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Momentum Equation of the Boundary Layer on Rotating Bodies for the Case of Dissociated Gas Flow Along a Porous Contour

This paper studies a laminar steady boundary layer on a rotating body with a porous contour at an axisymmetrical compressible fluid flow. The ionized gas flows in the conditions of equilibrium dissociation. The momentum equation of the considered flow problem is obtained. First, the primary porosity parameter and then the corresponding set of porosity parameters are defined. It has been shown that the general similarity method in V. N. Saljnikov's version can be applied after introduction of necessary purposeful transformations.

Keywords: *Boundary layer, axisymmetrical flow, momentum equation, porosity parameter, general similarity method.*

1. INTRODUCTION

Development of technology brought about complex problems of boundary layer flow, which had to be solved. At the same time, with advancement of science, investigators developed different methods for solution of complex flow problems, i.e. methods for solution of the corresponding equation systems. Mathematical models of boundary layer flow problems of homogenous incompressible or compressible fluid, especially dissociated and ionized gas, are very complex equation systems. These are the systems of nonlinear partial simultaneous equations whose solutions cannot be obtained in a general closed analytic form [1, 2].

It is, therefore, understandable that investigators, while considering these boundary layer flow problems, first looked for the ways to transform the corresponding partial equations to a system of simple differential equations. This way the so called similar i.e. auto-model solutions were obtained. Using special transformations of variables [2], partial differential equations are brought into a system of simple differential equations.

After the so called parametric methods [2], a general similarity method was developed [3]. This method was successfully applied to solution of MHD boundary layer flow problems [4], as well as to solution of dissociated and ionized gas flow problems [5, 6]. Nowadays, general similarity method is mainly used or, alternatively, the corresponding equations are directly numerically solved.

The application of the general similarity method for solution of boundary layer equations of different flow problems is based on the use of the *momentum equation* and the use of the corresponding *sets of parameters*. These parameters represent the so called *similarity parameters*. The corresponding transformations of physical quantities are also introduced. This way, the governing equation system is transformed into a form which is independent from the given velocity distribution on the outer edge of the boundary layer.

When this modern method is applied to different problems of compressible fluid flow (dissociated or ionized gas), the corresponding *previous* transformations are followed by general similarity transformations. As shown in details in [7], by means of the previous transformations the dynamic equation of the compressible fluid boundary layer is brought to the same form as the corresponding equation of incompressible fluid.

In this paper, we derive the momentum equation of the boundary layer on a rotating body with a porous contour when dissociated gas flows in the conditions of equilibrium dissociation. A set of porosity parameters of the considered problem is also defined. The paper is a part of our current broader investigations, the goal of which is to apply the general similarity method to a dissociated gas boundary layer on rotating bodies and to solve the obtained system of generalized equations.

2. GOVERNING EQUATIONS

In order to derive the momentum equation when dissociated gas flows along rotating bodies (Fig. 1), as with other cases of fluid flow, we start from the continuity equation and from the corresponding boundary layer dynamic equation. For the considered case of axisymmetrical flow in the boundary layer, a completely new equation system (with the corresponding boundary conditions [9, 10]) has the following form:

$$\frac{\partial}{\partial x}(\rho u\, r)+\frac{\partial}{\partial y}(\rho v r)=0\ ,$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\rho_e u_e\frac{\mathrm{d}u_e}{\mathrm{d}x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)\ ,\qquad (1)$$

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$$\rho u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial y}=$$

$$=-u\rho_e u_e\frac{\mathrm{d}u_e}{\mathrm{d}x}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left[\frac{\mu}{Pr}(1+l)\frac{\partial h}{\partial y}\right],$$

$$u=0,\quad \underline{v=v_w(x)},\quad h=h_w\quad \text{for}\quad y=0,$$

$$u\to u_e(x)\ \left(\frac{\partial u}{\partial y}=0\right),\quad h\to h_e(x)\quad \text{for}\quad y\to\infty.$$

The given equations represent the continuity equation, dynamic and energy equation, respectively, in which the function $l(p,h)$ depends on Lewis number Le and on the enthalpy of the atomic and molecular component of the equilibrium dissociated gas [10].

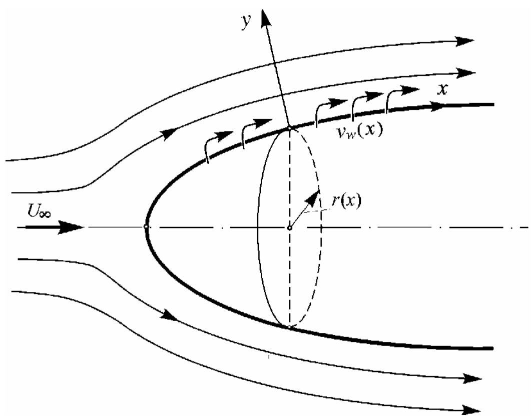

Figure 1. The dissociated gas flow along the rotating body

In the equations of the system (1) the usual notation in the boundary layer theory is used for physical quantities. The symbols stand for: x - the longitudinal coordinate measured along the contour of the body, y - transversal coordinate perpendicular to the contour of the body, $u(x,y)$ - longitudinal projection of the velocity in the boundary layer, $v(x, y)$ - transversal projection, ρ - density, h - enthalpy, μ - dynamic viscosity. In the system (1) Pr denotes Prandtl number, while $r=r(x)$ represents the radius of the cross section of the rotating body which is perpendicular to the rotating axis. Therefore, the contour of the body is given by the function $r(x)$. The subscript e stands for physical quantities at the outer edge of the boundary layer, while the subscript w denotes the quantities on the wall of the body within the fluid. Therefore, $v_w(x)$ denotes the given velocity at which gas flows perpendicularly through the solid porous wall ($v_w>0$ or $v_w<0$). Everywhere on the body, the thickness of the boundary layer $\delta(x)$ is assumed to be significantly less than the radius of the rotating body, i.e. $\delta \ll r$. Therefore, this thickness can be ignored compared to the radius of cross section of the body. This assumption cannot be applied to long thin bodies [9, 12].

For the planar steady flow of equilibrium dissociated compressible fluid in the boundary layer, the corresponding equation system, as known [10], differs only in the continuity equation. This equation does not contain the radius $r(x)$ of the cross section of the rotating body. Hence, for both flows, the continuity equation can be written in a more general form

$$\frac{\partial}{\partial x}(\rho u r^j)+\frac{\partial}{\partial y}(\rho v r^j)=0\,, \tag{2}$$

where $j=0$ for the planar and $j=1$ for the axisymmetrical flow.

In order to derive the momentum equation of the considered flow problem, the same procedure as with the incompressible flow is performed. The continuity equation (2) is multiplied by $u_e(x)$ while the dynamic equation is multiplied by r, i.e., by $r^j=r^j(x)$. Then, subtracting these equations, we get a new one that is integrated from the inner to the outer edge of the boundary layer. New variables are introduced and the equation of the form $\mathrm{d}Z^{**}/\mathrm{d}s=F_{ot}/u_e$ is obtained. However, in this equation, not all the terms of the so called characteristic function F_{ot} are nondimensional. This phase of investigation has shown that we should start with the continuity equation written in a more general form

$$\frac{\partial}{\partial x}\left[\rho u\left(\frac{r}{L}\right)^j\right]+\frac{\partial}{\partial y}\left[\rho v\left(\frac{r}{L}\right)^j\right]=0\,, \tag{3}$$

which for the axisymmetrical flow ($j=1$) and $L=\text{const}$ comes down to the first equation of the system (1). In that equation, L is a characteristic constant length (and at the numerical calculation we can take that $L=1$ [11]).

3. MOMENTUM EQUATION OF THE CONSIDERED FLOW PROBLEM

If the continuity equation of the form (3) is multiplied by the velocity $u_e(x)$ at the outer edge of the boundary layer, we will obtain

$$\frac{\partial}{\partial x}\left[\rho u\left(\frac{r}{L}\right)^j u_e\right]+\frac{\partial}{\partial y}\left[\rho v\left(\frac{r}{L}\right)^j u_e\right]=\\ =\rho u\left(\frac{r}{L}\right)^j\frac{\mathrm{d}u_e}{\mathrm{d}x}\ . \tag{4}$$

The dynamic equation of the governing system (1), after multiplication by $(r/L)^j$ and based on the continuity equation (3), comes down to

$$\frac{\partial}{\partial x}\left[\rho u\left(\frac{r}{L}\right)^j u\right]+\frac{\partial}{\partial y}\left[\rho v\left(\frac{r}{L}\right)^j u\right]=\\ =\rho_e u_e\left(\frac{r}{L}\right)^j\frac{\mathrm{d}u_e}{\mathrm{d}x}+\left(\frac{r}{L}\right)^j\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)\ . \tag{5}$$

Subtracting the equation (5) from the equation (4), we obtain that

$$\frac{\partial}{\partial x}\left[\rho u\left(\frac{r}{L}\right)^j (u_e-u)\right]+\frac{\partial}{\partial y}\left[\rho v\left(\frac{r}{L}\right)^j (u_e-u)\right]=$$
$$=\left(\frac{r}{L}\right)^j \frac{\mathrm{d}u_e}{\mathrm{d}x}(\rho u-\rho_e u_e)-\left(\frac{r}{L}\right)^j \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) .$$

When the previous equation, as with incompressible fluid, is multiplied by dy and integrated perpendicularly to the boundary layer from the inner $(y=0)$ to the outer edge of the boundary layer $(y\to\infty)$ we will obtain an equation in the form of a sum of integrals:

$$\int_0^\infty \frac{\partial}{\partial x}\left[\rho u\left(\frac{r}{L}\right)^j (u_e-u)\right]\mathrm{d}y+$$
$$+\int_0^\infty \frac{\partial}{\partial y}\left[\rho v\left(\frac{r}{L}\right)^j (u_e-u)\right]\mathrm{d}y=$$
$$=\int_0^\infty \left(\frac{r}{L}\right)^j \frac{\mathrm{d}u_e}{\mathrm{d}x}(\rho u-\rho_e u_e)\,\mathrm{d}y-\int_0^\infty\left(\frac{r}{L}\right)^j \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)\mathrm{d}y.$$

Taking into consideration the possibility of changing the order of the two operations - integration and differentiation, and bearing in mind the boundary conditions (1), the previous equation can be written as:

$$\frac{d}{dx}\left[\int_0^\infty \rho u\left(\frac{r}{L}\right)^j (u_e-u)\,\mathrm{d}y\right]-\rho_w v_w\left(\frac{r}{L}\right)^j u_e=$$
$$=\left(\frac{r}{L}\right)^j \frac{\mathrm{d}u_e}{\mathrm{d}x}\int_0^\infty(\rho u-\rho_e u_e)\mathrm{d}y+\left(\frac{r}{L}\right)^j\left(\mu\frac{\partial u}{\partial y}\right)_{y=0} . \tag{6}$$

In order to obtain the simplest form of the momentum equation, which would be formally the same as the corresponding momentum equation in incompressible fluid, it is necessary, for further solution of the integrals in the equation (6), to introduce new variables instead of physical coordinates x and y. The methodology of introduction and the necessity of these transformations are explained in details in [7]. Since this paper studies the axisymmetrical flow of dissociated gas, a new longitudinal variable $s(x)$ and a new transversal variable $z(x,y)$ are introduced in the form of relations

$$s(x)=\frac{1}{\rho_0\ \ \mu_0}\int_0^x \rho_w\ \ \mu_w\left(\frac{r}{L}\right)^{2j}dx,$$
$$z(x,y)=\frac{1}{\rho_0}\left(\frac{r}{L}\right)^j\int_0^y \rho\,\mathrm{d}y. \tag{7}$$

In the transformations (7), ρ_0 and μ_0 denote arbitrary known values of the density and dynamic viscosity at a certain point of the boundary layer, while ρ_w and μ_w are the given values of these quantities at the inner edge of the boundary layer. Krivtsova used these transformations in her studies ([10] for $j=0$ and [11] for $j\neq 0$), *for the case of a nonporous contour of the body within the fluid*.

We should point out that due to the factor $(r/L)^2$ and r/L (for $j=1$), these new previous variables (7) also contain Mangler-Stepanov's transformations [8]. Otherwise, transformations (7) without these factors are known in the literature as Dorodnicin's transformations modified by Lees [10].

Since, in some terms of the equation (6), the integration is performed transversally to the boundary layer, the variable z changes only due to the change of the coordinate y (for any x). Therefore, by means of the newly introduced variables (7), the equation (6) is brought to the form:

$$\frac{\mathrm{d}}{\mathrm{d}s}(u_e^2\Delta^{**})+u_e\frac{\mathrm{d}u_e}{\mathrm{d}s}\Delta^*=\left(\frac{r}{L}\right)^j\frac{\rho_w v_w u_e}{\rho_0\ \ \mathrm{d}s/\mathrm{d}x}+$$
$$+\left(\frac{r}{L}\right)^{2j}\frac{\rho_w\mu_w}{\rho_0^2\ \ \mathrm{d}s/dx}\ \frac{u_e\zeta}{\Delta^{**}}\ ,\qquad u_e=u_e(s), \tag{8}$$

in which the conditional displacement thickness $\Delta^*(s)$, conditional momentum loss thickness $\Delta^{**}(s)$ and nondimensional friction function $\zeta(s)$ are determined in the form of the known expressions

$$\Delta^*(s)=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{u}{u_e}\right)\mathrm{d}z,$$
$$\Delta^{**}(s)=\int_0^\infty\frac{u}{u_e}\left(1-\frac{u}{u_e}\right)\mathrm{d}z, \tag{9}$$
$$\zeta(s)=\left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})}\right].$$

After differentiation and multiplication of the equation (8) by $2\Delta^{**}/\nu_0 u_e^2$ $(\mu_0/\rho_0=\nu_0)$, we will obtain the equation:

$$\frac{d}{ds}\left(\frac{\Delta^{**2}}{\nu_0}\right)+\frac{2}{u_e}\frac{u_e'\ \Delta^{**2}}{\nu_0}\left(2+\frac{\Delta^*}{\Delta^{**}}\right)=$$
$$=\frac{2\zeta}{u_e}+\frac{2}{(r/L)^j}\ \frac{v_w\ \ \mu_0\Delta^{**}}{u_e\ \ \mu_w\nu_0}. \tag{10}$$

When, as with homogenous or electroconductive liquid or other cases of compressible fluid flow, we introduce the parameter of the form f and widely accepted notations for physical quantities in the theory of the boundary layer:

$$f=f_1=u_e'\ \ Z^{**}=f(s),\quad Z^{**}=\frac{\Delta^{**2}}{\nu_0},$$
$$H=\frac{\Delta^*}{\Delta^{**}}\ ,\qquad (u_e'=\mathrm{d}u_e/\mathrm{d}s), \tag{11}$$

the equation (10) can be written in its final form

$$\frac{\mathrm{d}Z^{**}}{\mathrm{d}s}=\frac{F_{ot}}{u_e} \quad . \tag{12}$$

In the obtained momentum equation, the characteristic function of the boundary layer F_{ot} is determined as

$$F_{ot}=2\left[\zeta-(2+H)f\right]+\frac{2}{(r/L)^j}\frac{v_w\,\Delta^{**}}{\nu_0}\frac{\mu_0}{\mu_w} \quad . \tag{13}$$

For $j=0$, this function is the same as the function F_{dp} at the planar flow [7]. In the case of a boundary layer flow along a nonporous wall $(v_w=0)$, the expression for the characteristic function F_{ot} comes down to the same form as the known expression [2], which corresponds to incompressible fluid flow $(F_{ot}=F)$. Furthermore, by means of (11), the momentum equation (12) can be written in its two forms

$$\frac{\mathrm{d}f}{\mathrm{d}s}=\frac{u_e'}{u_e}F_{ot}+\frac{u_e''}{u_e'}f,\quad \frac{\Delta^{**'}}{\Delta^{**}}=\frac{u_e'}{u_e}\frac{F_{ot}}{2f},\quad (f=f_1). \tag{14}$$

They are formally the same as the corresponding forms of this equation for incompressible fluid (where $'$ stands for a derivation per *s*).

If with the axisymmetrical flow of compressible fluid, we define the *porosity parameter* in the following way:

$$\Lambda=-\frac{1}{(r/L)^j}\frac{v_w\quad\Delta^{**}}{\nu_0}\frac{\mu_0}{\mu_w}=\Lambda(s),\qquad (j=1), \tag{15}$$

the characteristic boundary layer function F_{ot} can be written as the final expression:

$$F_{ot}=2\left[\ \zeta-(2+H)f\right]-2\Lambda . \tag{16}$$

It is obvious that the obtained expression (16) is formally the same as the corresponding expression for the characteristic function of incompressible fluid [2]. The defined parameter characterizes injection or ejection of the dissociated gas in the boundary layer. The expression for this parameter can be written in the form of

$$\Lambda=-\frac{\mu_0}{\mu_w}\ v_w\ \frac{1}{(r/L)^j}\ \frac{\Delta^{**}}{\nu_0}=-\frac{V_w\ \ \Delta^{**}}{\nu_0}\ ,$$

$$V_w=\frac{\mu_0}{\mu_w}\ v_w\frac{1}{(r/L)^j}=V_w(s),\qquad (j=1),$$

where $V_w(s)$ can be called the conditional injection velocity.

4. INTRODUCTION OF A SET OF POROSITY PARAMETERS OF LOITSIANSKII TYPE

As already stated in the Introduction, the general similarity method is based on the use of the appropriate sets of parameters of Loitsianskii type. Therefore, in this section we will define a set of porosity parameters at the axisymmetrical flow along a porous contour of the rotating body. Because of the relation $Z^{**}=\Delta^{**2}/\nu_0$, the first, i.e. the primary porosity parameter can also be defined by the expression

$$\Lambda=-\frac{V_w}{\sqrt{\nu_0}}\ \ Z^{**^{1/2}}=\Lambda_1(s) \tag{17}$$

from which we get

$$Z^{**}=\frac{\Lambda^2\nu_0}{V_w^2}\ .$$

Based on the previous relation and the momentum equation (12) written in the form of $\frac{\mathrm{d}}{\mathrm{d}s}\ (\ \Lambda^2\nu_0/V_w^2\ )=\frac{F_{ot}}{u_e}$, and after the differentiation, it is relatively easy to obtain the equation

$$\frac{\mathrm{d}\Lambda}{\mathrm{d}s}=\frac{\mathrm{d}\Lambda_1}{\mathrm{d}s}=\frac{u_e'}{u_e f_1}\left(\frac{1}{2}F_{ot}\Lambda_1-u_e\frac{V_w'}{\sqrt{\nu_0}}Z^{**^{3/2}}\right)\ .$$

Accepting the second addend in the brackets of the previous equation as a new parameter Λ_2,

$$\Lambda_2=-\ u_e\frac{V_w'}{\sqrt{\nu_0}}Z^{**^{3/2}},$$

this equation reduces to:

$$\frac{u_e}{u_e'}f_{\ 1}\frac{\mathrm{d}\Lambda_1}{\mathrm{d}s}=\frac{1}{2}F_{ot}\ \Lambda_1+\Lambda_2=\chi_1 .$$

If by a usual Loitsianskii procedure, as with other fluid flow problems, the parameter Λ_2 is differentiated per the variable *s*, we will get the equation

$$\frac{u_e}{u_e'}\ \ f_1\ \ \frac{\mathrm{d}\Lambda_2}{\mathrm{d}s}=\left(f_1+\frac{3}{2}\ \ F_{ot}\right)\Lambda_2+\Lambda_3=\chi_2\ ,$$

in which Λ_3 denotes the third porosity parameter determined by the expression

$$\Lambda_3=-u_e^2\frac{V_w''}{\sqrt{\nu_0}}\quad Z^{**^{5/2}}\ .$$

By differentiation of the parameter Λ_3 per the variable *s* we will obtain the corresponding equation

$$\frac{u_e}{u_e'}\ f_1\ \frac{\mathrm{d}\Lambda_3}{\mathrm{d}s}=\left(2f_1+\frac{5}{2}F_{ot}\right)\Lambda_3+\Lambda_4=\chi_3\ ,$$

where

$$\Lambda_4=-u_e^3\ \ \frac{V_w'''}{\sqrt{\nu_0}}\quad Z^{**^{7/2}}\ ,$$

represents a new parameter.

According to the shown procedure, it is clear that the general porosity parameter $\Lambda_k(s)$ can be written in the form of the expression

$$\Lambda_k = - u_e^{k-1} \left(\frac{V_w}{\sqrt{\nu_0}} \right)^{(k-1)} Z^{** \, k-1/2}, \quad (k = 1,\ 2, 3, ...) \quad (18)$$

Furthermore, each of the parameters of the previous set, as concluded in our analysis, satisfies a recurrent differential equation

$$\begin{gathered} \frac{u_e}{u_e'} \; f_1 \; \frac{\mathrm{d}\Lambda_k}{\mathrm{d}s} = \\ = \left\{ (k-1) f_1 + \left[(2k-1)/2 \right] F_{ot} \right\} \Lambda_k + \Lambda_{k+1} \, . \end{gathered} \quad (19)$$

This equation can be also written as

$$\frac{u_e}{u_e'} \, f_1 \, \frac{\mathrm{d}\Lambda_k}{\mathrm{d}s} = \chi_k \, , \quad (20)$$

where

$$\chi_k = \left\{ (k-1) f_1 + \left[(2k-1)/2 \ \right] F_{ot} \right\} \Lambda_k + \Lambda_{k+1} \, . \quad (21)$$

At the end of this section, it should be pointed out that the set of porosity parameters Λ_k (18), defined in this paper is formally the same as the corresponding set of parameters in the case of the incompressible fluid flow [2]. The corresponding recurrent relations (19) also have the same form. Obviously, the porosity parameters given in this paper are a function of a newly introduced variable s instead of the physical variable x (which is the case with incompressible fluid flow). Defining of a set of porosity parameters (18) creates a possibility to apply the general similarity method to the considered flow problem, i.e. to bring the governing equations to the so called generalized form and to solve them numerically.

5. TRANSFORMATION OF THE GOVERNING EQUATIONS

Transformations of the variables (7) should be applied to the boundary layer equations system of the dissociated gas flow on the rotating bodies, i.e. to the continuity equation (3) and to the dynamic and energy equations of the system (1) with the corresponding boundary conditions. As with other flow problems, a stream function $\psi(x, y)$ is introduced. The form of the continuity equation (3) suggests that the stream function should be introduced by the following relations:

$$u = \frac{\rho_0}{\rho} \frac{1}{(r/L)^j} \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\rho_e}{\rho} \frac{1}{(r/L)^j} \frac{\partial \psi}{\partial y} \, .$$

However, after new variables (7) have been introduced, the function $\left[\psi(x, y) \to \psi(s, z) \right]$ should be defined in accordance with the equations:

$$\begin{gathered} u = \frac{\partial \psi}{\partial z} \ , \\ \tilde{v} = \frac{\rho_0 \ \mu_0}{\rho_w \ \mu_w \ (r/L)^{2j}} \left[u \, \frac{\partial z}{\partial x} + v \frac{\rho}{\rho_0} \left(\frac{r}{L} \right)^j \right] = -\frac{\partial \psi}{\partial s} \ . \end{gathered} \quad (22)$$

These relations also come from the continuity equation of this compressible fluid flow problem. Note that the second relation (22) for $j = 0$, comes down to the relation Krivtsova once used in her investigations of the planar dissociated gas flow [10].

Changing the variables and using the relation (22), the governing equation system reduces to:

$$\begin{gathered} \frac{\partial \psi}{\partial z} \frac{\partial^2 \psi}{\partial s \partial z} - \frac{\partial \psi}{\partial s} \frac{\partial^2 \psi}{\partial z^2} = \frac{\rho_e}{\rho} u_e \frac{\mathrm{d}u_e}{\mathrm{d}s} + \nu_0 \, \frac{\partial}{\partial z} \left(Q \, \frac{\partial^2 \psi}{\partial z^2} \right), \\ \frac{\partial \psi}{\partial z} \frac{\partial h}{\partial s} - \frac{\partial \psi}{\partial s} \frac{\partial h}{\partial z} = - \frac{\rho_e}{\rho} u_e \frac{du_e}{ds} \frac{\partial \psi}{\partial z} + \\ + \nu_0 \, Q \left(\frac{\partial^2 \psi}{\partial z^2} \right) + \nu_0 \frac{\partial}{\partial z} \left[\frac{Q}{Pr} (1+l) \frac{\partial h}{\partial z} \right], \\ \frac{\partial \psi}{\partial z} = 0 \ , \qquad \underline{\frac{\partial \psi}{\partial s} = -\frac{\mu_0}{\mu_w} v_w \, \frac{1}{(r/L)^j}} = -\tilde{v}_w = -V_w, \\ h = h_w \qquad \text{for} \qquad z = 0 \, , \\ \frac{\partial \psi}{\partial z} \to u_e(s), \qquad h \to h_e(s) \qquad \text{for} \qquad z \to \infty \, . \end{gathered} \quad (23)$$

In the obtained equations, the nondimensional function Q is determined as

$$\begin{gathered} Q = \frac{\rho \ \mu}{\rho_w \ \mu_w}, \qquad Q = 1 \qquad \text{for} \qquad z = 0, \\ Q = \frac{\rho_e \ \mu_e}{\rho_w \ \mu_w} = Q(s) \qquad \text{for} \qquad z \to \infty \ . \end{gathered} \quad (24)$$

6. CONCLUSION

Based on the obtained equations (23), we can draw some conclusions important for further investigation of the boundary layer fluid flow. They are:

- The obtained equations (23) are completely the same as the equations obtained in [10] for the planar problem of the dissociated gas flow. This conclusion is expected because it was made possible by the transformations (7) that contain the factors $(r/L)^2$ and r/L. Obviously, the boundary conditions are different.
- The transformed dynamic equation (23) has almost the identical form as the corresponding equation for the case of the incompressible fluid flow [2] expressed by means of the stream function $\psi(x, y)$. In the case of an isothermal flow of incompressible fluid ($\rho = \text{const.}$, $\mu = \text{const.}$, $Q = 1$) these equations are identical.
- Due to the boundary conditions at the outer edge of the boundary layer ($\partial \psi / \partial s = -V_w$ for $z = 0$), by dividing the stream function into two addends, these conditions can be brought to a form that applies to a nonporous wall of the body within the fluid (as is the case with incompressible fluid [2]).
- This paper defines a set of porosity parameters for the case of the dissociated gas flow along rotating bodies ($j = 1$) which enables application of the general similarity method (Saljnikov's version) to this complicated compressible fluid flow problem.

In conclusion, the essential contribution of this paper lies in defining the primary porosity parameter Λ (15), obtained from the momentum equation (13) derived in the paper, as well as in introduction of the corresponding set of parameters Λ_k (18). This set of parameters is a generalization of the already known set of parameters of Loitsianskii type for the case of incompressible fluid flow along a porous contour.

Finally, we should especially point out that in our further investigations we will use the introduced set of porosity parameters (18) with the aim to apply the general similarity method to the considered problem of compressible fluid flow, i.e., to solve the generalized equations numerically.

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ИМПУЛСНА ЈЕДНАЧИНА ГРАНИЧНОГ СЛОЈА НА ОБРТНИМ ТЕЛИМА ПРИ СТРУЈАЊУ ДИСОЦИРАНОГ ГАСА ПОРЕД ПОРОЗНЕ КОНТУРЕ

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Истражује се ламинарни стационарни гранични слој на обртним телима са порозном контуром при осносиметричном струјању стишљивог флуида. Реч је о струјању дисоцираног гаса у условима равнотежне дисоцијације. У раду је изведена импулсна једначина разматраног проблема струјања. Дефинисан је основни параметар порозности а затим и одговарајући скуп параметара порозности. Показано је да се за решавање разматраног проблема струјања, после увођења неопходних сврсисходних трансформација, може да примени метода уопштене сличности у верзији В. Н. Саљникова.