Optimal Control of Redundant Robots in Human-Like Fashion

Mihailo Lazarević
Assistant Professor
University of Belgrade
Faculty of Mechanical Engineering

This paper suggests a new optimal control of a redundant robotic system. It is achieved using suitable kinematic and dynamic criteria based on biological principles, i.e. in human-like fashion. Here, a dynamical model of robotic system is given in the form of Langrange`s equations of second kind in covariant form. Several criteria are introduced which are the function of generalized coordinates, velocities, accelerations and control vectors, respectively. Finally, the effectiveness of suggested optimal control in human-like fashion is demonstrated with a robot with four degrees of freedom as the illustrative example.

Keywords: *redundant, robot, optimal control, biological analog*

1. INTRODUCTION

Industrial robots perform various tasks improving the quality and efficiency of manufacturing. Some complex industrial – and especially nonindustrial tasks – recently induced a new approach to robot design and control in order to achieve very stable, fast, and accurate systems. For example, these are industrial assembly, high-speed manipulation, robotized surgery, etc. They also replace human workers in tasks that may jeopardize human safety and health. Such demanding tasks could efficiently be solved if a robot was configured as redundant. A robotic manipulator is called kinetically redundant if it has more degrees of freedom (DOF) than required for a realization of a prescribed task in a task space. The operation usually involves a prescribed motion task and it is clear that the level of redundancy depends on the task. So, with one task the system can be redundant, while with some other it is not. The kinematic redundancy in a manipulator structure yields increased dexterity and versatility and also allows avoiding collisions with obstacles by the choice of appropriate configurations. Also, the acceleration of massive segments in redundant mechanism leads to drive overload and required redundancy, [1],[2]. The main difficulty of redundant robots is that the task cannot define the joint motions uniquely. In other words, the main question is how to choose a suitable mechanism configuration from the infinite number of possible configurations called "self-motions", which match each position of the manipulation object, for a prescribed point of the end-effector in a task space. There are two major approaches to solve this problem. One is to impose certain mathematical constraints on changes in kinematic parameters. For example, Bailleul [3] complemented the given set of equations by choosing additional set of constraints, and this method is referred to as the "extended Jacobian method". The other approach is based on the possibility of optimizing manipulator motion, provided that the motion of the end-effector is prescribed. The standard methods to deal with this problem are divided into two groups, i.e. global and local methods, according to the criterion of what is needed to know in advance about the operational space trajectory in order to find an appropriate solution [4]. So, the optimization problem for redundant robots is stated as following: given a prescribed motion of the end-effector of the manipulation robot, find the motion of the robot so as to minimize either a scalar function of the state variables at each time instant (the local optimization),[5] or a functional that depends on the motion as a whole (the global optimization) [6]. Different kinematic or dynamic optimization criteria could be introduced to achieve the unique solution of the inverse kinematics, such as: the kinetic energy, the sum of squared generalized velocities, total driving power, potential energy, etc.. For the local schemes many researchers have traditionally used the generalized or pseudoinverse of the manipulator Jacobian matrix as a central tool in redundancy resolution.

Most of these works are based on the local (i.e. at any given moment) optimization of certain objective functions. Whitney [7] has minimized the kinetic energy of the manipulator. Liegeois [5] used the appropriate vector from the null space to improve the pseudoinverse control i.e. presented a gradient projection method by using the homogeneous solution, a vector in the null space of the Jacobian. Hanafusa et al. [8] used the pseudoinverse of the Jacobian matrix to obtain optimal joint velocities while avoiding obstacles. Konstantinov et al. [9] used the concept of the generalized inverse to deal with inequality constraints on the joint values. Klein [10] investigated redundancy as an effective tool to avoid obstacles while optimizing the joint rates and Kazerounian [11] used the local level of joint rates as a weight factor to minimize the power consumption. Utilization of redundancy for singularity avoidance has

Received: September 2004, Accepted: May 2005.
Correspondence to: Mihailo Lazarević
Faculty of Mechanical Engineering,
Kraljice Marije 16, 11120 Belgrade 35, Serbia and Montenegro
E-mail: mlazarevic@mas.bg.ac.yu

also been attacked by several authors ([12],[13], [14]).Also, optimization of driving forces was also considered by Kazerounian and Nedungadi, [15]. A major disadvantage of the local method is its unpredictable behavior resulting from poor choices of cost which can lead to instability but, need only information about the instantaneous position of end-effector.

Also, a few papers have been presented on global optimization concepts. Uchiyama et al. [16] optimized an integrant-type performance criterion on the determinant of the Jacobian to increase the dexterity of the arm while executing the trajectory. Nakamura and Hafanusa [17] have developed solutions for global optimization of general objective function based on Pontryagin's maximum principle and Suh and Hollerbach [18] offered a solution for global torque optimization by using the calculus of variations. Disadvantages of global schemes are complex and currently applied to off-line programming,[11],[19]. Moreover, new classification of redundancy is suggested by Lazarević [20]. For the purpose of systematization and a clear insight in features and capabilities of redundant systems, a general overview of previous results is proposed, which are related to these systems.So, redundancy which is used in robotic systems can be considered from different points of view. For instance, redundancy can be treated from the *kinematical* point of view in following cases: higher degree of mobility-measure of manipulability, the avoidance of mechanical limits in robot joints and joint velocities, the avoidance of obstacles, the avoidance of kinematical singularities, etc.If it is taken in consideration of dynamics of redundant system, one can treat the redundancy from the *dynamical* point of view which can allow: minimizing consumption of kinetic energy, minimizing of driving joint torques, increasing of dynamical capabilities, minimizing time of prescribed motion etc. Besides these two approaches there are cases where it is not possible to clearly observe redundancy i.e. where a quality control of redundancy is important. In such cases redundancy can be studied from the *control* point of view. For example, there are cases where accuracy or precisely prescribed motions are important, tasks with degree of priority, applying distributed positioning [DP], appearance of algorithmic singularities etc.

The other idea for obtaining unique solution of the inverse kinematics is to imitate human behavior.This is especially convenient for tasks that are similar to those performed by humans (e.g., assembly in industry, health services and different jobs at home). From a mechanical point of view, any human or animal represents a redundant mechanism, [21]. The main role of redundancy is to provide the flexibility of maneuvering space.For instance, this property allows the arm to avoid obstacles while performing a manipulation task. In some tasks, the redundancy does not need, but it exists and should be compensated. In biological systems it is noticed that this problem is usually solved by using the concept of synergy, [22]. Such behaviors of organisms can be only explained by the existence of inherent optimization laws in self-organized systems governing the acquisition of motor skills. In this paper, resolving redundancy and obtaining optimal control using kinematic and dynamic optimization criteria based on biological analog will be presented. In that way, proposed local kinematic and dynamic criteria will be suitable for on-line use in robotics.

2. FUNDAMENTALS OF KINEMATICS AND DYNAMICS OF REDUNDANT ROBOTS

Here, a redundant robot arm is considered as an open linkage consisting of $n+1$ rigid bodies interconnected by n one-degree-of–freedom (DOF) joints (Fig.1). The joints are modeled as a kinematic pairs of V`s class and so the arm has n degrees of freedom.

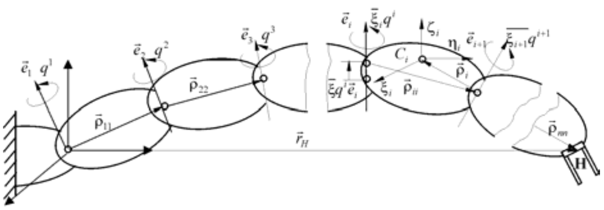

Figure 1. Redundant robot as open linkage with n one-degree-of - freedom joints

Let, the position of the redundant robot be defined by the vector joint of (internal) generalized coordinates q of dimension n, $\{q\}=\left(q^1,q^2,...,q^n\right)^T$. The vector of global (external) coordinates of dimension defines the position of the terminal device of redundant robot $m<n, m\leq 6$, $\{\overline{q}\}=\left(q^1,q^2,...,q^m\right)^T$. The kinematic model of the presented redundant robot is given by the following expression:

$$\overline{q}^i(t)=f^i(q^1,q^2,...,q^n),\ i=1,2,...,m \qquad (1)$$

Moreover, equation (1) is well known as the *direct kinematic problem* (calculation of $\overline{q}^i(t)$ for given $q(t)$) and has a unique solution. However, the *inverse kinematics* (calculation of $q(t)$ for given $\overline{q}^i(t)$) has an infinite number of solutions since equation (1) represents a set of m equations with n variables due to the redundancy.The dimension of redundancy is $n_r=n-m$. Dynamical model of robotic system can be described in covariant form:

$$\sum_{\alpha=1}^{n} a_{\alpha\gamma}(q)\ddot{q}^{\alpha}+\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Gamma_{\alpha\beta,\gamma}(q)\dot{q}^{\alpha}\dot{q}^{\beta}=Q_{\gamma}^{a}+Q_{\gamma}^{u}, \quad \gamma=1,2,...,n \qquad (2)$$

where kinetic energy of robotic system is given by:

$$E_k = \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha\beta}\dot{q}^{\alpha}\dot{q}^{\beta} = \frac{1}{2}(\dot{q})\left[a_{\alpha\beta}\right]\{\dot{q}\},$$
$$\alpha,\beta = 1,2,...,n, \tag{3}$$
$$\left[a_{\alpha\beta}\right]\in R^{n\times n}.$$

Coefficients $a_{\alpha\beta} = a_{\beta\alpha}$ of square form are covariant coordinates of basic metric tensor $\left[a_{\alpha\beta}\right]\in R^{n\times n}$:

$$a_{\alpha\beta} = \sum_{i=1}^{n} m_i\left(\vec{T}_{\alpha(i)}\right)\left\{\vec{T}_{\beta(i)}\right\} + \sum_{i=1}^{n}\left(\vec{\Omega}_{\alpha(i)}\right)\left[J_{Ci}\right]\left\{\vec{\Omega}_{\beta(i)}\right\} \tag{4}$$

Also, $\Gamma_{\alpha\beta,\gamma} = \Gamma_{\beta\alpha,\gamma}$, $\alpha,\beta,\gamma = 1,2,...,n$ present Christoffel symbols of first kind as:

$$\Gamma_{\alpha\beta,\gamma} = \frac{1}{2}\left(\frac{\partial a_{\beta\gamma}}{\partial q^{\alpha}} + \frac{\partial a_{\gamma_{\alpha}}}{\partial q^{\beta}} - \frac{\partial a_{\alpha\beta}}{\partial q^{\gamma}}\right), \tag{5}$$

or [23]

$$\Gamma_{\alpha\beta,\gamma} = \sum_{i=1}^{n} m_i \frac{\partial^2 \vec{r}_{Ci}}{\partial q^{\alpha}\partial q^{\beta}}\cdot\frac{\partial \vec{r}_{Ci}}{\partial q^{\gamma}} + \sum_{i=1}^{n}\int_{(Vi)}\frac{\partial^2 \vec{\rho}_i}{\partial q^{\alpha}\partial q^{\beta}}\cdot\frac{\partial \vec{\rho}_i}{\partial q^{\gamma}}dm_i\ , \tag{6}$$

At last, Q^u_{γ} control part of generalized forces Q and Q^a_{γ} non-control part of Q are given such as, [24]:

$$Q^a_{\gamma} = \sum_{\gamma=1}^{n}\left(\vec{F}_{R(i)}\cdot\vec{T}_{\gamma(i)} + \vec{M}_{CiR(i)}\cdot\vec{\Omega}_{\gamma(i)}\right), \tag{7}$$

$$Q^u_{\gamma} = \left(\bar{\xi}_{\gamma}\vec{M}_{\gamma} + \vec{P}_{\gamma}\xi_{\gamma}\right)\cdot\vec{e}_{\gamma}, \quad \gamma = 1,2,...,n. \tag{8}$$

In prismatic joints forces are acting, while in revolute joints torques are acting. For complete dynamics of redundant robot, taking into account dynamic actuators is also necessary.For example, here DC motors will be adopted as driving units. The dynamics of the motors, the motor that drives the joint $()^{\alpha}$ satisfies the electrical and mechanical equilibrium equations, [25]:

$$u_{\alpha} = R_{\alpha}i_{\alpha} + L_{\alpha}\left(di_{\alpha}/dt\right) + K_{E\alpha}\dot{\varphi}_{\alpha}, \tag{9}$$

$$K_{M\alpha}i_{\alpha} = I_{\alpha}\ddot{\varphi}_{\alpha} + B_{\alpha}\dot{\varphi}_{\alpha} + M_{\alpha}, \tag{10}$$

where u_{α} is the input control voltage, i_{α} is the current in the motor winding, R_{α}, L_{α}, are resistance and inductivity, respectively, φ_{α} is the angle of the motor shaft, $K_{E\alpha}$, $K_{M\alpha}$ are the back e.m.f and torque constants, I_{α} is the motor moment of inertia, B_{α} is the viscous friction coefficient, and M_{α} is the motor output torque. Between the motor shaft and the joint shaft, there usually exists some transmission, which can be modeled as a linear relation between the motor variables and joint variables:

$$\varphi_{\alpha} = N_{\alpha}q_{\alpha},\ M_{\alpha} = Q_{\alpha}/N_{\alpha}, \tag{11}$$

In addition, robot is completely controllable if (necessary and sufficient conditions) the following inequalities are satisfied, [26]:

$$g_i = \sup\left|Q^a_{\gamma}\right| < h_{\gamma}, \quad \gamma = 1,2,...,n\ . \tag{12}$$

3. RESOLVING REDUNDANCY USING LOCAL OPTIMIZATION OF A KINEMATIC CRITERION

3.1 Resolving redundancy using local optimization of a geometrical-based kinematic criterion

Generally, there are two approaches to solve inverse kinematical problem. One is to impose $l = n - m$ additional set of constraints, i.e:

$$f^{\nu}(q^1, q^2,...,q^n) = 0,\ \nu = 1,2,...,l = n - m\ . \tag{13}$$

Now, one can have a system of algebraic equations

$$\bar{q}^{\lambda}(t) = f^{\lambda}(q^1, q^2,...,q^n),\ \lambda = 1,2,...,m\ , \tag{14}$$

$$0 = f^{\lambda}(q^1, q^2,...,q^n),\ \lambda = m+1, m+2,...,n\ , \tag{15}$$

which is closed and it is possible to solve a given system of equations.One of the possible ways of solving previous problem is linearization around working point i.e

$$\left\{\dot{\bar{q}}(t)\right\} = \left[\frac{\partial f^{\nu}}{\partial q^{\alpha}}\right]_{\substack{\nu=1,2,..,m\\ \alpha=1,2,...,n}}\{\dot{q}(t)\} = \left[J_{OS}(q)\right]\{\dot{q}(t)\} \tag{16}$$

Differentiation equation (13) in respect to time yields

$$\{0\} = \left[\frac{\partial f^{\nu}}{\partial q^{\alpha}}\right]_{\substack{\nu=1,2,...,l\\ \alpha=1,2,...,n}}\{\dot{q}\} = \left[J_{DOP}\right]\{\dot{q}\} \tag{17}$$

Combining equations (16) and (17) one can form extended system as follows:

$$\left\{\dot{\bar{q}}^{\bullet}(t)\right\} = \begin{Bmatrix}\dot{q}(t)\\ 0\end{Bmatrix} = \begin{bmatrix}J_{OS}(q)\\ J_{DOP}(q)\end{bmatrix}\{\dot{q}(t)\} = \left[J_{EXT}(q)\right]\{\dot{q}(t)\} \tag{18}$$

If $\det\left[J_{EXT}(q)\right] \neq 0$ then exists $\left[J_{EXT}\right]^{-1}$ and one determine

$$\{\dot{q}(t)\} = \left[J_{EXT}(q)\right]^{-1}\left\{\dot{\bar{q}}(t)\right\} \tag{19}$$

Moreover, also in time instant $t + \Delta t$, it follows:

$$\{q(t+\Delta t)\} = \{q(t)\} + \{\dot{q}(t)\}\Delta t \tag{20}$$

The other approach is based on possibility of using a biological concept-sinergy. This notation introduces some relationships between the motions of different DOFs [1], [27], [28], [29]. From the standpoint of mechanism operation, synergy means that a group of DOFs operates together behaving as one-DOF subsystem. A representative example is the human finger that has four DOFs where each joint can move separately. But, in practical operation the tip joints (3 and 4) work together forming one-DOF subsystem (Fig. 2).

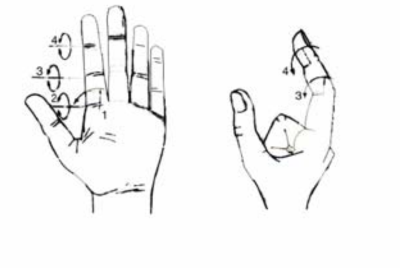

Figure 2. Human fingers motions using synergy

In fact, such behavior implies that is obeys the optimization at the coordination level where the goal is to minimize efforts in terms of synergy patterns. Speaking mathematically, the synergy imposes specific constraints on the control variables of joints, [30],[31]. As a result, one can suppose that an additional set of constraints can be realized using optimization procedure with applying suitable quadratic criterion as:

$$I=\sum_{\nu=1}^{l}\left(f^{\nu}(q^{1},q^{2},...,q^{n})\right)^{2}\rightarrow\min_{q} I\,. \tag{21}$$

It is assumed that functions f^{ν} define in area Ω where first partial derivatives are continuous in respect to $\left(q^{1},q^{2},q^{3},...,q^{n}\right)$. Also, an assumption that only one solution exists is introduced. Applying the theory of optimization one can obtain necessary conditions of optimality in this case

$$\begin{aligned}&\frac{\partial I}{\partial q^{\alpha}}=2\sum_{\nu=1}^{l}f^{\nu}(q^{1},q^{2},...,q^{n})\frac{\partial f^{\nu}}{\partial q^{\alpha}}=0\,.\\&\alpha=1,2,...,n\end{aligned} \tag{22}$$

Taking into account previous assumption as

$$\frac{\partial f^{\nu}}{\partial q^{\alpha}}\neq 0,\quad \forall\nu=1,2,...,l\quad \alpha=1,2,...,n\quad . \tag{23}$$

From equation (11), it yields:

$$f^{\nu}(q^{1},q^{2},...,q^{n})=0,\quad \nu=1,2,...,l=n-m \tag{24}$$

Sufficient conditions of optimality to achieve a minimum are

$$\frac{\partial^{2}I}{\partial\left(q^{\alpha}\right)^{2}}\geq 0 \tag{25}$$

and in proposed case it yields

$$\frac{\partial^{2}I}{\partial\left(q^{\alpha}\right)^{2}}=\sum_{\nu=1}^{l}\left(\frac{\partial f^{\nu}}{\partial q^{\alpha}}\right)^{2}\geq 0 \tag{26}$$

For practical use, one can introduce a Δq based local kinematic criterion in form weighted minimum norm least-squares:

$$I=\frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}w_{\alpha\beta}\Delta q^{\alpha}\Delta q^{\beta}\rightarrow\min_{\Delta q^{j}}, \tag{27}$$

and constraints, which is linearized of equation (13) as:

$$\Delta\bar{q}^{i}=\sum_{\alpha=1}^{n}\frac{\partial f^{i}}{\partial q^{\alpha}}\Delta q^{\alpha},\ i=1,2,...,m \tag{28}$$

Optimal solution can be obtained using optimimization method with unknown Langrange multipliers. The augmented objective function I_{a} is defined as:

$$\begin{aligned}I_{a}=&\frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}w_{\alpha\beta}\Delta q^{\alpha}\Delta q^{\beta}+\\&+\sum_{i=1}^{m}\lambda_{i}\left(\Delta\bar{q}^{i}-\sum_{\alpha=1}^{n}\frac{\partial f^{i}}{\partial q^{\alpha}}\Delta q^{\alpha}\right)\rightarrow\min_{\Delta q_{j},\lambda}\end{aligned} \tag{29}$$

Necessary conditions for optimality are:

$$\begin{aligned}&\frac{\partial I_{a}}{\partial\Delta q^{j}}=0,\ j=1,2,...,n\\&\frac{\partial I_{a}}{\partial\lambda_{i}}=0,\ i=1,2,..,m\end{aligned} \tag{30}$$

or,

$$\begin{aligned}&\sum_{\alpha=1}^{n}w_{\alpha\gamma}\Delta q^{\alpha}=\sum_{i=1}^{m}\frac{\partial f^{i}}{\partial q^{\gamma}}\lambda_{i},\quad \gamma=1,2,...,n\\&\Delta\bar{q}^{i}=\sum_{\alpha=1}^{n}\frac{\partial f^{i}}{\partial q^{\alpha}}\Delta q^{\alpha},\qquad i=1,2,...,m\end{aligned} \tag{31}$$

After solving first of these equations in respect to Δq^{α} one can obtain:

$$\Delta q^{\alpha}=\sum_{\alpha=1}^{n}\sum_{i=1}^{m}w^{\bullet}_{\alpha\gamma}\frac{\partial f^{i}}{\partial q^{\gamma}}\lambda_{i},\quad \gamma=1,2,...,n \tag{32}$$

where is $\left[w_{\alpha\gamma}\right]^{-1}=\left[w^{\bullet}_{\alpha\gamma}\right]$, $\alpha,\gamma=1,2,...,n$ and

$$\Delta\bar{q}^{i}=\sum_{i=1}^{m}\lambda_{i}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}w^{\bullet}_{\alpha\gamma}\frac{\partial f^{i}}{\partial q^{\alpha}}\frac{\partial f^{i}}{\partial q^{\beta}},\quad \gamma=1,2,...,n \tag{33}$$

Previous equations (31) can be presented in condensed form:

$$\begin{aligned}&\{\Delta q\}=[W]^{-1}[J]^{T}\{\lambda\}\\&\{\Delta\bar{q}\}=[J]\{\Delta q\}\end{aligned}, \tag{34}$$

after solving in respect to $\lambda,\ \Delta q$ it yields:

$$\{\lambda\}_{opt}=\left([J][W]^{-1}[J]^{T}\right)^{-1}\{\Delta\bar{q}\}$$

$$\{\Delta q\}_{opt} = [W]^{-1}[J]^T\left([J][W]^{-1}[J]^T\right)^{-1}\{\Delta\bar{q}\} = \\ = \left[J_W^{PI}\right]\{\Delta\bar{q}\} \quad (35)$$

where $\left[J_W^{PI}\right]$ denotes a generalized pseudoinverse Jacobean matrix $[J]$ and it is presented as follows:

$$\left[J_W^{PI}\right] = [W]^{-1}[J]^T\left([J][W]^{-1}[J]^T\right)^{-1}. \quad (36)$$

Vectors of joint velocities and joint accelerations can be calculated applying, for instance, method of finite differences as:

$$\dot{q}_{opt}(t) = \frac{q_{opt}(t) - q_{opt}(t-\Delta t)}{\Delta t} = \frac{\Delta q_{opt}(t)}{\Delta t},$$

$$\ddot{q}_{opt}(t) = \frac{\dot{q}_{opt}(t) - \dot{q}_{opt}(t-\Delta t)}{\Delta t}. \quad (37)$$

At last, optimal control part Q_{opt}^u of generalized forces Q directly follows from dynamical model of robotic system in covariant form (2) where are previously calculated

$$a_{\alpha\gamma}(q_{opt}), \quad \Gamma_{\alpha\beta,\gamma}(q_{opt}), \quad \gamma = 1,2,...,n,$$

$$\alpha = 1,2,...,n, \quad \beta = 1,2,...,n, \quad (38)$$

3.2 Resolving redundancy by using local optimization of a velocity-based kinematic criterion

Also, the most commonly used optimization criterion is given in the form of scalar quadratic function:

$$I = \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} w_{\alpha\beta}\dot{q}^\alpha\dot{q}^\beta \to \min_{\dot{q}^j}, \quad (39)$$

or in a condensed form

$$I = \frac{1}{2}(\dot{q})[W]\{\dot{q}\} \to \min_{\dot{q}} \quad (40)$$

where $[W]$ is a symmetric positive-definite weighting matrix. Justification of introducing this criterion can be observed by the fact that if one replaces matrix $[W]$ with basic metric tensor $\left[a_{\alpha\beta}\right] \in R^{n\times n}$, a criterion that presents a kinetic energy of redundant robot such as:

$$E_k = \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} a_{\alpha\beta}\dot{q}^\alpha\dot{q}^\beta = \frac{1}{2}(\dot{q})\left[a_{\alpha\beta}\right]\{\dot{q}\}, \\ \alpha,\beta = 1,2,...,n \quad (41)$$

is obtained, where matrix $\left[a_{\alpha\beta}\right]$ is also positive-definite. As a result, solution with minimal consumption of kinetic energy is obtained. Also, in the paper [32] it is shown that a joint angle synergy in control of arm movements exits.

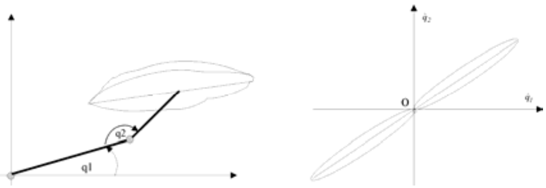

Figure 3. Human arm motions using synergy between shoulder and elbow

A simplified strategy for control of an anthropomorphic manipulator with two DOFs was analyzed. It is found that a synergy exists between joint angles in analyzed movements, given as a linear scaling parameter between the elbow and the shoulder angular velocities (see Fig.3). As a consequence, a biological velocity-based quadratic kinematic criterion is proposed here. Moreover, criterion is given by the following expression, [33]:

$$I = \frac{1}{2}\left\{\left(\sum_{\alpha=1}^{n} w_\alpha\dot{q}^\alpha\right)^2 + (\dot{q})[S]\{\dot{q}\}\right\} \to \underset{\dot{q}_j}{ext}, \quad (42)$$

where are w_α, $\alpha = 1,2,...,n$ presents weighted coefficients and matrix $[S] = \mathrm{diag}\left[a_{\alpha\beta}\right]_{q=\mathrm{const}}$, $\alpha,\beta = 1,2,...,n$ appropriate dimensions. Introducing into account constraints (1) which can be written in the Jacobian form of first order:

$$\dot{\bar{q}}^i = \sum_{\alpha=1}^{n}\frac{\partial f^i}{\partial q^\alpha}\dot{q}^\alpha \Rightarrow \{\dot{\bar{q}}\} = [J]\{\dot{q}\} \quad (43)$$

optimal trajectories can be obtained using the same procedure as in part 3.1. The augmented criterion is now:

$$I_a = \frac{1}{2}\left(\sum_{\alpha=1}^{n} w_\alpha\dot{q}^\alpha\right)^2 + \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} s_{\alpha\beta}\dot{q}^\alpha\dot{q}^\beta + \\ + \sum_{i=1}^{m}\lambda_i\left(\dot{\bar{q}}^i - \sum_{\alpha=1}^{n}\frac{\partial f^i}{\partial q^\alpha}\dot{q}^\alpha\right) \to \min_{\dot{q},\lambda} \quad (44)$$

Also, one can use a similar form of necessary conditions:

$$\frac{\partial I_a}{\partial \dot{q}^j} = 0, \; j = 1,2,...,n \\ \frac{\partial I_a}{\partial \lambda_i} = 0, \; i = 1,2,..,m \quad (45)$$

or

$$a_{jj}\dot{q}^j + w_j\left(\sum_{\alpha=1}^{n} w_\alpha\dot{q}^\alpha\right) + \sum_{i=1}^{m}\frac{\partial f^i}{\partial q^j}\lambda_i = 0, \\ j = 1,2,...,n \quad (46)$$

$$\dot{\bar{q}}^i = \sum_{\alpha=1}^{n} \frac{\partial f^i}{\partial q^\alpha} \dot{q}^\alpha, \; i = 1,2,...m \tag{47}$$

Taking into account $w_\alpha w_j = w^{\bullet}_{\alpha j},\ \alpha, j = 1,2,...n$, one can introduce in condensed form equation (46) as:

$$\left(\left[W^{\bullet}\right] + [S]\right)\{\dot{q}\} = [W]\{\dot{q}\} = [J]^T \{\lambda\} \tag{48}$$

and

$$\{\dot{\bar{q}}\} = [J]\{\dot{q}\} \tag{49}$$

Solving previous set of equations (48),(49) in respect to λ , $\dot{q}$ one can get:

$$\{\lambda\}_{opt} = \left([J][W]^{-1}[J]^T\right)^{-1}\{\dot{\bar{q}}\}$$

$$\{\dot{q}\}_{opt} = [W]^{-1}[J]^T\left([J][W]^{-1}[J]^T\right)^{-1}\{\dot{\bar{q}}\} = = \left[J_W^{PI}\right]\{\dot{\bar{q}}\}, \tag{50}$$

where $\left[J_W^{PI}\right]$ is weighted generalized pseudoinverse of the Jacobian matrix. Vector of joint motion $q_{opt}(t)$ can be solved by numerical integration and vector of joint acceleration is determined after differentiating vector $\dot{q}_{opt}(t)$ with respect to time i.e

$$\ddot{q}_{opt}(t) = \frac{d\dot{q}_{opt}(t)}{dt} = \left[\dot{J}_W^{PI}\right]\{\dot{\bar{q}}\} + \left[J_W^{PI}\right]\{\ddot{\bar{q}}\} \tag{51}$$

or, taking into account (see Appendix A), one can write

$$\ddot{q}_{opt} = J_W^{PI}\left[\ddot{\bar{q}} - \dot{J}\dot{q}_{opt}\right] + \dot{J}_W^{PI}\left[I - JJ_W^{PI}\right]\dot{\bar{q}} \tag{52}$$

So, using the dynamic model of the redundant robot one can obtain vector of generalized forces (control) Q_{uopt} such as:

$$\sum_{\alpha=1}^{n} a_{\alpha\gamma}(q_{opt})\ddot{q}_{opt}^\alpha + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \Gamma_{\alpha\beta,\gamma}(q_{opt})\dot{q}_{opt}^\alpha \dot{q}_{opt}^\beta = Q_\gamma^a + Q_{opt}^u,$$
$$\gamma = 1,2,...,n \tag{53}$$

3.3 Resolving redundancy by using local optimization of a acceleration-based kinematic criterion

Now, resolving redundancy using local optimization of the kinematic criterion in respect to internal acceleration is proposed here. Also, [34],[35] justifiableness introducing of criterion in following way is shown:

$$I_l = \frac{1}{2}\ddot{q}^T W \ddot{q} \to \underset{\ddot{q}}{\text{ext}}\,(\text{min}) \tag{54}$$

Suitable choice can be realized for matrix W . The weighting matrix W can be chosen to be the identity matrix $W = I_n$ or basic metric tensor $\left[a_{\alpha\beta}\right] \in R^{n\times n}$ from dynamical model of redundant robot. Authors in paper [4] show that local minimum of criterion in respect to internal acceleration is equal to weak global optimum of integral criterion in respect to internal velocity. Let an acceleration-based kinematic criterion be introduced, i.e. it is proposed in the form of weighted minimum norm least-squares

$$I = \frac{1}{2}\left\{\left(\sum_{\alpha=1}^{n} w_\alpha \ddot{q}^\alpha\right)^2 + (\ddot{q})[S]\{\ddot{q}\}\right\} \to \underset{\ddot{q}_j}{ext}, \tag{55}$$

where w_α , $\alpha = 1,2,...,n$ presents weighted coefficients and matrix $[S] = \text{diag}\left[a_{\alpha\beta}\right],\ \alpha,\beta = 1,2,...,n$. Taking into account constraints (1) which can be written in the Jacobian form of second order:

$$\ddot{\bar{q}}^i = \sum_{\alpha=1}^{n} \frac{\partial f^i}{\partial q^\alpha}\ddot{q}^\alpha + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} \frac{\partial^2 f^i}{\partial q^\alpha \partial q^\beta}\dot{q}^\beta \dot{q}^\alpha \Rightarrow$$
$$\{\ddot{\bar{q}}\} = [J]\{\ddot{q}\} + \{A(q,\dot{q})\} \tag{56}$$

In the same manner, optimal trajectories can be obtained using optimization method with unknown Langrange multipliers.The augmented objective function I_a is defined as:

$$I_a = \frac{1}{2}\left(\sum_{\alpha=1}^{n} w_\alpha \ddot{q}^\alpha\right)^2 + \frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} s_{\alpha\beta}\ddot{q}^\alpha\ddot{q}^\beta +$$
$$+ \sum_{i=1}^{m}\lambda_i\left(\ddot{\bar{q}}^i - \sum_{\alpha=1}^{n}\frac{\partial f^i}{\partial q^\alpha}\ddot{q}^\alpha - \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{\partial^2 f^i}{\partial q^\alpha \partial q^\beta}\dot{q}^\beta\dot{q}^\alpha\right) \to \min_{\ddot{q},\lambda} \tag{57}$$

Necessary conditions for optimality are:

$$\frac{\partial I_a}{\partial \ddot{q}^j} = 0, \quad j = 1,2,...,n$$
$$\frac{\partial I_a}{\partial \lambda^i} = 0, \quad i = 1,2,...,m \tag{58}$$

In the same manner as in part 3.2 one can obtain

$$a_{jj}\ddot{q}^j + w_j\left(\sum_{\alpha=1}^{n} w_\alpha \ddot{q}^\alpha\right) + \sum_{i=1}^{m}\frac{\partial f^i}{\partial q^j}\lambda_i = 0, \quad j = 1,2,...,n \tag{59}$$

$$\ddot{\bar{q}}^i = \sum_{\alpha=1}^{n}\frac{\partial f^i}{\partial q^\alpha}\ddot{q}^\alpha + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\frac{\partial^2 f^i}{\partial q^\alpha\partial q^\beta}\dot{q}^\beta\dot{q}^\alpha \quad i = 1,2,...,m \tag{60}$$

Alternatively, in matrix form such as:

$$[W]\{\ddot{q}\} = [J]^T\{\lambda\} \tag{61}$$

and

$$\{\ddot{\bar{q}}\} = [J]\{\ddot{q}\} + \{A(q,\dot{q})\} \tag{62}$$

where is also, (see eq. (48)) $\left(\left[W^{\bullet}\right]+[S]\right)=[W]$. Solving previous set of equations (50),(51) in respect to λ, $\ddot{q}$, one can get:

$$\{\lambda\}_{opt}=\left([J][W]^{-1}[J]^{T}\right)^{-1}\left\{\ddot{\bar{q}}-A(q,\dot{q})\right\},$$

$$\{\ddot{q}\}_{opt}=[W]^{-1}[J]^{T}\left([J][W]^{-1}[J]^{T}\right)^{-1}\left\{\ddot{\bar{q}}-A(q,\dot{q})\right\}=,$$

$$=\left[J_{W}^{PI}\right]\left\{\ddot{\bar{q}}-A(q,\dot{q})\right\}. \tag{63}$$

Vectors of joint motion $q_{opt}(t)$ and joint velocity $\dot{q}_{opt}(t)$ can be solved by numerical integration using second relation of (63). Substituting $\ddot{q}_{opt}(t)$, $\dot{q}_{opt}(t)$, $q_{opt}(t)$ in dynamical model (2) it is obtained expression for $Q_u^{opt}(t)$.

3.4 Resolving redundancy by using local optimization of the energy-based dynamic criterion

Here, the following criterion appropriate for on-line use in robotics is suggested.

$$I=\frac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}w_{\alpha\beta}Q_u^{\alpha}Q_u^{\beta}+\left|\sum_{\alpha=1}^{n}w_{\alpha}\dot{q}^{\alpha}Q_u^{\alpha}\right|\to\min_{Q_u}. \tag{64}$$

Minimizing criterion of optimality which is a function of $\dot{q},Q_u$ in respect to Q_u, one can optimize involving Q_u. In that way, one can realize tendency to obtain control vector and energy consumption with less possible participation in proposed motions. The second term in criterion corresponds to the demand of minimizing energy consumption and improving stability of trajectories. Also, equality constraints (1), (55) are given as:

$$\left\{\ddot{\bar{q}}\right\}=[J]\{\ddot{q}\}+\{A(q,\dot{q})\} \tag{65}$$

$$\left[a_{\alpha\beta}\right]\{\ddot{q}\}+\{\Gamma(q,\dot{q})\}=\left\{Q^{a}\right\}+\left\{Q^{u}\right\}, \tag{66}$$

where equation (66) can be written as follows:

$$[J]\{\ddot{q}\}=[J]\left[a_{\alpha\beta}\right]^{-1}\left(\{Q_u\}+\{Q_a\}-\{\Gamma(q,\dot{q})\}\right)=$$

$$=[J]\left[a_{\alpha\beta}\right]^{-1}\left(\{Q_u\}-\{D\}\right) \tag{67}$$

Now,one can obtain augmented objective cost function as:

$$I_a=\frac{1}{2}(Q_u)\cdot[W]\{Q_u\}+\left|(\dot{q})\{Q_u\}\right|+$$

$$+(\lambda)\left\{\ddot{\bar{q}}-A(q,\dot{q})-[J]\left[a_{\alpha\beta}\right]^{-1}\left(\{Q_u\}-\{D\}\right)\right\}\to\min_{Q_u} \tag{68}$$

Necessary conditions for optimality are:

$$\frac{\partial I_a}{\partial\lambda}=0\Rightarrow\ddot{\bar{q}}=[J)]\{\ddot{q}\}+\{A(q,\dot{q})\} \tag{69}$$

$$\frac{\partial I_l}{\partial Q_u}=0\Rightarrow[W]^{T}\{Q_u\}+\operatorname{sgn}(\dot{q})\operatorname{sgn}(Q_u)\{\dot{q}\}-$$

$$-\left[a_{\alpha\beta}\right]^{-1}[J]^{T}\{\lambda\}=0 \tag{70}$$

Solving (70) it yields expression for □:

$$\{\lambda\}=\left([J]\left[a_{\alpha\beta}\right]^{-1}[W]^{-1}\left[a_{\alpha\beta}\right]^{-T}[J]^{T}\right)^{-1}$$

$$\left\{\begin{matrix}\left\{\ddot{\bar{q}}\right\}-\{A(q,\dot{q})\}+[J]\left[a_{\alpha\beta}\right]^{-1}\{D\}+\\+[J]\left[a_{\alpha\beta}\right]^{-1}[W]^{-1}\{\dot{q}\}\operatorname{sgn}(\dot{q})\operatorname{sgn}(Q_u)\end{matrix}\right\} \tag{71}$$

Also, one can obtain expression for Q_u, after substituting (71) in previous equation (70), as:

$$\{Q_u\}+[W]^{-T}\{\dot{q}\}\operatorname{sgn}(\dot{q})\operatorname{sgn}(Q_u)-$$

$$-\left[J_{aW}^{PI}\right]\left\{\ddot{\bar{q}}-A(q,\dot{q})+\left[J_a\right]\{D\}\right\}=0 \tag{72}$$

where is $\left[J_a\right]=[J]\left[a_{\alpha\beta}\right]^{-1}$ and pseudoinverse of $\left[J_a\right]$ is:

$$\left[J_{aW}^{PI}\right]=[W]^{-1}\left([J]\left[a_{\alpha\beta}\right]^{-1}\right)^{T}\cdot$$

$$\left([J]\left[a_{\alpha\beta}\right]^{-1}[W]^{-1}\left[a_{\alpha\beta}\right]^{-T}[J]^{T}\right)^{-1} \tag{73}$$

Solving previous equation (72) in respect to Q_u one can get Q_{uopt}. Now, vector $\ddot{q}$ is obtained as follows :

$$\{\ddot{q}\}=\left[a_{\alpha\beta}\right]^{-1}\left(\left\{Q_{uopt}\right\}-\{D\}\right) \tag{74}$$

At last, using (73) it is possible to obtain optimal vector of joint velocities $\dot{q}_{opt}(t)$ and the joint motions $q_{opt}(t)$ by suitable numerical integration.

4. EXAMPLE

The effectiveness of suggested optimal control in human-like fashion is demonstrated with a robot with three segments and four degrees of freedom as the illustrative example (Fig.4).

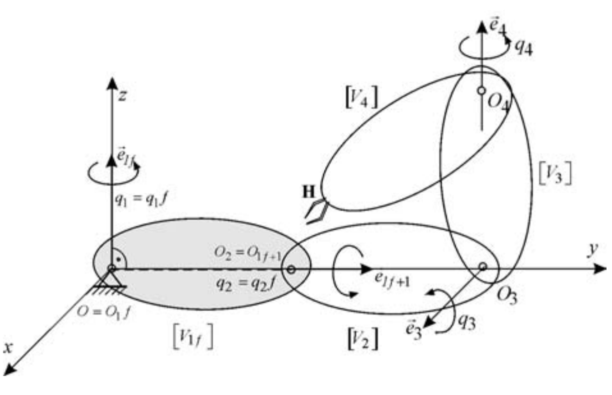

Figure 4. Robot with three segments and four DOFs

i	$\vec{\rho}_{ii}$	$\vec{\rho}_i$	$\vec{e}_i$
1	$(0,0,0)^T$	$(0,0,0)^T$	$(0,0,1)^T$
2	$(0,1,0)^T$	$(0,-0.5,0)^T$	$(0,1,0)^T$
3	$(0,0,1)^T$	$(0,0,-0.5)^T$	$(1,0,0)^T$
4	$(1,0,0)^T$	$(-0.5,0,0)^T$	$(0,0,1)^T$

Table 1. Quantities which define geometry of robot

Moreover, robot contains one cinematic pair of fourth class and two cinematic pair of fifth class. Two segments connected with cinematic pair of fourth class can be presented as open loop chain with three segments (one fictional and two real) and cinematic pairs of fifth class (see Fig 4). Also, in this case, appropriate Rodrigo's matrices of transformation are [24]:

$$\left[A_{0,1}\right]=[I]+\left[e_1^{d(1)}\right]^2\left(1-\cos q^1\right)+\left[e_1^{d(1)}\right]\sin\left(q^1\right) \tag{75}$$

where are

$$\left\{e_1^{(1)}\right\}=(0,0,1)^T,\ \left[e_1^{d(1)}\right]=\begin{bmatrix}0&-1&0\\1&0&0\\0&0&0\end{bmatrix}, \tag{76}$$

One can obtain

$$\left[A_{0,1}\right]=\begin{bmatrix}\cos q^1&-\sin q^1&0\\ \sin q^1&\cos q^1&0\\0&0&1\end{bmatrix}, \tag{77}$$

In the same manner, it yields

$$\left[A_{1,2}\right]=[I]+\left[e_2^{d(2)}\right]^2\left(1-\cos q^2\right)+\left[e_2^{d(2)}\right]\sin\left(q^2\right) \tag{78}$$

$$\left\{e_2^{(2)}\right\}=(0,1,0)^T,\ \left[e_2^{d(2)}\right]=\begin{bmatrix}0&0&1\\0&0&0\\-1&0&0\end{bmatrix} \tag{79}$$

$$\left[A_{1,2}\right]=\begin{bmatrix}\cos q^2&0&\sin q^2\\0&1&0\\-\sin q^2&0&\cos q^2\end{bmatrix} \tag{80}$$

and

$$\left[A_{2,3}\right]=[I]+\left[e_3^{d(3)}\right]^2\left(1-\cos q^3\right)+\left[e_3^{d(3)}\right]\sin\left(q^3\right), \tag{81}$$

$$\left\{e_3^{(3)}\right\}=(1,0,0)^T,\ \left[e_3^{d(3)}\right]=\begin{bmatrix}0&0&0\\0&0&-1\\0&1&0\end{bmatrix}, \tag{82}$$

$$\left[A_{2,3}\right]=\begin{bmatrix}1&0&0\\0&\cos q^3&-\sin q^3\\0&\sin q^3&\cos q^3\end{bmatrix}. \tag{83}$$

$$\left[A_{3,4}\right]=[I]+\left[e_4^{d(4)}\right]^2\left(1-\cos q^4\right)+\left[e_4^{d(4)}\right]\sin\left(q^4\right) \tag{84}$$

$$\left\{e_4^{(4)}\right\}=(0,0,1)^T,\ \left[e_4^{d(4)}\right]=\begin{bmatrix}0&-1&0\\1&0&0\\0&0&0\end{bmatrix}, \tag{85}$$

$$\left[A_{3,4}\right]=\begin{bmatrix}\cos q^4&-\sin nq^4&0\\ \sin q^4&\cos q^3&-0\\0&0&1\end{bmatrix}. \tag{86}$$

Also, one can obtain the following expressions:

$$\begin{aligned}\left[A_{0,2}\right]&=\left[A_{0,1}\right]\cdot\left[A_{1,2}\right],\\ \left[A_{0,3}\right]&=\left[A_{0,2}\right]\cdot\left[A_{2,3}\right],\\ \left[A_{0,4}\right]&=\left[A_{0,3}\right]\cdot\left[A_{3,4}\right].\end{aligned} \tag{87}$$

Therefore, direct kinematics is given:

$$\begin{aligned}&\begin{Bmatrix}\bar{q}^1\\ \bar{q}^2\\ \bar{q}^3\end{Bmatrix}=\begin{Bmatrix}x_H\\ y_H\\ z_H\end{Bmatrix}=\sum_{j=1}^{n=4}\left[A_{0,j}\right]\left(\left\{\rho_{jj}^{(j)}\right\}+\xi_j q^j\left\{e_j^{(j)}\right\}\right)=\\ &=\left[A_{0,1}\right]\left\{\rho_{11}^{(1)}\right\}+\left[A_{0,2}\right]\left\{\rho_{22}^{(2)}\right\}+\left[A_{0,3}\right]\left\{\rho_{33}^{(3)}\right\}+\left[A_{0,4}\right]\left\{\rho_{44}^{(4)}\right\}\\ &=\left[A_{0,1}\right]\begin{Bmatrix}0\\0\\0\end{Bmatrix}+\left[A_{0,2}\right]\begin{Bmatrix}0\\1\\0\end{Bmatrix}+\left[A_{0,3}\right]\begin{Bmatrix}0\\0\\1\end{Bmatrix}+\left[A_{0,4}\right]\begin{Bmatrix}1\\0\\0\end{Bmatrix}\end{aligned} \tag{88}$$

Using eqs.(77-88) follows:

$$\begin{aligned}x_H&=-\sin q^1+\sin q^1\sin q^3+\cos q^1\cos q^3\sin q^2+\\ &+\cos q^1\cos q^4\cos q^2+\sin q^4\cos q^1\sin q^2\sin q^3+\\ &-\sin q^4\sin q^1\cos q^3,\\ y_H&=\cos q^1-\cos q^1\sin q^3+\sin q^1\sin q^2\cos q^3+\\ &+\sin q^1\cos q^2\cos q^4+\sin q^4\cos q^1\cos q^3+\\ &+\sin q^4\sin q^1\sin q^2\sin q^3,\\ z_H&=\cos q^2\cos q^3+\cos q^2\sin q^4\sin q^3-\sin q^2\cos q^4.\end{aligned} \tag{89}$$

The end-off effector is required to move along a trajectory (Fig.5)defined as:

$$\begin{Bmatrix}x_H(t)\\ y_H(t)\\ z_H(t)\end{Bmatrix}=\begin{Bmatrix}x_0+\alpha(t)\left(x_k-x_0\right)\\ y_0+\alpha(t)\left(y_k-y_0\right)+2\beta(t)\\ z_0+\alpha(t)\left(z_k-z_0\right)-\beta(t)\end{Bmatrix}, \tag{90}$$

where $(x_0=1,\ y_0=1,\ z_0=1)\,[m]$ is an a initial point, $(x_k=1.2,\ y_k=0.6,\ z_k=1.1)\,[m]$ is a final point and

$\alpha(t),\beta(t)$ are the specified time dependent parameters given as:

$$\begin{aligned}&\alpha(t)=\frac{6t^2}{T^2}-\frac{8t^3}{T^3}+\frac{3t^4}{T^4},\ \alpha(t)\in[0,1]\\&\beta(t)=\frac{t}{T}\left(1-\frac{t}{T}\right)^2\end{aligned}, \qquad (91)$$

Moreover, $t_0=0,\ t_k=T=1\,\mathrm{s}$ is a time of execution of movement of proposed robotic mechanism. These forms $\alpha(t),\beta(t)$ allow the anthropomorphic trajectories which imply that in $t_k=1\,\mathrm{s}$ velocity $v(t_k)=0.$ Differentiating eq. (89) in respect to time, one can get:

$$\begin{Bmatrix}\dot{x}_H(t)\\\dot{y}_H(t)\\\dot{z}_H(t)\end{Bmatrix}=[J]_{3\times4}\begin{Bmatrix}\dot{q}^1\\\dot{q}^2\\\dot{q}^3\\\dot{q}^4\end{Bmatrix} \qquad (92)$$

where are

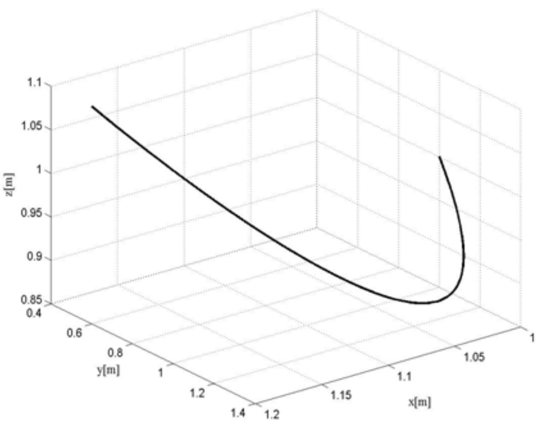

Figure 5. Proposed trajectory of end-off effector

$$\begin{aligned}j_{11}&=\cos q^1\left(\sin q^3-1\right)-\sin q^1\sin q^2\cos q^3-\\&-\sin q^1\cos q^1\cos q^2-\sin q^1\sin q^2\sin q^3\sin q^4-\\&-\sin q^4\cos q^1\cos q^3,\end{aligned}$$

$$\begin{aligned}j_{12}&=\cos q^1\cos q^2\cos q^3-\cos q^1\cos q^4\sin q^2+\\&+\sin q^4\cos q^1\cos q^2\sin q^3,\end{aligned} \qquad (93)$$

$$\begin{aligned}j_{13}&=\sin q^1\cos q^3-\cos q^1\sin q^3\sin q^2+\\&+\sin q^1\sin q^4\sin q^3+\sin q^2\sin q^4\cos q^3\cos q^1,\end{aligned}$$

$$j_{14}=-\cos q^1\cos q^2\sin q^4-\sin q^1\cos q^3\cos q^4,$$

$$\begin{aligned}j_{21}&=\sin q^1\left(\sin q^3-1\right)-\sin q^1\sin q^4\cos q^3+\\&+\cos q^1\begin{pmatrix}\sin q^2\cos q^3+\cos q^2\cos q^4\\+\sin q^2\sin q^3\sin q^4\end{pmatrix},\end{aligned}$$

$$j_{22}=\sin q^1\begin{pmatrix}\cos q^2\cos q^3-\sin q^2\cos q^4\\+\cos q^2\sin q^3\end{pmatrix}, \qquad (94)$$

$$\begin{aligned}j_{23}&=\cos q^1\left(\cos q^3-\sin q^3\sin q^4\right)+\\&+\sin q^1\sin q^2\left(\cos q^3-\sin q^3\right),\end{aligned}$$

$$\begin{aligned}j_{24}&=-\sin q^1\cos q^2\sin q^4+\cos q^4\cos q^1\cos q^3+\\&+\cos q^4\sin q^1\sin q^2\sin q^3\end{aligned}$$

$$j_{31}=0,$$

$$\begin{aligned}j_{32}&=-\sin q^2\left(\cos q^3+\sin q^4\sin q^3\right)-\\&-\cos q^2\cos q^4,\end{aligned} \qquad (95)$$

$$j_{33}=\cos q^2\left(\sin q^4\cos q^3-\sin q^3\right),$$

$$j_{34}=\cos q^2\sin q^3\cos q^4+\sin q^2\sin q^4.$$

In this example, weighted coefficients are $w_\alpha^\bullet=1,\quad \alpha=1,2,...,n$ and matrix $[S]=$ $=\mathrm{diag}\left[a_{\alpha\beta}\right]_{q_i=0,i=1,2,3,4},\ \alpha,\beta=1,2,...,4$ is given as:

$$[S]=\mathrm{diag}\{a_{\alpha\alpha}\}\Big|_{\substack{\alpha=1,2,3,4\\q_i(0)=0}}=\begin{bmatrix}8/3&0&0&0\\0&49/12&0&0\\0&0&4/3&0\\0&0&0&1/3\end{bmatrix} \qquad (96)$$

and finally, matrix $[W]$ is:

$$[W]=\begin{bmatrix}11/3&1&1&1\\1&61/12&1&1\\1&1&7/3&1\\1&1&1&4/3\end{bmatrix}, \qquad (97)$$

Solving set of equations (48),(49) in respect to $\lambda,\dot{q}$ one can get:

$$\dot{q}_{opt}=[W]^{-1}[J]^T\left([J][W]^{-1}[J]^T\right)^{-1}\left\{\dot{\bar{q}}\right\}=\left[J_W^{PI}\right]\left\{\dot{\bar{q}}\right\}, \qquad (98)$$

At last, vector of joint motion $q_{opt}(t)$ is obtained by numerical integration (method of finite differences), (Fig.6):

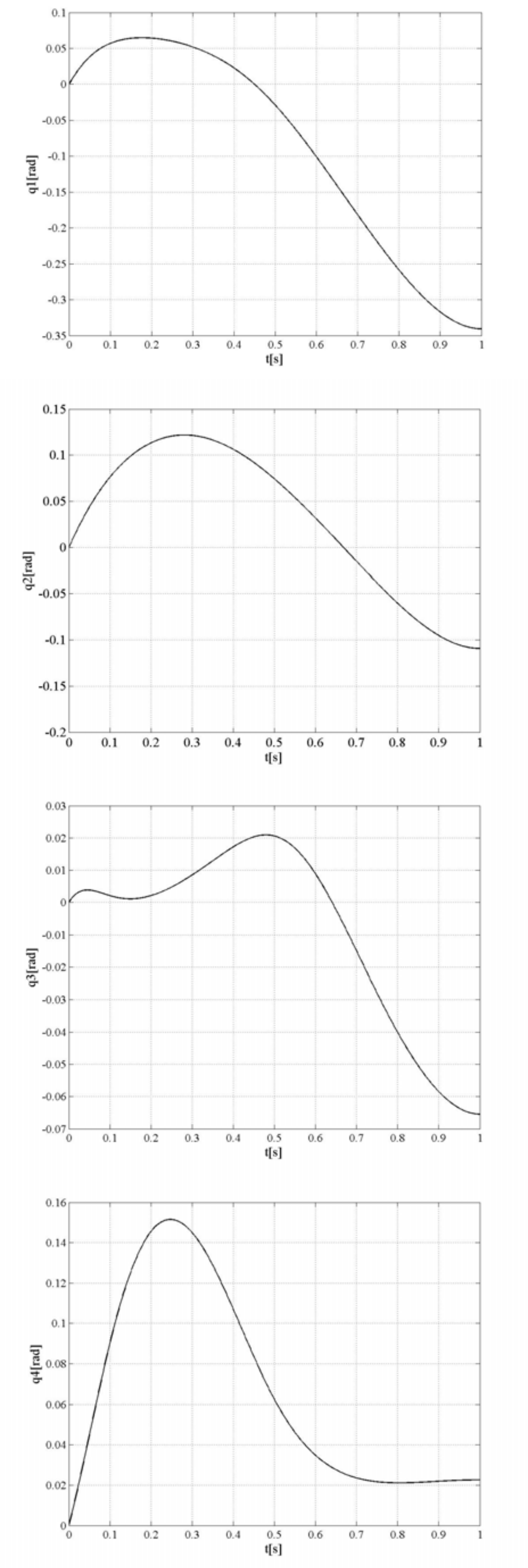

Figure 6 Optimal trajectories $q_i(t), i = 1,2,3,4$

5. CONCLUSION

In this paper, a new approach of control of redundant system is presented using suitable biological analogous, which appear in human control strategies. It suggests joint geometrical, velocity, acceleration and control vector based on criterion which is established by optimization law. Also, the dynamical model of robotic system is given in covariant form of Langrange`s equations of second kind which is now suitable to obtain vector of control applying (numerical)-symbolic programming. The effectiveness of the proposed optimal control is illustrated by simulation results of a redundant robot with 4 DOFs (Fig.6).

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Appendix A

In this appendix, for simplicity, a matrix notation is omitted. Here, weighted generalized pseudoinverse of the Jacobian matrix is presented as:

$$J_W^{PI} = W^{-1}J^T(JW^{-1}J^T)^{-1} \tag{A0}$$

Using (A0) and after differentiating it yields

$$\dot{J}_W^{PI} = -J_W^{PI}\dot{J}J_W^{PI} + (J^TWJ)^{-1}\dot{J}^TW(I - JJ_W^{PI}) \tag{A1}$$

Multiplying both sides with JJ_W^{PI} it follows

$$\begin{aligned}
\dot{J}_W^{PI}JJ_W^{PI} &= \left(\dot{J}^TWJ\right)^{-1}J^TWJJ_W^{PI} + \\
&+\left(J^TWJ\right)^{-1}J^TWJJ_W^{PI} + \left(J^TWJ\right)^{-1}\dot{J}^TWJJ_W^{PI} \\
&= \left(\dot{J}^TWJ\right)^{-1}J^TW + \left(J^TWJ\right)^{-1}J^TW + \\
&+\left(J^TWJ\right)^{-1}\dot{J}^TWJJ_W^{PI} = \\
&= \dot{J}_W^{PI} - \left(J^TWJ\right)^{-1}\dot{J}^TW + \left(J^TWJ\right)^{-1}\dot{J}^TWJJ_W^{PI}
\end{aligned} \tag{A2}$$

or

$$\dot{J}_W^{PI} = \left(J^T W J\right)^{-1} \dot{J}^T W \left[I - J J_W^{PI} \right] + \dot{J}_W^{PI} J J_W^{PI} \qquad \text{(A3)}$$

Also,

$$J_W^{PI} J = I \Rightarrow \dot{J}_W^{PI} J + J_W^{PI} \dot{J} = 0 / \cdot J_W^{PI}$$

$$\Rightarrow \dot{J}_W^{PI} J J_W^{PI} = -J_W^{PI} \dot{J} J_W^{PI} \qquad \text{(A4)}$$

After substituting (A4) in (A3) one obtains expression which will be used in the following i.e.

$$\dot{J}_W^{PI} = -J_W^{PI} \dot{J} J_W^{PI} + (J^T W J)^{-1} \dot{J}^T W (I - J J_W^{PI}) \qquad \text{(A5)}$$

Now, relationship for $\ddot{q}_{opt}$ is :)

$$\ddot{q}_{opt} = J_W^{PI} \ddot{\bar{q}} + \dot{J}_W^{PI} \dot{\bar{q}} =$$

$$= J_W^{PI} \ddot{\bar{q}} - \left[J_W^{PI} \dot{J} J_W^{PI} + \left(J^T W J\right)^{-1} \dot{J}^T W \left(I - J J_W^{PI}\right) \right] \dot{\bar{q}} \qquad \text{(A6)}$$

taking in to account

$$\dot{J}_W^{PI} = \left(J^T W J\right)^{-1} \dot{J}^T W \left[I - J J_W^{PI} \right] + \dot{J}_W^{PI} J J_W^{PI} \qquad \text{(A7)}$$

Taking the relationship (A7),and (A6) gives:

$$\ddot{q}_{opt} = J_W^{PI} [\ \ddot{\bar{q}} - \dot{J} \dot{q}_{opt}\ ] +$$

$$+ \{ -J_W^{PI} \dot{J} J_W^{PI} + (J^T W J)^{-1} \dot{J}^T W (I - J J_W^{PI}) \} \dot{\bar{q}} \qquad \text{(A8)}$$

or,

$$\ddot{q}_{opt} = J_W^{PI} \left[\ddot{\bar{q}} - \dot{J} \dot{q}_{opt} \right] + \dot{J}_W^{PI} \left[I - J J_W^{PI} \right] \dot{\bar{q}} \qquad \text{(A9)}$$

ОПТИМАЛНО УПРАВЉАЊЕ РЕДУНДАНТНИМ РОБОТИМА НА НАЧИН СЛИЧАН ЧОВЕКУ

Михаило Лазаревић

У овом раду је предложен један нови вид управљања редундантним роботским системом. То је остварено применом погодног кинематичког и динамичког критеријума заснованим на биолошким принципима тј. на начину који је сличан и својствен човеку. Овде је динамички модел роботског система дат у форми Лангранжевих једначина друге врсте у коваријатном облику.Неколико критеријума је уведено који су функција генералисаних координата, брзина, вектора убрзања као и вектора управљања респективно.Коначно, ефикасност предложеног оптималног управљања на начин сличан човеку је демонстрирана на роботу са четири степена слободе.