Geometry Optimization of Star Shaped Propellant Grain With Special Attention to Minimization of Stress and Strain

Predrag Miloš
Research Assistant
University of Belgrade
Faculty of Mechanical Engineering

Apart from always permanent demands for achieving as much as possible higher rocket motor internal ballistic performances, reliability, service-life, operating temperature and requirements for handling, transportation and storage are also of great importance. By the rule, the most critical part of the rocket is propellant grain. In order to improve those characteristics, it is very important to minimize stresses and strains that occur in the propellant grain, especially at the most critical moment of rocket motor ignition. Principally, this can be done in several ways, for example: by radical change of propellant grain geometry, change of propellant composition, change of propellant grain placement in rocket motor, etc., but those possibilities are either spent or are too complicated and expensive. Because of that, it is preferable, to solve the problem or to increase reliability and operating temperature range wherever it is possible with minimal, inexpensive and simple solution.
In this paper, it is presented a specific methodology for optimization of star shape propellant grains in the sense of minimizing stress and strain without compromising the required internal ballistic performances.

***Keywords:** solid rocket motor, grain design, star grains, rocket propellant.*

1. INTRODUCTION

The star geometry has been a popular grain geometry in the professional solid propulsion industry for years and is still used extensively today. The star a is radial burning cylindrical grain with distinctive geometric properties. Design flexibility of the star configuration protects the chamber wall from consequences of gas temperature and erosion, thereby eliminating the need for wholesale case insulation. With seven variables available, it is quite easy to achieve desirable volumetric fraction and relatively neutral burning. Neutrality is provided in two dimensions by the interaction of the regressive burning star wedges and the progressive burning tube.

The design of solid propellant grain that provides neutral burning is important to optimize rocket motor performance. The star configuration has been widely used to achieve this goal. For that reason it is very important to optimally design propellant grain both from internal ballistic and from structural point of view. As demand for loading factor of propellant grain is higher, achieving this goal is harder.

The star geometry does possess some notable advantages over other grain geometry's such as:

- Can possess high surface area exposure
- Great for fast powerful burns
- Can possess close to neutral burn
- Proven track record
- Low heat exposure to the combustion chamber

Some disadvantages may include:

- Existence of sliver
- Existence of stress concentration in star valley
- Effect of erosive burning may be very high in some cases
- Mandrels maybe difficult to make.

The demonstration of possibilities for optimizing the star shape propellant grain in such manner that stresses and strains are minimized without compromising internal ballistic performances will be presented on the example of real existing rocket motor. The main idea is to introduce elliptical instead of classic circular fillet of star valley in order to minimize stress concentration factor. Also, web thickness is optimized. New mandrel for production of propellant grain is even simpler, regardless of elliptical fillet, due to four-star-point configuration.

2. PROPELLANT GRAIN CONFIGURATION

Both propellant grain configurations, the existing and a new improved one, are of the same type, star shaped. Also, outside diametar and length are absolutely the same. The only difference is in cross section appearance, but they have identical port area, which means both propellant grains possess the same amount of identical solid rocket propellant. Propellant grain placement in rocket motor is the same, so in this way the influence of geometry on rocket motor performances is isolated.

Received: March 2007, Accepted: May 2007
Correspondence to: Predrag Miloš
Faculty of Mechanical Engineering,
Kraljice Marije 16, 11120 Belgrade 35, Serbia
E-mail: beker@eunet.yu

2.1 Existing geometry

The existing geometry is classic five-star points with the following main cross section geometry.

Table 1. Main dimensions of cross section

	Designation	Value	Unit
D	Outside diameter	116.50	mm
W	Web thickness	28.85	mm
r1	Tip radius	8.00	mm
r2	Internal radius	2.00	mm
η	Separation angle	23.00	°
Rp	Port radius	15.40	mm
N	Number of star points	5	

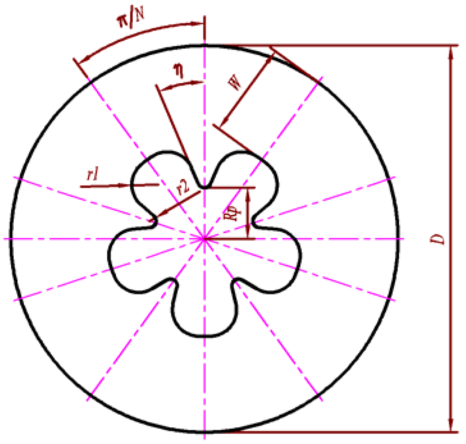

Figure 1. Cross section of existing propellant grain

2.2 New improved geometry

In this case we have 4-star points and elliptical tip geometry.

Table 2. Main dimensions of cross section

	Designation	Value	Unit
D	Outside diameter	116.50	mm
W	Web thickness	27.25	mm
ea	Major ellipse axis	10.00	mm
eb	Minor ellipse axis	6.00	mm
r2	Internal radius	5.00	mm
η	Separation angle	0.00	°
N	Number of star points	4	

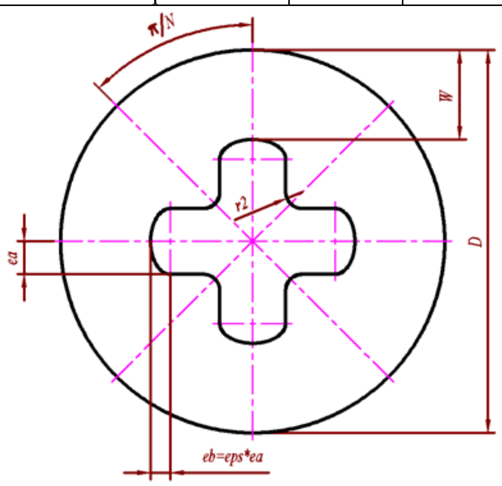

Figure 2. Cross section of improved propellant grain

The value for parameter *eps* (ratio of minor to major ellipse axis) was chosen in such manner that stress concentration factor is minimal. Since the semi-circle star tip ($eps = 1$) has one point of stress concentration and the geometry with $eps = 0$ has two points of concentration, it is clear that at some intermediate value of *eps* a transition from one point of concentration to two points of concentration must occur, and that at this transition the stress should be uniformly distributed around the tip. That transition will represent a minimum stress configuration. A configuration of minimum stress occurs for a value of *eps* in the range $0,35 < eps < 0,7$, and at a value of *eps* where the transition from one concentration point to two concentration points is occurring. To find the optimum value for the *eps* in our case, we will use program package (Pro/Mechanica).

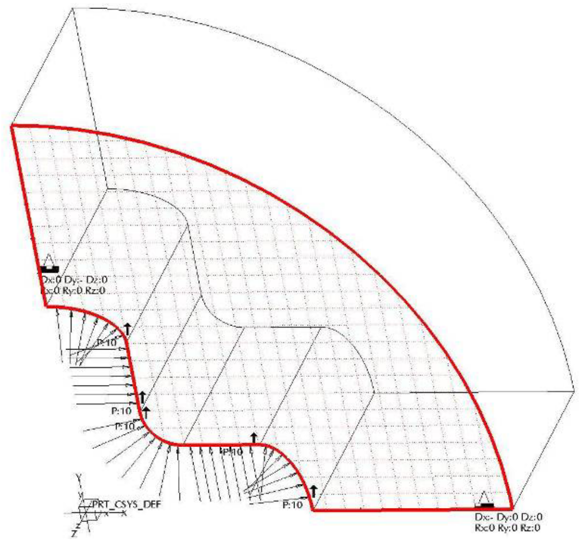

Figure 3. Pro/Mechanica simulation model

As can be seen from the Fig. 3 internal pressure is specified load and model was considered as 2D plain strain (infinitely thick). Because of the model symmetry, one quarter of cross section is enough. Parameter *eps* was varied from 1 to 0.3 with 0.1 steps.

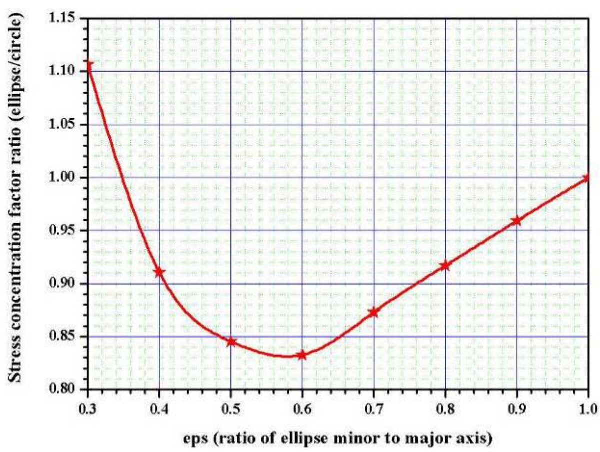

Figure 4. Elliptical star tip calculation results

From the Fig. 4 it can be seen that minimal stress concentration factor will occur for the $eps \approx 0,58$. Accepted value is 0.6 and in that case stress concentration factor ratio between elliptic and semi-circle star tip is 0.83.

3. STRESS AND STRAIN ANALYSIS

Causes of operational failures of solid rocket motors are varied, but the major are connected to the structural integrity of the propellant grain. A structural analysis, when coupled with appropriate failure data of the component materials through a failure analysis, defines the limiting environment in which a solid rocket motor may be expected to perform satisfactorily.

The loads encountered by a solid rocket motor are normally classified as two types: specified loads and induced loads or derived loads. Specified loads are fixed by mission requirements demanded by the prime contractor. These loads are typically the operational temperature environment, acceleration, vibration, shock, transportation and handling loads and the physical environment (aging conditions, humidity, etc.). Induced loads arise from particular selection of the propellant, processing techniques and grain configuration satisfying the mission objectives of the motor. Induced loads are typically cure shrinkage, pressure, flight and certain combined loads.

There are two very distinct stages in the operational life of a propellant grain:

- The stage before firing: this includes manufacturing followed by various transportation and storage phases.
- The firing which lasts from a few milliseconds to several seconds.

To esteem correctly the safety factor in propellant grain it is first necessary to make analysis of loads it is exposed to. In case of most rocket motors, the propellant grain is exposed to these loads:

- The load that is the consequence of great pressure difference along the central port length. These loads are the greatest at motor start.
- Pressurization loads arise during ignition of a solid propellant rocket motor and act until motor burnout. The pressurization loads imposed on a solid rocket motor are determined by the propellant properties, the grain configuration and the stiffness of the motor case. The hoop stresses and strain at the inner bore are usually the critical design parameter for pressurization loading, particularly for low temperature firings where the propellants have less elongation capabilities than at high temperatures.
- Inertial loads at start as a result of axial acceleration.
- Axial and radial thermal loads. Those loads are the consequence of thermal spreading (shrinking) of propellant grain at various temperatures.
- Axial and radial transport loads which are the results of rocket transportation and are defined by basic technical requirements.
- Centrifugal loads as the result of rocket rotation.
- Remaining stresses that are realized as a consequence of technological processing of propellant grain.
- Cyclical loads due to cyclical (climatic) temperature changes.
- Loads due to its own weight.

In our case of console supported grain the most critical load is one that grain bears in the ignition phase, when exposed to internal pressure in central port, it leans on the wall. For this case of load of greatest importance is the clearance value between grain and combustion chamber at a certain temperature.

3.1 Mathematical model

Parameter H, directly connected with stress concentration factor in star valley is defined by empiric relationship derived from photo-elastic tests:

$$H = \frac{1}{\sqrt[3]{N}} \sqrt{\frac{\psi + 1}{\psi - 1}} \left(1 + 2\sqrt{\frac{a}{\rho}} \right). \qquad (1)$$

Parameter ψ is determined by the relationship of external and internal diameters and is defined as:

$$\psi = \frac{b}{a} = \frac{\frac{D}{2}}{\frac{D}{2} - W}. \qquad (2)$$

For the internally pressurized grain, the hoop stress at the star valley is related to (1) by the equation:

$$\left(\sigma_\theta^a\right)_s = (H-1)\frac{\delta_r}{b}\frac{\psi^2-1}{2}E_T, \qquad (3)$$

where δ_r is clearance between grain and combustion chamber at a certain temperature:

$$\delta_r = \frac{(D_{ic} - D)}{2}. \qquad (4)$$

Inner diameter of the combustion chamber at a certain temperature is:

$$D_{ic} = D_{ic0}\left[1 + \alpha_c (T - T_0)\right]. \qquad (5)$$

Outer diameter of the propellant grain at a certain temperature is:

$$D = D_0\left[1 + \alpha_p (T - T_0)\right]. \qquad (6)$$

Maximal tangential deformation at the star valley is defined by equation:

$$(\varepsilon_\theta^a)_s = \left(\frac{3}{4}H\frac{\psi^2-1}{2}\right)\left(\frac{\delta_r}{b}\right). \qquad (7)$$

The pressure in central port, which is necessary to make the grain lean on combustion chamber wall, is:

$$p_0 = \frac{\psi^2-1}{2}E_T\frac{\delta_r}{b}. \qquad (8)$$

Referring to (3), photo-elastic parameter in the case of elliptical tip geometry will be:

$$H_{elip} = elip(H_c - 1) + 1. \qquad (9)$$

With the knowledge of propellant mechanical characteristics according to the temperature, it is now possible to calculate stresses and strains for star perforated grain.

The best way to present what new geometry improves is to find the ratio of stresses and strains. In this way, we are comparing only geometrical parameters of the grain cross-sections without influence of the propellant mechanical characteristics.

The ratio of stresses between radius and elliptical fillet:

$$SR = \frac{(H-1)(\psi^2-1)}{(H_{elip}-1)(\psi_{elip}^2-1)} = 1.421 . \qquad (10)$$

The ratio of elongation between radius and elliptical fillet:

$$ER = \frac{H(\psi^2-1)}{H_{elip}(\psi_{elip}^2-1)} = 1.404 . \qquad (11)$$

The ratio of the pressure of radius with elliptical fillet:

$$PR = \frac{\psi^2-1}{\psi_{elip}^2-1} = 1.346 . \qquad (12)$$

As can be seen the significant improvement is achieved. The old design has 42% higher stresses at the critical moment of motor starting at every motor working temperature. This means that with new a design the motor can operate at lower temperature and with higher reliability safety factor. This result is achieved with the change of two major design parameters. One is the optimal type of the star tip to reduce stress concentration factor and the other one is web thickness to reduce the load.

4. INTERNAL BALLISTIC VERIFICATION

After determining the new optimized shape of propellant grain cross-section it is necessary to check if the internal ballistic of motor is satisfactory. In other words, if the main motor parameters remain unchanged or in acceptable limits. In this case, the main motor parameters of our interest are:

- Maximal motor working pressure during operating time
- Total motor impulse
- Motor thrust at the start of motor burning
- Total and effective motor burning time
- Thrust vs. time trace history

To this end, we will use the existing computer program for internal ballistic calculation. Our theoretical results for the “old” – 5-star-point case will be first compared to the experiment for validation of calculation, and after words with “new” – 4-star-point case. The first comparison is done where both configurations use the same propellant with the same burning rate. In second case, the propellant in a new designed cross section has by 9% higher burning rate (mean diameter of oxidizer particle size is smaller), but mechanical properties remain the same.

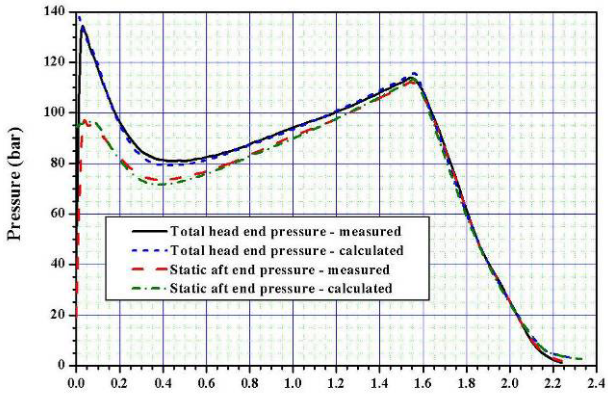

Figure 5. Pressure vs. time history for old star shape grain

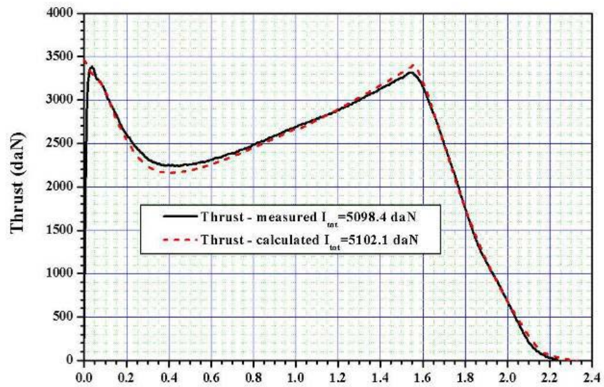

Figure 6. Thrust vs. time history for old star shape grain

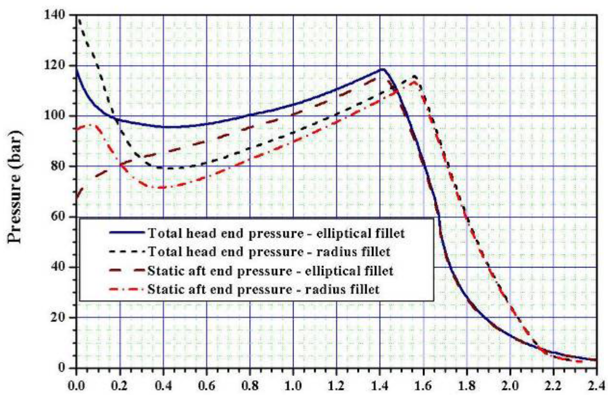

Figure 7. Pressure vs. time comparison between old and new design

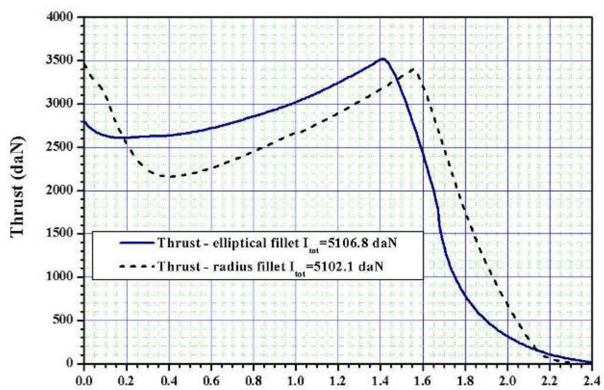

Figure 8. Thrust vs. time comparison between old and new design

From previous figures, 5 and 6, it can be concluded that the program used for internal ballistic calculation gives excellent results and can be used in further analysis.

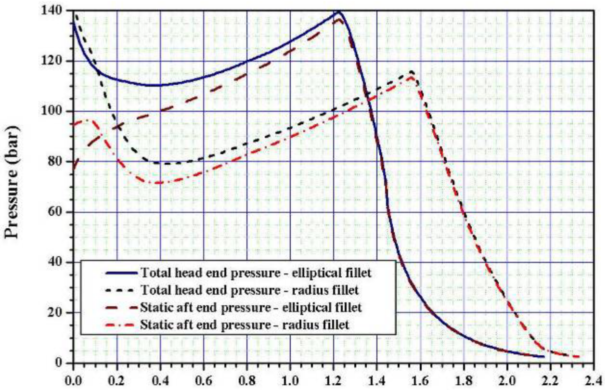

Figure 9. Pressure vs. time comparison between old and new design with higher burning rate propellant

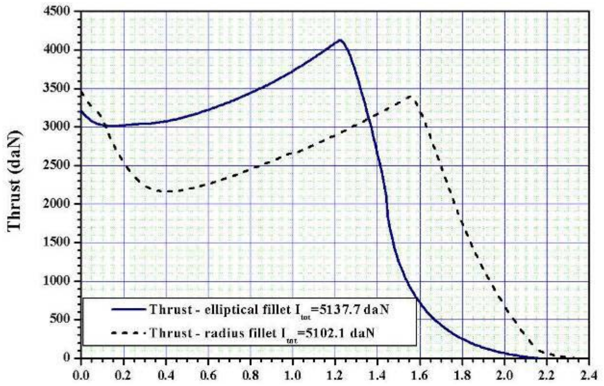

Figure 10. Thrust vs. time comparison between old and new design with higher burning rate propellant

The new star design with the usage of the same propellant gives satisfactory results. Motor burning time and obtained total impulse is practically the same for both configurations. Maximal pressure for elliptic fillet design is lower than maximal pressure for original design, more than 20 bars (figure 7.). Only a small disadvantage is in the fact that at start the thrust is lower ≈22% but after 0.2 s is equal to the thrust of radius fillet configuration (figure 8.). Because of lower maximal pressure that lack in thrust can be overcome by increasing the propellant burning rate (by decreasing the oxidizer particle size), of course up to the same limit as in original design, 140 bars. In that case starting thrusts are almost the same and due to the higher average operating motor pressure a new design has slightly bigger total impulse, 0.7% (figure 10).

5. CONCLUSION

By the presented example it is shown that it is possible to significantly improve the design (from mechanical point of view) of star perforated propellant grains only with optimization of the star tip geometry. The main geometry difference is in the fact that instead of classical round star tip elliptical tip geometry was introduced. In this way stress is uniformly distributed around the tip, which means that a minimum stress configuration is achieved. The ratio of ellipse minor to major axis (*eps*), which gives minimal concentration factor, should be calculated for each independent case. Parameter *eps* is mainly the function of the number of star points, ratio of outer grain to star tip radius, and ratio of star tip to star tip fillet radius. In most cases a configuration of minimum stress occurs for a value of *eps* in the range 0.35÷0.70.

Even better results could be obtained with optimization of web thickness, number of star points and separation angle.

Of course, it should be kept in mind that required internal ballistic performances must be satisfied. In our case we achieved that as can be seen on figures in 7. and 8. Furthermore, with a new design we have opportunity to even increase motor total impulse by introducing the propellant which differs from original only in burning rate law (by decreasing the oxidizer particle size we increase the propellant burning rate), but keeps the maximal pressure on the same level (figures 9. and 10.).

The evidence for the right choice of the design idea and dimensions of the improved geometry are conducted ground and flight tests which demonstrate increment in operating temperature range (lower minus temperatures from -20^{0}C to over -30^{0}C) without lacking in ballistic performances.

REFERENCES

[1] Fitzgerald, J.E.: *Handbook for the Engineering Structural Analysis of Solid Propellants*, University of Utah Lake City, Utah, 1971.

[2] Billheimer, J.S.: *The Ballistic Interface in Grain Configuration for Structural Analysis*, Rep. AFRPLTR-69-57 Solid Rocket Structural Integrity Information Center Univ. of Utah, Vol. 6, No. 2, pp. 1-40, 1969.

[3] Hartfield, R., Jenkins, R., Burkhalter, J., and Foster, W.: *A Review of Analytical Methods for Solid Rocket Motor Grain Analysis*, AIAA 2003-4506 39th Joint Propulsion Conference and Exhibit, Huntsville, AL, July 2003.

[4] Massa, L., Jackson, and T., Buckmaster, J.: *Numerical Simulation of AP-Based Propellant Combustion*, Proceedings of AIAA Joint Propulsion Conference AIAA 2006-4588, July 2006.

NOMENCLATURE

H_c	Photo-elastic parameter in case of semi-circle tip geometry
H_{elip}	Photo-elastic parameter in case of elliptical tip geometry
a	Radius to star valley
b	Outside radius of proppelant grain
E_T	Tensile modul of propellant at temperature T
T	Temperature
D_{ic0}	Inner diameter of the combustion chamber at temperature T_0
D_0	Outer diameter of the propellant grain at temperature T_0

Greek symbols

ψ	Grain radius ratio
ρ	Tip radius, $\rho = r1$ in case of semi-circle and $\rho = ea$ in case of elliptical tip geometry

α_c Coefficient of linear expansion for combustion chamber material

α_p Coefficient of linear expansion for propellant

ГЕОМЕТРИЈСКА ОПТИМИЗАЦИЈА ПОГОНСКОГ ПУЊЕЊА ТИПА ЗВЕЗДА СА ПОСЕБНИМ ОСВРТОМ НА МИНИМИЗАЦИЈУ НАПОНА И ИЗДУЖЕЊА

Предраг Милош

Поред стално присутних захтева за постизање што бољих унутрашњьо-балистичких перформанси ракетних мотора од велике важности су такође и поузданост, век употребе, температурни опсег примене и захтеви везани за руковање транспорт и складиштење. По правилу најкритичнији део ракете је погонско пуњење. Да би се унапредиле предходно набројане карактеристике веома је важно смањити напоне и издужења која се јављају у погонском пуњењу нарочито у најкритичнијем тренутку припаљивања ракетног мотора. Начелно је то могуће урадити на више начина на пример радикалном променом геометрије погонског пуњења, променом погонске материје, променом начина смештаја погонског пуњења у самом мотору итд., али такве могућности су или већ потрошене или су сувише компликоване и скупе. Из тог разлога је пожељно, где год је то могуће, са минималним, јефтиним и простим захватима решити проблем или повећати поузданост и температурски опсег примене.

У овом раду је презентована методологија за оптимизацију погонских пуњења типа звезда у смислу смањења напона и издужења без угрожавања тражених унутрашњо-балистичких перформанси.