Figure 10 shows very clearly that gear vibration are free and repeats after new tooth entering into the mesh. With the increase of the rotation speed there is a decrease in the difference of frequencies. Teeth mesh frequency f increases with speed of rotation and the natural frequency f_r remains the same (corresponds to teeth in mesh stiffness and rotation masses). Besides due to the frequent restoration of the excitation, the quantity of the absorbed energy increases as well as the level of the free vibrations, as can be seen from the measurement results.

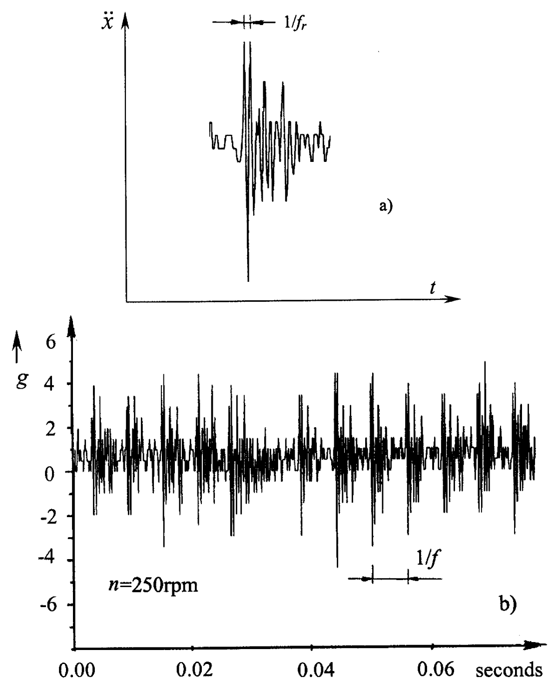

Figure 10. Free gear vibration after excitations by the entrance of teeth into the mesh: a) separate free damped vibration one teeth pair collision, b) successive repetition of collisions and free vibration

Excitation of natural vibration are caused by fluctuation of deformation and by teeth impacts. Deformations fluctuates because teeth in mesh stiffness fluctuates. Besides, the wear and flank profile shape corrections lead to a change in the deformation time function, thus the deformation process becomes complex and dependent on many conditions. The deformation time function is in relation to this, also a complex quasi periodical phenomenon.

The tested gears in the back to back system are exposed to the loading of 130Nm. The deformation (displacement) in the direction of the contact line x occurs under the action of normal force. Due to teeth in mesh stiffness fluctuation, deformations fluctuate from x_{min}=4.22 μm to x_{max}=7 μm, with amplitude x_{a0}=1.39μm.

Teeth impact is repeated at each tooth entrance into mesh. The gear teeth get in mesh deformed under outside loading. Due to this deformation a change occurs in the position of the gear in relation to the theory. The driven gear lags behind by the deformation magnitude, and the driving gear goes ahead of the theoretically correct position. Due to this the contact of the teeth begins earlier i.e. in front of the theoretically correct position A (Fig. 11*b*). As the contact point speeds at that moment are not equal, the contact starts with an impact. The teeth collision speed v_c depends on the magnitude of the teeth deformation (load) and on the possible discrepancies (differences) in the pace of the conjugated gears. Besides, the teeth collision speed increases in proportion to the increase of the gear rotation speed. The collision force F_c is proportional to the collision speed v_c, equivalent mass of the meshed gears and added masses which rotate together with them m_e and the equivalent momentary stiffness of both teeth in the momentary contact point c', i.e. $F_c = v_c\sqrt{c'\,m_e}$.

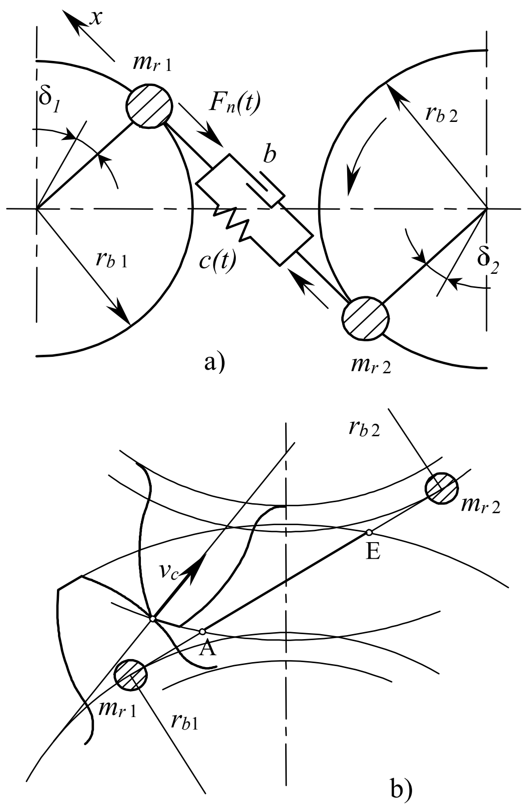

Figure 11. The gear meshing: a) simplified model, b) teeth collision at the mesh entrance

The disturbance energy is absorbed by the elastic deformations and collisions of teeth following each entrance of teeth into mesh. The disturbance energy is a form of potential energy i.e. represents the deformation work obtained by teeth deformation. At each entrance of teeth into mesh the deformation increases, and at its exit, it decreases. The amplitude of the of the deformation work fluctuates is proportional to the amplitudes of force and deformation. The force amplitude can be presented through the mean teeth in mesh stiffness c_γ and static deformation amplitude x_{a0} i.e. F_a=$c_\gamma x_{a0}$. Deformation work is proportional to half of the product of force and deformation, i.e. $F_a x_{a0}/2 = c_\gamma x_{a0}^2/2$. By teeth mesh frequency f increase, absorbed potential energy increases. All absorbed energy does not transform into vibration energy. The part of that energy can be defined as a part of mentioned "static" deformation work. It is possible to suppose that the ratio between those two parts of work is a parameter of the system and that its value is constant. That value can be marked with letter A which presents how much of energy in Nm are loses per one Nm of "static" potential energy in the time of one second. In that relation the measure unit for constant A is seconds. Potential energy is

$$E_p = A\frac{c_\gamma x_{a0}^2}{2} f = \frac{c_\gamma x'^2}{2} \quad ; \quad x' = A\,x_{a0} f \ . \qquad (1)$$

Product Af presents a dimensionless parameter. With the increase of the teeth mesh frequency, f, there is an increase in the absorbed potential energy .

4. CALCULATION MODEL OF GEAR VIBRATION CAUSED BY TEETH IMPACT

Based on the previously performed analyses of the gear vibration measurement results and on the basis of the presentation of the excitation which leads to vibrations, it is possible to conclude the following. Gear vibrations are mainly free damped and restored at each entrance of a new teeth pair into the mesh – singular process. When the mesh frequency f comes close to or becomes equal to the natural frequency, resonance occurs and free vibrations become stronger.

The measurement results i.e. the frequency spectra of the measured vibrations show that the gears vibrate with natural frequencies and that the level of these free vibrations increases proportionally to the increase in the rotation speed. The harmonics with frequencies which depend on the teeth mesh frequency are significantly lower and have a small effect on the total vibration level. An exception to this are only the harmonics with frequencies which are similar to some of the natural ones (Fig. 7, 8 and 9). It is possible to conclude from this that these vibrations are mainly free and that they depend on the quantity of absorbed excitation energy. The need to analyze the oscillation energy balance imposes Lagrange's equations as a suitable solution for this purpose. The analysis of the potential energy has been given in the previous section. The kinetic energy of the vibration is $E_k = m_e \dot{x}'^2/2$. If the damping i.e.

energy of the losses is omitted at the beginning, Lagrange's oscillation equation of the second kind is

$$\frac{1}{\mathrm{d}t}\frac{\partial E_k}{\partial \dot{x}'}+\frac{\partial E_p}{\partial x'}=0 \quad ; \quad m_e\ddot{x}'+c_\gamma x'=0 \ . \qquad (2)$$

Then follows the possibility of determining the acceleration as per equation

$$\ddot{x}'=A\frac{c_\gamma x_{a0}}{m_e}f \ . \qquad (3)$$

For the given loading the "static" displacement amplitude x_{a0} would be the same i.e. constant. The magnitude of the acceleration $\ddot{x}'$ would be increasing linearly with the increase of the teeth mesh frequency f, as per line $\ddot{x}'$ in Figure 12. Besides the effect of damping, as well as the other effects which determine the conditions of teeth meshing, it is possible to cover by the value of constant A. These can be discrepancies of the dimensions of the values of the clearances in mesh, mesh conditions and transmission errors, etc. For the measurement results at (T=30 Nm) this constant amounts to $A=8\cdot10^{-4}$ s, and for T=130 Nm the value of this constant is, $A=4.42\cdot10^{-4}$ s.

By increasing of the teeth mesh frequency $f=nz/60$, the frequency of the disturbance restoration increases as compared to the natural frequency f_r which does not change. Depending on the difference in frequencies, the vibration level increases as compared to the free ones defined by equation (3). The increase is the greatest in the resonance field, and is very little when frequency f are small. As the excitation frequency overlaps with the natural, the displacement due to natural vibrations phase overlaps the displacement (deformation) of teeth due to loading. At each coming entrance of teeth into meshing, the displacement speed and acceleration become greater. If $F_a'=c_\gamma x'$ is thereby the amplitude of the force due to displacement x', which represents the excitation and if $F_a''=c_\gamma x''+b\dot{x}''$ is the amplitude of the response, the ratio is

$$\frac{F_a''}{F_a'}=\frac{\sqrt{1+4\zeta^2(f/f_r)^2}}{\sqrt{\left(1-(f/f_r)^2\right)^2+4\zeta^2(f/f_r)^2}}=\frac{\ddot{x}''}{\ddot{x}'} \ . \qquad (4)$$

This relation is obtained by the transformations of the ratio of the time functions of the indicated forces. As it represents the relation of the force due to vibrations and due to the excitation. The equation (4) is a known formula [3] for the determination of the isolation degree of the vibrations of the elastically supported masses. As force is the product of mass and acceleration, by mass elimination, the expression (4) is transformed into the acceleration ratio. In this form this is the ratio of the gear acceleration response and the acceleration which is expressed at the entrance of the teeth into the meshing. This is thus an increase of the acceleration due to natural vibrations (equation 3). By adding up equations (3) and (4), according to singular system principle, the final expression is obtained $\ddot{x}=\ddot{x}'+\ddot{x}''$ i.e.

$$\ddot{x}=A\frac{c_\gamma x_a}{m_e}f\left[1+\frac{\sqrt{1+4\zeta^2(f/f_r)^2}}{\sqrt{\left(1-(f/f_r)^2\right)^2+4\zeta^2(f/f_r)^2}}\right] \ . \qquad (5)$$

The dimensionless damping parameter $\zeta=b/(2\sqrt{c_\gamma m_e})$ is obtained on the basis of the damping coefficient $b=2\zeta\sqrt{c_\gamma m_e}$ which is according to this expression

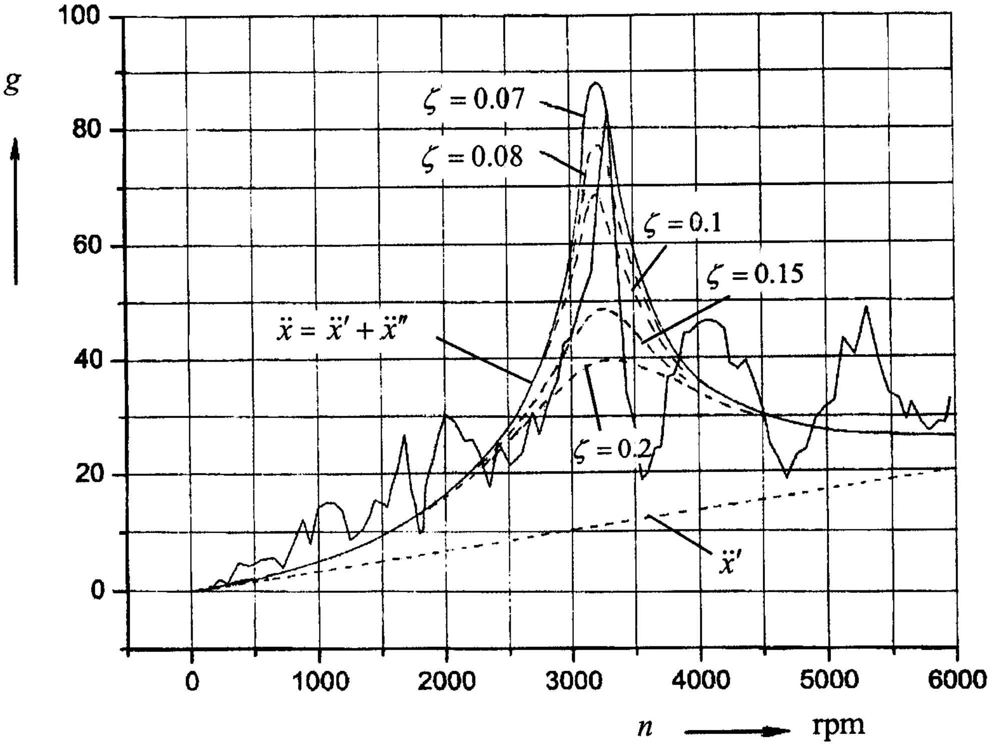

Figure 12. The approximation of the measurement vibration results by analitical curve

with the dimensions [Ns/m]. It is possible with this parameter to adjust in the resonance the vibration level to the measured value. For the details from the diagram in Figure 5, where the acceleration level for the resonance is $\ddot{x} = 840\,\mathrm{m/s^2}$, the damping is of small value and amounts to approximately b=4925 Ns/m.

Gear acceleration, $\ddot{x}$, and the diagram (presented in Figure 12) are calculated according to the equation (5). Line for dimensionless parameter ζ=0.07 is very close to the line obtained by measurement, which means that suggested model is proofed. Besides that, according to the new model, in supercritical range vibration level gradually increases.

The model is presented using singularly system theory. Those systems consist of two parts: continual and with singularities. Power transmission is continual, and teeth mesh present a singular process. In mathematical manner, solution consist also, of two parts [6]: algebraic $\ddot{x}'$ (obtained using simplified Lagrange equation) and differential part $\ddot{x}''$ (obtained using Laplace response function). This solution corresponds to measured results in all three frequency ranges: sub-critical, critical and supercritical. Besides this, obtained dimensionless parameter ζ=0.07 is equal to the same parameter obtained in the number of measurements. Those two things are the proof that the suggested model is correct.

The model presented by equation (5) and Figure 12 contains only one natural frequency of the system. It is possible to extend this model using other natural frequencies f_{r2}, f_{r3}, etc. The total level of acceleration for those conditions will be presented as a sum of $\ddot{x}''$ for all natural frequencies. Obtained curve, presented at Figure 12, will not be smooth in the region from 4000-6000 rpm. Additional resonances make it possible to follow experimental curve. The two picks at experimental curve are caused by resonances with natural frequencies f_{r2} and f_{r3}. Using supplemented model, based on the same approach, it is possible by calculation to determine damping parameters for other resonances i.e. modal damping coefficients. Besides the suggested supplementation, it is possible to make the model wider in one more direction. Excitation displacement is present by the sum of sin-time-functions

$$x_0 = x_{m0} + \sum_{i=1}^{\infty} x_{a0i} \sin\left(2\pi f_i t + \psi_i\right).$$

Presented model (equat. 5) contains only the first member of the order $x_{a01} = x_{a0}$ and $f_1 = f$. Presented model can include higher order members with frequencies $f_2=2f$, $f_3=3f$, etc. In this way the presented smooth curve (Fig. 12) will be richer by sub critical resonances. Step by step, that curve can get very close to measured one. But very important thing to mention is that in the area of extremely high speeds of rotation calculated results follow the measured ones, opposite to the problem presented in Figure1.

5. CONCLUSION

Based on the results of the experiments, analyses and mathematical modeling, the following answers can be given.

1. Gear vibration are free damped whereby the excitation is restored every time when a new pair of teeth enters the mesh. The measurement results prove this. Vibrations with natural frequencies dominate the vibration spectrums. The suggested mathematical model gives a slight increase of vibrations in the supercritical frequency range, in accordance with the measurement results.

2. Suggested mathematical model is defined as a singular system. Power transmission consist of two processes: continuous torque transmission and discontinuous (singular) teeth mesh. Model (equation) consists, also, of two parts: algebraic (obtained from simplified Lagrange equation) and differential (based on Laplace response function).

3. The suggested model of free vibrations is based on teeth impact and on disturbance energy absorbed by teeth deformation after impact. The results of that model well approximate the measured in all the three ranges, the sub critical, resonant and supercritical.

4. The internal dynamic forces in teeth mesh, vibration and noise are consequences of the same causes. Those several causes are: change in teeth deformation, teeth impact, gear inertia due to measure and teeth shape deviation, etc.

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ВИБРАЦИЈЕ ЗУПЧАНИКА У НАДКРИТИЧНОМ ПОДРУЧЈУ ФРЕКВЕНЦИЈА СПРЕЗАЊА ЗУБАЦА

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Вибрације зупчаника су изучаване током низа година али врло мало у надкритичном подручју фреквенција спрезања зубаца. Вибрације су углавном третиране као принудне изазване променом крутости зубаца у спрези. У надкритичном подручју ниво измерених вибрација се благо повећава са повећавањем брзине ротације зупчаника. Осим тога оне су слободне пригушене са обнављањем побуде сударима зубаца. Полазна основа овога рада је да се вибрације зупчаника третирају као слободне пригушене, а да се побуда обнавља сударом при сваком уласку у спрегу новог пара зубаца. Спрезање се сматра сингуларним процесом, а нови модел прорачуна је развијен на приципима на којима су засновани сингуларни системи. Нови модел добро апроксимира резултате мерења у докритичном, критичном и надкритичном подручју фреквенција спрезања зубаца.