

Numerical Simulation of Condensation Induced Water Hammer

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A numerical model for the simulation and analysis of the water hammer in the pipe two-phase flow is developed. The modelling is based on one-dimensional homogeneous model of two-phase flow, tracking of the interface between steam volume and water column and modelling of the direct condensation of steam on subcooled liquid. The mass, momentum and energy conservation equations are solved by the method of characteristics. For these three equations, there are three characteristic directions: two of them are determined by the pressure wave propagation and the third one by the fluid particle propagation. The fluid particle and the steam-water interface tracking are obtained through the energy conservation equation solving in space, with the accuracy of the third degree. The value of thermodynamic quality is used to determine whether the observed computational region is filled with water, two-phase mixture or steam. The term in the energy conservation equation, which contains information about the heat exchanged between steam and liquid phase through condensation, is determined by integration of superficial heat flux over steam-water interface. The model is applied to the simulation and analysis of the air-water interface propagation in the experimental apparatus of oscillating manometer and the condensation induced water hammer in a vertical pipe for draining of steam into the pool filled with subcooled water.

Keywords: water hammer, modelling, condensation, method of characteristics.

1. INTRODUCTION

A direct contact between subcooled liquid and steam can lead to the condensation induced water hammer. The condensation occurs at the interface between water and steam. During the phase change from vapour to liquid, the specific volume drastically decreases and the steam pressure drops, causing the water column movement towards the steam. The steam condensation continues towards the propagating water-steam interface. The water column accelerates until it hits into the obstacle (e.g. a closed end of the pipe, a closed valve or another water column). The water column velocity is decreased to zero when the water column hits the obstacle, which leads to the high pressure peak at the surface of water splashing. In case of intensive condensation, a high velocity of the water column can be developed and a destructive pressure increase can be induced. According to available measured data, maximal pressure peaks induced by the intensive condensation and subsequent water hammer splashing could reach values of 10 MPa and even higher, although the initial pipe pressure is equal to the atmospheric pressure [2,3]. Such a pressure peak can fracture a cast-iron valve or blow out a steam gasket. Amplitudes of the pressure wave, induced by the direct condensation

of steam onto the head of the water column, may cause a damage of the equipment, which could result in the serious impact on the safety of employees.

There are several characteristic two-phase flows with a high possibility of water hammer occurrence: the steam – water flow in the horizontal pipe, the steam – subcooled water flow in the vertical pipe, the pressurized water inflow in the vertical pipe filled with steam, and the hot water inflow in the cold low pressure pipeline. In the time period between 1969 and 1981 over one hundred reported water hammer incidents occurred in nuclear power plants in USA [3]. The feedwater line rupture at the plant Indian Point No. 2 in 1973 was also consequence of a condensation induced water hammer [3]. In the district heating system in Fort Wainwright, Alaska [2], and in the steam pipeline of the fire-tube shell boiler in the Serbian Clinical Centre [5], the same cause led to the hot water and steam explosion, to the cracking of valve and the steamline burst.

These numerous examples from the practice of thermal power equipment and plants pose a strong need for the development of thermal-hydraulic methods that are able to investigate and predict the possibility of water hammer occurrence and support the implementation of necessary safety measures that should prevent this kind of accidents.

In here presented research, the two-phase flow that leads to the condensation induced water hammer is described with homogeneous two-fluid model. The problem is considered as one-dimensional. The model consists of mass, momentum and energy conservation equations. These equations form quasi-linear system of

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hyperbolic partial differential equations. The method of characteristics is applied for the numerical solution of the obtained system of equations.

2. MODELLING APPROACH

3.1 Mathematical model of one-dimensional transient flow of a homogeneous fluid

One-phase and two-phase transient flows in pipes are described by the homogeneous model. Both velocity and thermal equilibrium are assumed in case of two-phase flow [4]. The following form of the mass, momentum and energy conservation equations is applied:

- mass conservation

$$\frac{D\rho}{Dt} + \rho \frac{\partial u}{\partial x} = 0, \quad (1)$$

- momentum conservation

$$\frac{Du}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{fu|u|}{2D_H} + f_t \text{sign}(u) \left| \frac{\partial u}{\partial t} \right| + g \sin \theta = 0, \quad (2)$$

- energy conservation

$$\frac{Dh}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} - \frac{fu^2|u|}{2D_H} - f_t u \text{sign}(u) \left| \frac{\partial u}{\partial t} \right| + \int_{x_i-\varepsilon}^{x_i+\varepsilon} \frac{\dot{q}_A a_i}{\rho} \delta(x-x_i) dx = 0. \quad (3)$$

The last term in the energy equation (3) describes heat flux per unit of mass due to phase change, where

$$\dot{q}_A = h_{\text{cond}}(T_g - T_l) \quad (4)$$

is the heat flux per unit of interface area, a_i is the specific water-steam interface area, $\delta(x-x_i)$ is Delta function that has characteristic $\int_{x_i-\varepsilon}^{x_i+\varepsilon} \delta(x-x_i) dx = 1$,

where ε is the short thickness at the interface surrounding. The interface is located at x_i . The fourth terms on the left hand sides of the momentum and energy equations take into account the influence of the transient friction [4].

Additional equation is equation of state

$$\rho = \rho(p, h). \quad (5)$$

After mathematical transformations, including application of material derivative, combining (1), (3) and (5), introducing sonic velocity in one-phase fluid flow as a function of pressure and enthalpy

$$c = \sqrt{\frac{1}{\left(\frac{\partial \rho}{\partial p}\right)_h + \frac{1}{\rho} \left(\frac{\partial \rho}{\partial h}\right)_p}}, \quad (6)$$

and grouping of all partial differentials over time and coordinate on the left hand side of equations, the system of quasi-linear partial differential equations is obtained

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + c^2 \rho \frac{\partial u}{\partial x} = X, \quad (7)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = Y, \quad (8)$$

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} - \frac{1}{\rho} \left(\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} \right) = Z. \quad (9)$$

where

$$X = -c^2 \left(\frac{\partial \rho}{\partial h} \right)_p \left(\frac{fu^2|u|}{2D_H} + f_t u \text{sign}(u) \left| \frac{\partial u}{\partial t} \right| - g u \sin \theta - \int_{x_i-\varepsilon}^{x_i+\varepsilon} \frac{\dot{q}_A a_i}{\rho} \delta(x-x_i) dx \right), \quad (10)$$

$$Y = -\frac{fu|u|}{2D_H} - f_t \text{sign}(u) \left| \frac{\partial u}{\partial t} \right| - g \sin \theta, \quad (11)$$

$$Z = \frac{fu^2|u|}{2D_H} + f_t u \text{sign}(u) \left| \frac{\partial u}{\partial t} \right| - \int_{x_i-\varepsilon}^{x_i+\varepsilon} \frac{\dot{q}_A a_i}{\rho} \delta(x-x_i) dx. \quad (12)$$

This system of equations is solved for the appropriate initial and boundary conditions by the method of characteristics. Three characteristic paths are used, where two correspond to the pressure wave propagation and the third to the propagation of the fluid particle enthalpy front. The spatial step of integration is constant within one pipe segment. The time step of integration is determined according to the Courant criterion:

$$\Delta t \leq \min \left(\frac{\Delta x}{c_J + |u_J|} \right), \quad J = 1, 2, \dots, n. \quad (13)$$

The mass (1) and momentum (2) conservation equations are solved along the characteristic paths C^+ and C^- , in Figure 1, while energy conservation equation is solved along C^p path.

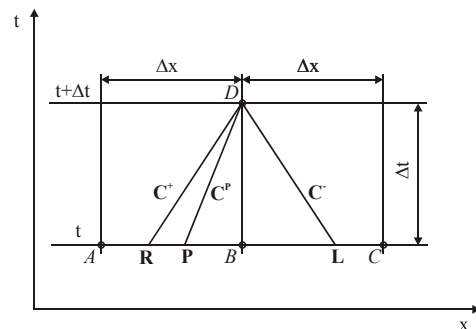


Figure 1. Spatial coordinate (x) – time (t) plane and characteristic paths ($u_p > 0$)

3.2 Numerical solution of the energy conservation equation

The energy conservation equation is solved numerically by non-standard application of the method of characteristics. This method transforms the partial

differential equation (3) into the ordinary differential equation

$$dh - \frac{1}{\rho} dp = Z dt, \quad (14)$$

along C^P characteristic path, determined by the fluid particle propagation

$$\frac{dt}{dx} = \frac{1}{u}. \quad (15)$$

The source term Z in the enthalpy equation is defined with (12). Total derivatives in (14) are approximated with finite differences, giving

$$(h_D - h_P) - v_P(p_D - p_P) = Z_P \Delta t \quad (16)$$

where indexes D and P denote nodes in Figure 1.

Points A, B and C in Figure 1 are three successive nodes in the flow channel, which are used for calculation of dependant variables at time level t . The point D in Figure 1 denotes the node where disturbance arrives at time level $t+\Delta t$, starting from initial nodes R and L at time level t ; hence, D denotes the intersection point of the characteristic path C^P and the x -axis at time level t . Coordinate x_P is determined using the slope of the characteristic path C^P (17) and the linear interpolation of the velocity between nodes A and B (18) for positive flow direction, and between nodes B and C (19) for negative flow direction.

$$\frac{\Delta t}{x_B - x_P} = \frac{1}{u_P}, \quad (17)$$

$$\frac{x_B - x_P}{x_B - x_A} = \frac{u_B - u_P}{u_B - u_A} \quad \text{for} \quad \frac{dt}{dx} = \frac{1}{u_B} \geq 0, \quad (18)$$

$$\frac{x_B - x_P}{x_B - x_C} = \frac{u_B - u_P}{u_B - u_C} \quad \text{for} \quad \frac{dt}{dx} = \frac{1}{u_B} < 0. \quad (19)$$

The following expressions for the determination of point P x -axis coordinate are derived using (17) – (18).

$$x_P = x_B - \frac{u_B \Delta t}{1+a} \quad \text{for} \quad \frac{dt}{dx} = \frac{1}{u_B} \geq 0, \quad (20)$$

where

$$a = (u_B - u_A) \frac{\Delta t}{\Delta x} \quad (21)$$

and

$$x_P = x_B - \frac{u_B \Delta t}{1+e} \quad \text{for} \quad \frac{dt}{dx} = \frac{1}{u_B} < 0, \quad (22)$$

where

$$e = (u_B - u_C) \frac{\Delta t}{\Delta x}. \quad (23)$$

For the calculation of enthalpy in node D, denoted as h_D in (16), the initial value of enthalpy at point P, h_P , is needed. Enthalpy h_P is determined by the Lagrange's interpolation polynomial (LIP) of the third degree. To

form the LIP of the third degree, it is necessary to determine coordinates of four nodes. The choosing of nodes is applied according to flow direction, one node downstream and two nodes upstream of the observed node, taking into account the observed node (denoted by i in Fig. 2).

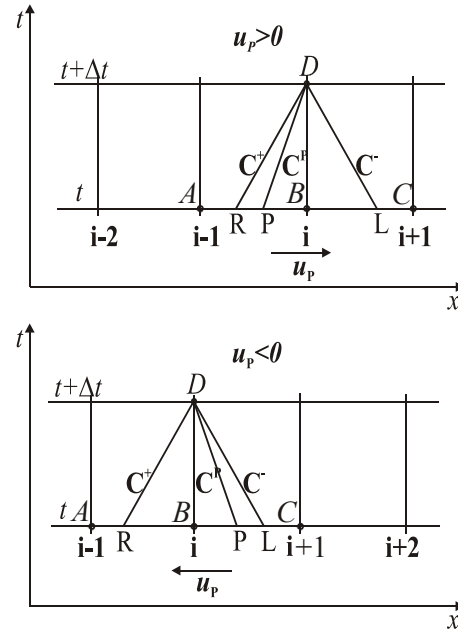


Figure 2. Determination of the nodes for LIP of the third degree, for positive ($u_p > 0$) and negative ($u_p < 0$) flow

3. NUMERICAL RESULTS

3.1 Air-water interface propagation in oscillating manometer

The apparatus, shown in Figure 3, consists of the U-tube manometer [1] connected at the top. The total length of the manometer is $L = 20$ m and its diameter is $d = 1$ m. At the beginning of the simulation the system was filled with water and air so that initial levels of the water were equal in both arms of the manometer, which was 5 m from the bottom. All parts of the fluid system have initial velocity $u_0 = 2.1$ m/s, but zero acceleration. The movement of the interface, denoted by x , is measured from the initial state when water levels in manometer are equal. The aims of this calculation are:

- to determine the ability of the numerical solution method to preserve mass in the system, which is constant,
- to track the propagation of the air-water interface and
- to model the oscillating period, which is analytically predictable.

The initial value of the temperature is 50 °C and it is equal in the whole system. Pressure at the interface, as well as in the above air region, is equal to atmospheric pressure. In the part of the U-tube which is filled with water, the pressure has different values in accordance with the hydrostatic pressure change.

The developed numerical model does not have implemented appropriate equation for determination of

thermodynamic values of state of air, it only has relations for water and steam. Hence, it is necessary to adjust initial parameters. Numerical simulation is performed with steam, instead of air, at the pressure at which the steam has the same density as air has in the experimental apparatus. The value of the steam enthalpy is a bit higher than the saturation enthalpy, in order to keep steam out of the two-phase area during calculation. The difference in the pressure in the experiment and in the numerical simulation practically does not induce change in the water density. The condensation is not considered.

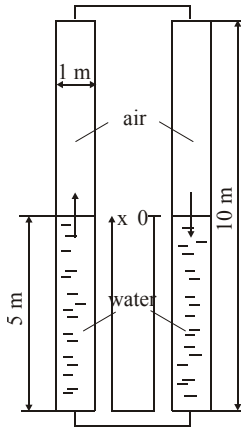


Figure 3. Schematic view of the oscillation manometer

Table 1. Initial parameters of water and steam used in numerical simulation of air-water interface propagation in oscillating manometer

Water	Steam
$p_1 = 0.1936 \text{ MPa}$	$p_2 = 0.1936 \text{ MPa}$
$h_1 = 209.3 \text{ kJ/kg}$	$h_2 > h''(p_2) = 2800 \text{ kJ/kg}$

Differential equation of the air-water interface propagation under the influence of the gravitational force and in the absence of the friction is

$$\ddot{x} + \frac{4g}{L}x = 0. \quad (24)$$

The equation is solved for the following initial conditions

$$x|_{t=0} = 0, \quad \dot{x}|_{t=0} = \frac{dx}{dt} = u_0 = 2.1 \text{ m/s} \quad (25)$$

and it is obtained

$$x = u_0 \sqrt{\frac{L}{4g}} \sin\left(t \sqrt{\frac{4g}{L}}\right), \quad (26)$$

and the water velocity is predicted as:

$$u = \frac{dx}{dt} = u_0 \cos\left(t \sqrt{\frac{4g}{L}}\right). \quad (27)$$

Comparison between results of the numerical simulation of the air-water interface propagation and analytical solution (26) is shown in Figure 4 and of the numerical simulation of the mass flow rate at the interface and analytical solution in Figure 5.

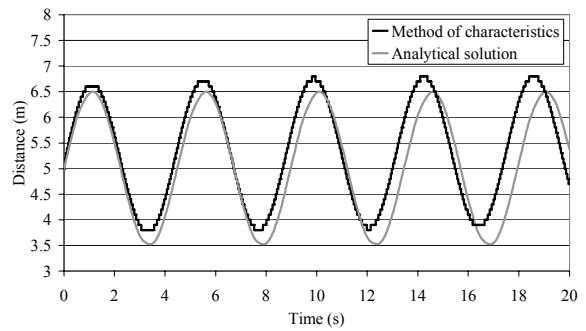


Figure 4. Propagation of the air-steam interface during time in U-tube manometer. Comparison of the analytical solution and numerical results obtained by the use of the method of characteristics

Water column starts to move with the velocity of 2.1 m/s, in the x direction Figure 3, pushing the air in the left arm of the manometer upwards. Gravitational force returns the water column downwards, simultaneously pushing the air in the right arm of the manometer towards the head of the arm. The procedure is repeating itself continually, because there is no friction on the tube walls of the manometer.

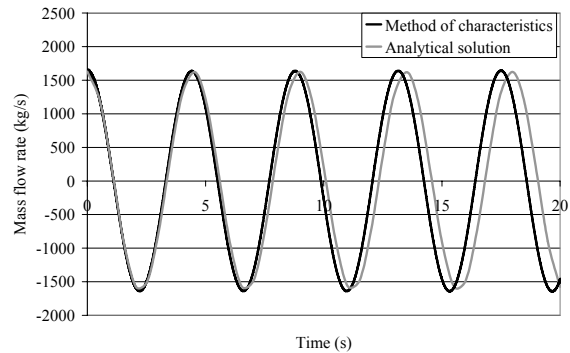


Figure 5. The change of the mass flow rate at the interface during time in U-tube manometer. Comparison of the analytical solution and numerical results obtained by the use of the method of characteristics

Obtained oscillating period is the same for each method in case of the interface propagation presented in Figure 4, while it differs in case of the mass flow rate in Figure 5, approximately 4 % higher period is obtained analytically. It causes increase of the time delay between amplitudes obtained by different methods, Figure 5. Amplitudes of oscillation, obtained by the method of characteristics and analytically, are equal in case of the mass flow rate at the interface. Numerical result of the interface propagation during time has higher amplitudes than analytical result Figure 4.

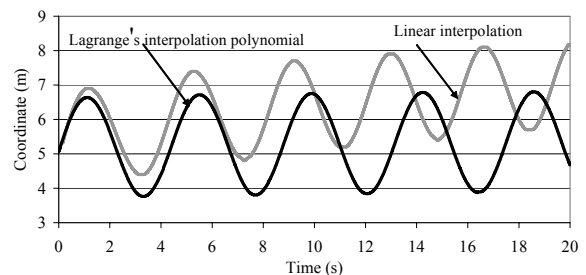


Figure 6. Comparison of the propagation of the air-steam interface during time in U-tube manometer obtained by the use of Lagrange's interpolation polynomial and linear interpolation for determination of the initial values of the enthalpy and pressure between nodes

Figure 6 illustrates significance of application of higher order interpolation polynomial in determination of initial values of enthalpy and pressure between nodes. The application of the linear interpolation is less accurate.

3.2 Condensation induced water hammer in vertical pipe for draining of the steam into the pool filled with subcooled water

Experimental apparatus shown in Figure 7 consists of a 0.7112 m long, 0.0381 m inner diameter vertical metal pipe with steam introduced from the top. The bottom of the pipe is submerged several centimetres in a large reservoir of water [6].

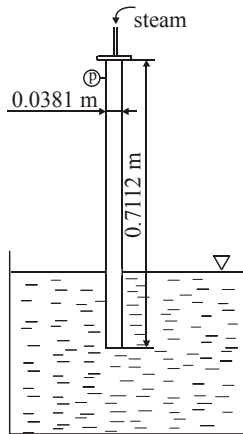


Figure 7. Schematic view of experimental apparatus

The pipe was initially full of water at the temperature 18 °C. Steam is introduced at a constant rate through a small pipe at the top of the pipe. Steam causes the warming of the pipe walls, suppressing the water downwards to the reservoir and eventually emptying the pipe.

The problem is observed from the moment when the vertical pipe is filled only with steam. Initial temperature of the water inside the reservoir is 49 °C [6].

When the water level in the vertical pipe comes to the open end of the pipe, steam comes into contact with subcooled water from the reservoir, which induces intensive steam condensation and occurrence of water hammer.

Table 2. Initial parameters of water and steam in case of numerical simulation of condensation induced water hammer in vertical pipe for draining of the steam into the pool filled with subcooled water

Water	Steam
$p_1 = 0.1023 \text{ MPa}$	$p_2 = 0.1023 \text{ MPa}$
$t_1 = 49 \text{ °C}$	$t_2 = t_{\text{sat}}(p_2)$
$h_1 = 209.35 \text{ kJ/kg}$	$h_2 = h''(p_2)$

Initial velocity in all parts of the system is 0 m/s.

Figures 8 and 9 present the change of pressure during transient in the vicinity of closed end of the vertical pipe filled with steam and submerged into the pool filled with subcooled water. Direct contact of the steam and subcooled water leads to intensive condensation. Steam pressure drops and water starts to run into the pipe. Due to pressure difference, condensation moves toward the end of the pipe where eventually water column hits. This

process happens in a very short period of time, water column hits the closed end of the pipe in approximately 0.3 s, therefore the name of this process – water cannon. First spike has the maximum value of the pressure. Practically, this spike is the most dangerous one. It has potential to cause serious damage of the equipment. After hitting at the closed end, pressure wave moves downwards lowering the pressure and heading towards the reservoir. Propagation of the reflected pressure wave causes pressure drop in the region above the water column, so that steam evaporates and two-phase mixture is formed. Water column starts moving upwards and in the 0.45 s it hits again the closed end, but this time with lower intensity. Elapsed time between two pressure peak rising coincides with the time elapsed between two strikes at the top end of the pipe.

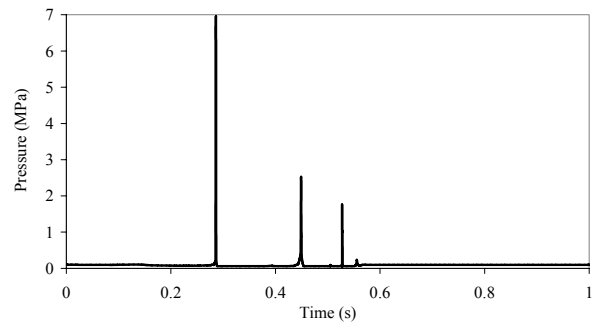


Figure 8. Pressure change during time in the vicinity of the close end of the vertical pipe. Numerical results obtained using developed computer code TEA. Steam condensation in the vertical pipe starts at 0 s

The results obtained using computer codes RELAP3/MODE5 [6] and developed code TEA are compared. It is important to accentuate that diagram in the Figure 9 comprises results of the whole experiment i.e. first phase – steam injection into the pipe, from 0 – 0.8 s, and second phase – water hammer from 0.8 s. The calculated peak pressure of the first pressure spike, and the time of its appearance show good agreement with available results from literature.

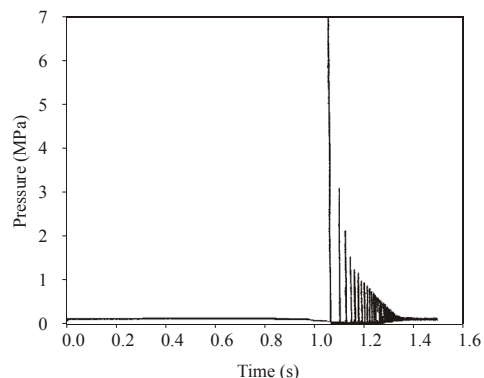


Figure 9. Pressure change during time in the vicinity of the close end of the vertical pipe. Numerical results obtained using RELAP5/MOD3 computer code. Steam condensation in the vertical pipe starts at 0.8 s

4. CONCLUSION

The thermal-hydraulic model for the simulation and analysis of the condensation induced water hammer is

based on the transient one-dimensional model of homogeneous fluid flow. The model is solved by the method of characteristics. The energy equation is solved by applying the Lagrange's interpolation polynomial of the third degree for the calculation of enthalpy values at the points of intersection of the characteristics paths and numerical grids. The developed method is tested for idealized oscillations of water column in the U-tube manometer, and for the condensation induced water hammer in the steam vertical pipe submerged in the water pool. Obtained results for both simulated experiments show acceptable agreement with available analytical and numerical results.

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NOMENCLATURE

a	interface area [m^2/m^3]
c	sonic velocity [m/s]
d	diameter [m]
D_H	hydraulic diameter [m]
f	friction coefficient
g	gravity constant [m/s^2]

h	specific enthalpy [J/kg]
h	heat transfer coefficient [$\text{W}/(\text{m}^2\text{K})$]
\dot{m}	mass flow rate [kg/s]
p	pressure [Pa]
\dot{q}_A	heat flux [W/m^2]
t	time [s]
u	velocity [m/s]
x	spatial coordinate

Greek symbols

ρ	density [kg/m^3]
θ	pipe inclination

Index

cond	condensation
g	gas phase, steam
i	interface
l	liquid phase
P	particle
t	transient
,	saturated water
''	saturated steam
0	initial time

НУМЕРИЧКА СИМУЛАЦИЈА ХИДРАУЛИЧКОГ УДАРА ИЗАЗВАНОГ КОНДЕНЗАЦИЈОМ ПАРЕ

**Сања Прица, Владимир Стевановић, Блаженка
Масловарић**

Развијен је нумерички модел за симулацију и анализу хидрауличког удара који је заснован на једнодимензионалном хомогеном моделу двофазног струјања, праћењу разделне површине стуба течности и парног мехура и моделирању директне кондензације паре на потхлађеној течности. Систем билансних једначина је решен применом методе карактеристика. Интеграција је вршена дуж три карактеристична правца: два правца одређена су правцем простирања таласа притиска а трећи простирањем флуидног делића. Праћење флуидног делића и разделне површине воде и паре извршено је решавањем енергетске једначине у простору са тачношћу трећег реда. Количина топлоте коју пара преда течности при кондензацији одређена је интеграцијом површинског топлотног флукса по разделној површини парне и течне фазе на месту контакта паре и потхлађене течности. Модел је примењен на симулацију и анализу хидрауличког удара изазваног у неколико експерименталних апаратура.