Second Harmonic Generation in Nonlinear Transformation of Electromagnetic Waves in Suddenly Created Cold Magnetized Plasma. Transversal Propagation

Zoran M. Trifković
Assistant Professor
University of Belgrade
Faculty of Mechanical Engineering

Božidar V. Stanić
Professor Emeritus
University of Belgrade
Faculty of Electrical Engineering

The nonlinear transformation of linearly polarized source electromagnetic wave in electron and electromagnetic plasma oscillations, stationary (rectification and space-varying) modes and travelling electron and electromagnetic plasma waves, due to a weak nonlinearity, has been analyzed by using the second order perturbation theory in radio approximation. The efficiency of excitation of the transversal second-harmonic electric wave modes with double wave number with respect to the wave number modes in the linear transformation has been studied for different values of source wave frequency, taking the electron cyclotron frequency as a parameter).

Keywords: *rapidly created plasma, stationary modes, second harmonic generation, nonlinear transformation, Laplace and Fourier transformation.*

1. INTRODUCTION

Rapidly created plasmas appear practically in all pulse gas discharges, laser created plasmas, lightning, and plasmas created by nuclear explosions. The transformation of electromagnetic (EM) waves in such time varying linear media has been the subject of interest in many recently published papers. The basic results of these contributions are summarized in [1].

In this paper we have assumed that for $t<0$ the linearly polarized (LP) source electromagnetic wave (EMW) with angular frequency ω_0 and the wave number $\boldsymbol{k_0}$ is propagating in free space in the positive z direction. The static magnetic field is assumed to be along positive y direction, $\boldsymbol{B_0}=\boldsymbol{y}B_0$, where $\boldsymbol{y}$ is unit vector in positive y-direction. At $t=0$ the entire free space is ionized with an electron plasma density increase from zero to some constant value N_0. The transformation of the source EM plane wave, due to a weak nonlinearity, has been analyzed by using the second order perturbation theory. The efficiency of excitation of newly created modes in anisotropic plasma medium has been obtained in a closed form and studied for different values of source wave frequency and for electron cyclotron angular frequency equal to the angular electron plasma angular frequency.

Received: December 2004, Accepted: Mart 2005.
Correspondence to: Zoran Trifković
Faculty of Mechanical Engineering,
Kraljice Marije 16, 11120 Belgrade 35, Serbia and Montenegro
E-mail: ztrifkovic@mas.bg.ac.yu

2. PROBLEM FORMULATION AND SOLUTION

Electric and magnetic fields of the linearly polarized source EMW for $t<0$ are given by

$$\boldsymbol{e_0}(z,t)=E_0\cos(\omega_0 t-k_0 z)\cdot\boldsymbol{x}\,, \qquad (1)$$

$$\boldsymbol{h_0}(z,t)=H_0\cos(\omega_0 t-k_0 z)\cdot\boldsymbol{y}\,, \qquad (2)$$

where $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$ are the unit vectors in positive direction of x, y and z axis, respectively, $H_0=\sqrt{\varepsilon_0/\mu_0}E_0$ is magnitude of magnetic field, and E_0 is magnitude of electric field. (μ_0 and ε_0 are permeability and permittivity of free space, respectively).

The EM $\boldsymbol{e}(z,t)$, $\boldsymbol{h}(z,t)$, electron velocity $\boldsymbol{u}(z,t)$ and electron density fields $n(z,t)$ in magneto-plasma medium have to satisfy the following equations:

$$\frac{\partial n(z,t)}{\partial t}+\nabla\left(n(z,t)\boldsymbol{u}(z,t)\right)=0\,, \qquad (3)$$

$$\nabla\times\boldsymbol{e}(z,t)=-\mu_0\frac{\partial\boldsymbol{h}(z,t)}{\partial t}\,, \qquad (4)$$

$$\nabla\times\boldsymbol{h}(z,t)=-N_0 q\boldsymbol{u}(z,t)+\varepsilon_0\frac{\partial\boldsymbol{e}(z,t)}{\partial t}\,, \qquad (5)$$

$$\frac{d\boldsymbol{u}(z,t)}{dt}=\frac{\partial\boldsymbol{u}(z,t)}{\partial t}+\left(\boldsymbol{u}(z,t)\nabla\right)\boldsymbol{u}(z,t)=$$

$$=-\frac{q}{m}\boldsymbol{e}(z,t)-\frac{q}{m}\boldsymbol{u}(z,t)\times\boldsymbol{B_0}(z,t)-\frac{q}{m}\boldsymbol{u}(z,t)\times\boldsymbol{h}(z,t)\,, \qquad (6)$$

The equation (6) is nonlinear due to the second term on its left-hand side and last term on its right-hand side. EM, electron velocity and electron density fields for weakly nonlinear plasma can be further expressed in the following form:

$$\boldsymbol{e}(z,t) = \boldsymbol{e_1}(z,t) + \boldsymbol{e_2}(z,t)\,, \tag{7}$$

$$\boldsymbol{h}(z,t) = \boldsymbol{h_1}(z,t) + \boldsymbol{h_2}(z,t)\,, \tag{8}$$

$$\boldsymbol{u}(z,t) = \boldsymbol{u_1}(z,t) + \boldsymbol{u_2}(z,t)\,, \tag{9}$$

$$n(z,t) = n_1(z,t) + n_2(z,t)\,, \tag{10}$$

where the subscript "1" refers to the linear and the subscript "2" to the weakly nonlinear field. Substituting (7-9) into (3-6) one obtains the following system of equations for linear and weakly nonlinear fields:

$$\nabla\times\boldsymbol{e_1}(z,t) + \mu_0 \frac{\partial \boldsymbol{h_1}(z,t)}{\partial t} = 0\,, \tag{4a}$$

$$\nabla\times\boldsymbol{h_1}(z,t) + N_0 q \boldsymbol{u_1}(z,t) - \varepsilon_0 \frac{\partial \boldsymbol{e_1}(z,t)}{\partial t} = 0\,, \tag{5a}$$

$$\frac{\partial \boldsymbol{u_1}(z,t)}{\partial t} + \frac{q}{m}\boldsymbol{e_1}(z,t) + \frac{q}{m}\boldsymbol{u_1}\times\boldsymbol{B_0} = 0 \tag{6a}$$

and

$$\frac{\partial n_2(z,t)}{\partial t} + \nabla\left(N_0 \boldsymbol{u_2}(z,t)\right) = -\nabla\left(n_1(z,t)\boldsymbol{u_1}(z,t)\right), \tag{3b}$$

$$\nabla\times\boldsymbol{e_2}(z,t) + \mu_0 \frac{\partial \boldsymbol{h_2}(z,t)}{\partial t} = 0\,, \tag{4b}$$

$$\nabla\times\boldsymbol{h_2}(z,t) + qN_0 \boldsymbol{u_2}(z,t) - \varepsilon_0 \frac{\partial \boldsymbol{e_2}(z,t)}{\partial t} = -qn_1(z,t)\boldsymbol{u}_1(z,t)\,, \tag{5b}$$

$$\frac{\partial \boldsymbol{u_2}(z,t)}{\partial t} + \frac{q}{m}\boldsymbol{e_2}(z,t) + \frac{q}{m}\boldsymbol{u_2}(z,t)\times\boldsymbol{B_0} = -\frac{q}{m}\boldsymbol{u}_1(z,t)\times\left(\mu_0\boldsymbol{h}_1(z,t) + \frac{n_1(z,t)}{N_0}\boldsymbol{B_0}\right) - \left(\boldsymbol{u}_1(z,t)\nabla\right)\boldsymbol{u}_1(z,t) - \frac{n_1(z,t)}{N_0}\cdot\frac{\partial \boldsymbol{u}_1(z,t)}{\partial t} - \frac{qn_1(z,t)}{mN_0}\boldsymbol{e_1}(z,t)\,, \tag{6b}$$

where q and m are electron charge and mass, respectively.

2.1 Linear transformation of source EMW

Linear plasma theory analysis, analyzed by using the first order perturbation theory, gives that two transmitted and two reflected wave modes are generated due to interaction between LP source EMW and suddenly created magneto-plasma medium, with the following frequencies [2] (see also Fig.1 in [3]):

$$\omega_{1,2} = \sqrt{a \pm \sqrt{a^2 - b}}\ ; \quad a = \omega_p^2 + (\omega_0^2 + \omega_b^2)/2\ ;$$

$$b = \omega_p^4 + (\omega_p^2 + \omega_b^2)\cdot\omega_0^2\ , \tag{11}$$

where ω_0, $\omega_p = \left(qN_0/\varepsilon_0 m\right)^{1/2}$ and $\omega_p = qB_0/(\varepsilon_0 m)$ are the source wave, electron plasma and electron cyclotron angular frequencies, respectively. Transmitted waves have the angular frequencies ω_1 and ω_2 ($\omega_1, \omega_2 > 0$), whereas the reflected waves have angular frequencies - ω_1, - ω_2. Dispersion relation and dielectric constant have the following forms, respectively:

$$\omega^4 - \left(2\omega_p^2 + \omega_b^2 + k^2c^2\right)\omega^2 + k^2c^2\left(\omega_p^2 + \omega_b^2\right) + \omega_p^4 = 0\,, \tag{12a}$$

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2\left[1 + \omega_b^2 \Big/ \left(\omega_p^2 - \omega^2\right)\right]}\,. \tag{12b}$$

Term $n_1(z,t)$ is given in [4].

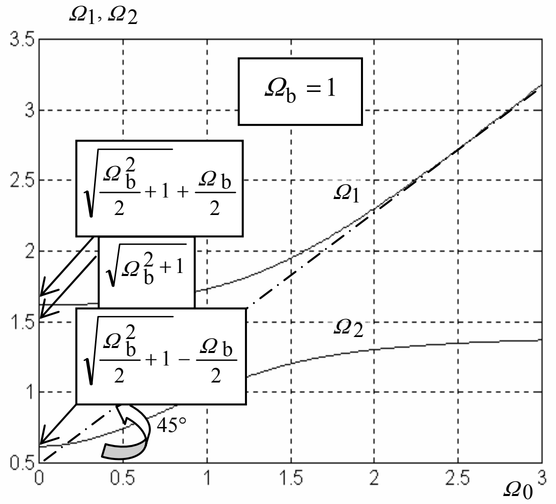

Figure 1. Normalized angular frequencies of newly created linear wave modes $\Omega_{1,2} = \omega_{1,2}/\omega_p$ versus normalized frequency of source wave $\Omega_0 = \omega_0/\omega_p$ for value of normalized cyclotron angular frequency $\Omega_b = \omega_b/\omega_p = 1$.

2.2 Nonlinear transformation of EM source wave

The weakly nonlinear fields are defined by Eqs. (3b)- (6b). The corresponding initial conditions are evaluated from the continuity of EM and velocity fields at $t = 0$ and assumption that newly created electrons in plasma are at rest at $t = 0$, i.e.:

$$\boldsymbol{e_2}(z,t=0^+) = \boldsymbol{h_2}(z,t=0^+) = \boldsymbol{u_2}(z,t=0^+) = 0 \tag{13}$$

In order to solve the system of partial differential equations (3b)-(6b), with the prescribed initial conditions given by Eq. (12), we shall apply the Laplace transform in time

$$\mathrm{L}(f(z,t)) = \int_0^\infty f(z,t)\exp(-st)\mathrm{d}t = F(z,s)\,, \tag{14a}$$

and, as the plasma is unbounded, Fourier transform in space

$$\mathrm{F}(f(z,t)) = \int_0^{\infty} f(z,t)\exp(\text{-j}k)\mathrm{d}z = F(k,s)\,. \quad (14b)$$

In the domain of complex frequency $s = \mathrm{j}\omega$ and wave number k, the EM and velocity fields are defined by the following system of linear algebraic equations, obtained from (3b)-(6b):

$$\mathrm{j}k\,E_{2x}(k,s) + \mu_0 s H_{2y}(k,s) = 0\,, \quad (3bx)$$

$$\text{-j}kH_{2y}(k,s) + N_0\,qU_{2x}(k,s) - \varepsilon_0\,s\,E_{2x}(k,s) = F(k,s)\,, \quad (4bx)$$

$$N_0 qU_{2z}(k,s)\text{-}\varepsilon_0 sE_{2z}(k,s) = G(k,s)\,, \quad (4by)$$

$$sU_{2x}(k,s)\text{-}\omega_b U_{2z}(k,s) + \frac{q}{m}E_{2x}(k,s) = H(k,s)\,, \quad (5bx)$$

$$sU_{2z}\ (k,s) + \omega_b V_{2x}(k,s) + \frac{q}{m}E_{2z}(k,s) = I(k,s)\,, \quad (5by)$$

$$sU_{2z}(k,s) + \frac{q}{m}E_{2z}(k,s) = J(k,s)\,, \quad (5bz)$$

where

$$F(k,s) = \mathrm{FL}\{\text{-}qn_1(z,t)u_{1x}(z,t)\}\,, \quad (15a)$$

$$G(k,s) = \mathrm{FL}\{\text{-}qn_1(z,t)u_{1z}(z,t)\}\,, \quad (15b)$$

$$H(k,s) = \mathrm{FL}\left\{\begin{array}{l} -u_{1z}(z,t)\dfrac{\partial u_{1x}(z,t)}{\partial t} + q\mu_0 u_{1z}(z,t)h_{1y}(z,t) \\ -\dfrac{q}{mN_0}n_1(z,t)e_{1x}(z,t) \\ -\dfrac{n_1(z,t)}{N_0}\cdot\dfrac{\partial u_{1x}(z,t)}{\partial t} + \dfrac{\omega_b}{N_0}n_1(z,t)u_{1z}(z,t) \end{array}\right\}, \quad (15c)$$

$$I(k,s) = \mathrm{FL}\left\{\begin{array}{l} -u_{1z}(z,t)\dfrac{\partial u_{1z}(z,t)}{\partial t} - q\mu_0 u_{1x}(z,t)h_{1y}(z,t) \\ -\dfrac{q}{mN_0}n_1(z,t)u_{1z}(z,t) \\ -\dfrac{n_1(z,t)}{N_0}\cdot\dfrac{\partial u_{1z}(z,t)}{\partial t} - \dfrac{\omega_b}{N_0}n_1(z,t)u_{1x}(z,t) \end{array}\right\}, \quad (15d)$$

$$J(k,s) = \mathrm{FL}\left\{\text{-}\frac{q\mu_0}{m}\left(u_{1x}(z,t)h_{1y}(z,t)\text{-}u_{1y}(z,t)h_{1x}(z,t)\right)\right\}. \quad (15e)$$

Operator FL in the above equations has the meaning of the Fourier-Laplace transformation.

In the domain of complex frequency s=jω and wave number k one should solve the obtained system of linear algebraic equations, (3bx)-(5bz), and perform the inverse Laplace and Fourier transforms. After these steps, nonlinear EM and electron fields in weakly nonlinear plasma are obtained in the following form:

$$e_{2x}(z,t) = E_{2x}^0 + \sum_{i=1}^{2} E_{2xi}^1\cos(\omega_{i\alpha}t) + \sum_{i=1}^{6} E_{2xi}^2\cos(\varphi_i t)$$
$$+\sum_{i=1}^{2} E_{2xi}^{2t,r}\cos(\omega_{i\beta}t \mp 2k_0 z) + \sum_{i=1}^{6} E_{2xi}^{4t,r}\cos(\varphi_i t \mp 2k_0 z)\,, \quad (16a)$$

$$h_{2y}(z,t) = H_{2y}^0\cos(2k_0 z) + \sum_{i=1}^{2} H_{2yi}^{3t,r}\cos(\omega_{i\beta}t \pm 2k_0 z) + \sum_{i=1}^{6} H_{2yi}^{4t,r}\cos(\varphi_i t \pm 2k_0 z)\,, \quad (16b)$$

$$e_{2z}(z,t) = E_{2z}^0\sin(2k_0 z) + \sum_{i=1}^{2} E_{2zi}^1\sin(\omega_{i\alpha}t) + \sum_{i=1}^{6} E_{2zi}^2\sin(\varphi_i t) + \sum_{i=1}^{2} E_{2zi}^{3t,r}\sin(\omega_{i\beta}t \mp 2k_0 z) + \sum_{i=1}^{6} E_{2zi}^{4t,r}\sin(\varphi_i t \mp 2k_0 z) \quad (17)$$

where

$$\varphi_1 = \omega_1\,;\ \varphi_2 = \omega_2\,;\ \varphi_3 = \omega_1\text{-}\omega_2\,;\ \varphi_4 = \omega_1 + \omega_2\,;$$
$$\varphi_5 = 2\omega_1\,;\ \varphi_6 = 2\omega_2\,;\ \omega_{i\alpha} = \omega_i(\omega_0 = 0)\,;$$
$$\omega_{i\beta} = \omega_i(\omega_0 \to 2\omega_0)\ \ i = 1,2\ , \quad (18)$$

(see Figs.2a and 2b). In Eqs. (16a)-(17) the t, r superscripts refer to transmitted and reflected wave modes, respectively. The upper sign in $\mp$ refers to the transmitted wave mode and the lower to the reflected.

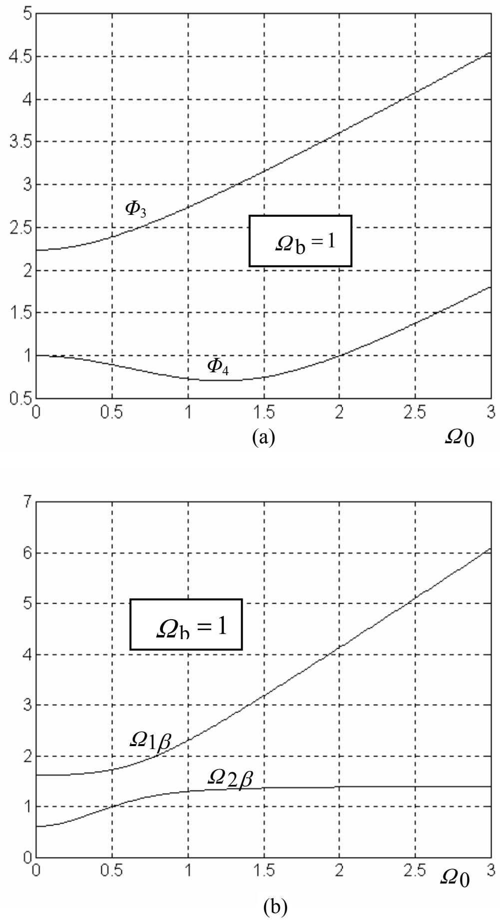

Figure 2. Normalized angular frequencies of newly created nonlinear wave modes $\Phi_{3,4} = \varphi_{3,4}/\omega_p$ (a) and $\Omega_{1,2\beta} = \omega_{1,2\beta}/\omega_p$ (b) versus normalized frequency of source wave $\Omega_0 = \omega_0/\omega_p$ for value of normalized cyclotron angular frequency $\Omega_b = \omega_b/\omega_p = 1$.

The effect of switching nonlinear magneto-plasma medium is creation of EM field with the following components: rectification electric mode E_{2x}^{0}, stationary space-varying magnetic mode with wave number $2k_0$, eight oscillating electric wave modes with angular frequencies $\omega_{i\alpha}$, $i=1,2$ and φ_i, $i\in[1,6]$, eight electric and magnetic transmitted and reflected wave modes with angular frequencies $\omega_{i\beta}$, $i=1,2$ and φ_i, $i\in[1,6]$,with wave number $2k_0$, and creation of electric electron plasma field with the following components: stationary space-varying mode with wave number $2k_0$, eight oscillating modes with angular frequencies $\omega_{i\alpha}$, $i=1,2$ and φ_i, $i\in[1,6]$, eight transmitted and reflected wave modes with angular frequencies $\omega_{i\beta}$, $i=1,2$ and φ_i, $i\in[1,6]$,with wave number $2k_0$.

In this paper the efficiency of excitation of electric second harmonics ($\pm\omega_{1,2}, 2k_0$) of EM field is analyzed. Distribution of amplitudes of these newly created wave modes in plasma, normalized to $E_{20}=qE_0^2/mc\omega_{\mathrm{p}}=586E_0^2/\omega_{\mathrm{p}}$ V/m, (for $E_0=100$ V/m and $\omega_{\mathrm{p}}=10^6$ Hz), versus the angular frequency of the source wave, normalized on electron plasma angular frequency ω_{p}, are presented in Figs.3a and 3b, respectively, thereon the electron cyclotron angular frequency is taken to be equal to angular plasma frequency.

3. NUMERICAL RESULTS

Inspecting carefully the Figs.1, 2a and 2b one concludes that resonant excitation is possible only for oscillating and wave modes having the angular frequency $\omega_{2\beta}$. For $\omega_{\mathrm{b}}=\omega_{\mathrm{p}}$, we get $\omega_{2\beta\,\mathrm{rezonant}}\approx 1,25\omega_{\mathrm{p}}$, i.e. the value of angular frequency frequency of the source wave which enables resonant excitation is approximately $\omega_{0\,\mathrm{rezonant}}\approx 0,63\omega_{\mathrm{p}}$.

3.1 Transmitted and reflected electric components ($\pm 2\omega_{1,2}$, $2k_0$) of EMW in plasma

Amplitudes of these components have the following form:

$$E_{2x5}^{4t,r}(\pm 2\omega_1, 2k_0)=\pm\frac{E_{20}}{32}\cdot X^{t,r}\left(\Omega_1,\Omega_0,\Omega_{\mathrm{b}}\right)\cdot \cdot\sum_{i=1}^{2}Y_{i\beta}\left(\Omega_{1\beta},\Omega_0,\Omega_{\mathrm{b}}\right)\cdot \cdot\left\{\begin{array}{l}\Omega_{\mathrm{b}}+\Omega_{1\beta}\left(\Omega_{1\beta}^2-1\right)\left[2\left(\Omega_1^2-1\right)\pm\Omega_1\Omega_{\mathrm{b}}\right]\\ +\Omega_1^2\Omega_{\mathrm{b}}^2+\Omega_{\mathrm{b}}\left(\Omega_{\mathrm{b}}^2-1\right)\left(\Omega_1^2-\Omega_{\mathrm{b}}^2-1\right)\\ +\Omega_{1\beta}\left(\Omega_1^2-1\right)\left(\Omega_{1\beta}^2-\Omega_{\mathrm{b}}^2-1\right)\end{array}\right\}, \quad (19)$$

$$E_{2x6}^{4t,r}(\pm 2\omega_2, 2k_0)=\pm\frac{E_{20}}{32}\cdot X^{t,r}\left(\Omega_1,\Omega_0,\Omega_{\mathrm{b}}\right)\cdot \cdot\sum_{i=1}^{2}Y_{i\beta}\left(\Omega_{1\beta},\Omega_0,\Omega_{\mathrm{b}}\right)\cdot \cdot\left\{\begin{array}{l}\Omega_0+\Omega_{i\beta}\left(\Omega_{i\beta}^2-1\right)\left[2\left(\Omega_2^2-1\right)\pm\Omega_2\Omega_{\mathrm{b}}\right]\\ +\Omega_2^2\Omega_{\mathrm{b}}^2+\Omega_{\mathrm{b}}\left(\Omega_2^2-1\right)\left(\Omega_2^2-\Omega_{\mathrm{b}}^2-1\right)\\ +\Omega_{i\beta}\left(\Omega_2^2-1\right)\left(\Omega_{i\beta}^2-\Omega_{\mathrm{b}}^2-1\right)\end{array}\right\}, \quad (20)$$

where the superscripts t, r refer to transmitted and reflected components, respectively. The upper sign in $\pm$ refers to transmitted and lower to reflected component, and

$$X^{t,r}\left(\Omega_{1,2},\Omega_0,\Omega_{\mathrm{b}}\right)=\frac{\Omega_0\Omega_{\mathrm{b}}\left(\Omega_{1,2}\pm\Omega_0\right)}{\Omega_{1,2}^2\left(\Omega_{1,2}^2-a\right)^2}, \quad (21)$$

$$Y_{i\beta}=\left[\Omega_{i\beta}\left(\Omega_{i\beta}^2-A\right)\right]^{-1}, \quad A=1+\frac{4\Omega_0^2+\Omega_{\mathrm{b}}^2}{2}, \quad (22)$$

where Ω_0, Ω_{b}, $\Omega_{1,2}$ and $\Omega_{i\beta}$ are normalized angular frequencies, on electron plasma angular frequency ω_{p}.

The relative power content of these wave modes is given by:

$$S_{2x5}^{4t,r}=E_{2y5}^{4t,r}\cdot H_{2y5}^{4t,r} \text{ and } S_{2x6}^{4t,r}=E_{2y6}^{4t,r}\cdot H_{2y6}^{4t,r}, \quad (23)$$

where

$$H_{2y5}^{4t,r}=\pm\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot\frac{\Omega_0}{\Omega_1}\cdot E_{2x5}^{4t,r},$$

$$H_{2y6}^{4t,r}=\pm\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot\frac{\Omega_0}{\Omega_1}\cdot E_{2x6}^{4t,r}. \quad (24)$$

Figs.3a and 3b show that the amplitudes of transmitted waves are much greater than the amplitudes of reflected waves.

The peak value of the amplitude of transmitted wave $E_{2x5}^{4t}(2\omega_1,2k_0)\approx 0,37E_{20}\approx 2,20$ V/m is obtained for value $\Omega_0=1,7$. Its relative power content, for that value of angular frequency of the source wave, is about 15% ($(S_{2x5}^{4t}/S_0)/E_{20}=0,025$, i.e. $(S_{2x5}^{4t}/S_0)\approx 0,15$, see Fig.4a). The relative power content of the corresponding reflected wave is negligible (see Fig.4b).

The peak value of the amplitude of transmitted wave $E_{2x5}^{4t}(2\omega_1,2k_0)\approx 0,34E_{20}\approx 1,99$ V/m is obtained for $\Omega_0=1,9$. Its relative power content, for this value of angular frequency of the source wave, is about 9% ($(S_{2x6}^{4t}/S_0)/E_{20}=0,016$, i.e. $(S_{2x6}^{4t}/S_0)\approx 0,09$, see Fig.4a). The relative power content of corresponding reflected wave is negligible (see Fig.4(b)).

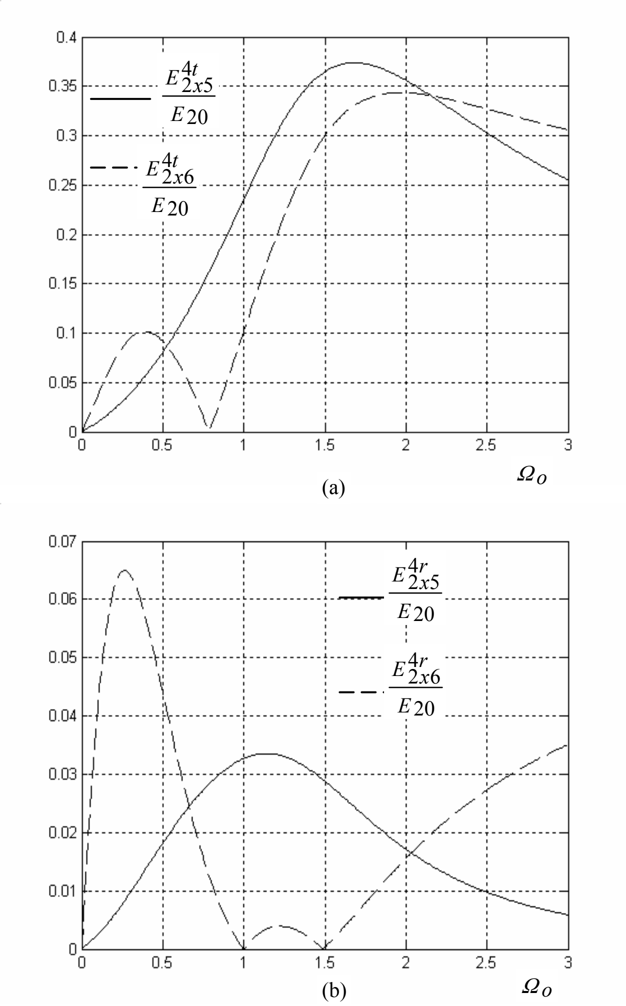

Figure 3. Normalized amplitudes of excited second-harmonic (transmitted (a) and reflected (b)) wave modes ($k = 2k_0$) with angular frequencies $\pm\omega_{1,2}$ versus normalized frequency of source wave $\Omega_0 = \omega_0 / \omega_p$ for value of normalized cyclotron angular frequency $\Omega_b = \omega_b / \omega_p = 1$.

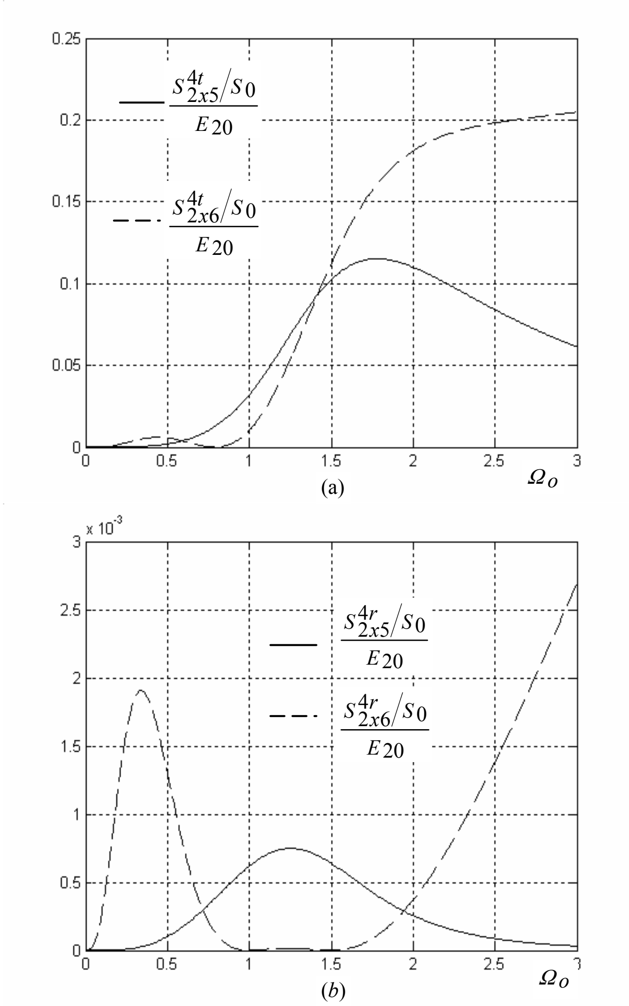

Figure 4. Normalized relative power content of the second-harmonic, transmitted (a) and reflected (b), wave modes ($k = 2k_0$) with angular frequencies $\pm\omega_{1,2}$ versus normalized frequency of source wave $\Omega_0 = \omega_0 / \omega_p$ for value of the normalized cyclotron angular frequency $\Omega_b = \omega_b / \omega_p = 1$.

4. CONCLUSION

The initial value problem of interaction of LP EM source wave with suddenly created weakly nonlinear magneto-plasma medium, in particular case of transverse propagation, is solved in the closed form in the paper. The nonlinearities caused by the interaction of newly created linearly magnetic and velocity modes in plasma and perturbation of electron density on direction of waves propagation are taken into account in the equation of continuity, Maxwell equation for space variation of magnetic field and equation of electron fluid motion. LP EM source wave in suddenly created *linear cold magneto-plasma* splits in two transmitted (with angular frequencies ω_1, ω_2) and two reflected (with angular frequencies $-\omega_1$, $-\omega_2$) traveling waves. In the case of weakly *nonlinear cold magneto-plasma* eight transversal EM and eight longitudinal electron oscillations ($k = 0$) with the following angular frequencies: $\varphi_1 = \omega_1$, $\varphi_2 = \omega_2$, $\varphi_3 = \omega_1 - \omega_2$, $\varphi_4 = \omega_1 + \omega_2$, $\varphi_5 = 2\omega_1$, $\varphi_6 = 2\omega_2$ and $\omega_{i\alpha} = \omega_i(\omega_0 = 0)$, ($i = 1,2$); sixteen transversal EM and sixteen longitudinal electron wave modes (eight transmitted plus eight reflected) ($k = 2k_0$) with angular frequencies φ_i, $i \in [1,6]$ and $\omega_{i\beta} = \omega_i(\omega_0 \to 2\omega_0)$, ($i = 1,2$), are excited. Additional stationary space-varying electron longitudinal and EM transversal wave mode ($k = 2k_0$) with zero value of angular frequency ($\omega = 0$) and stationary EM rectification mode ($\omega = 0$, $k = 0$) with angular frequencies are excited. The efficiency of creation of all of these modes can be controlled by the source wave angular frequency and the magnitude of external static magnetic field. The resonant transformation of LP source wave is noticed in the case when the angular frequency of newly created traveling and oscillating modes has value equal to $\omega_{2\beta}$.

REFERENCES

[1] Kalluri, D.K.,*Electromagnetics of Complex Media*, CRC, Boca Raton, 1999.

[2] Goteti, V.R. and Kalluri, D.K, Wave propagation in a switched magnetoplasma medium: Tranverse propagation, Radio Sci., 26, 61, 1990.

[3] Trifković, Z., Stanić, B., Frequency shift of a source EMW due to sudden changes of magnetized plasma. Transverse propagation, *ETRAN XLVII*, Contributed papers, Vol. 2, p. 213-215, 8-13 jun 2003, Herceg Novi.

[4] Trifković, Z., *Influnce analysis of time discontinuity on EMW transformation in cold weakly nonlinear magnetized plasma*, PhD Thesis, Faculty of Electrical Engineering, Belgrade, 2002.

ГЕНЕРИСАЊЕ ДРУГОГ ХАРМОНИКА ПРИ НЕЛИНЕАРНОЈ ТРАНСФОРМАЦИЈИ ЕЛЕКТРОМАГНЕТСКОГ ТАЛАСА У НАГЛО СТВОРЕНОЈ МАГНЕТИЗОВАНОЈ ХЛАДНОЈ ПЛАЗМИ. ТРАНСВЕРЗАЛНО ПРОСТИРАЊЕ

Зоран Трифковић, Божидар Станић

У раду је анализирана трансформација изворног линеарно поларизованог електромагнетског таласа, услед слабих нелинеарности, у електронске и електромагнетске осцилације, стационарне (просторно промењљиве и једносмерне) и таласне електромагнетске и електронске модове у плазми, применом пертурбационе теорије другог реда. Ефикасност екситације трансверзалних електричних модова, двоструко веће фреквенције и двоструко већег таласног броја од модова добијених у линеарној теорији, анализирана је за различите вредности угаоне фреквенције изворног таласа и за вредност угаоне електронске жиро-фреквенције која одговара вредности угаоне електронске пазмене фреквенције.