Calculation of Flow Parameters Inside the High Pressure Line of Fuel Injection System in Diesel Engines

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Hydrodynamical processes in the high pressure line of a fuel injection system in diesel engines are described by hyperbolical partial differential equations. In order to solve the equations, it is necessary to reduce them to one of the characteristic forms and then, applying the appropriate method, calculate the values of the fluid pressure and velocity inside the high pressure line. In this paper, the method of finite differences with the separation of the flux vector was used for solving the equations of the characteristic form. The method enables the calculation of the flow parameters inside the high pressure line in a very simple and efficient way, avoiding the transformation of the equations at each point of the calculation domain.

Keywords: *high pressure line, equation, pressure, velocity, fluid.*

1. INTRODUCTION

When calculating and modeling the working process in the high pressure line, apart from determining the physical properties of the working fluid, it is necessary to apply the corresponding mathematical tool for solving the partial differential equations which describe the hydrodynamical processes. Due to the extreme complexity of the partial differential equations, the numerical methods are used for their solving. One should bear in mind that partial differential equations are simultaneously used with integral and differential equations which define the process boundary conditions in the high pressure line. Therefore, the precise determinations of the flow parameters inside the high pressure line depend not only on the equations which describe the processes in the high pressure line and the calculation methods, but on the way in which the equations for boundary conditions are formed and solved as well. It can be concluded that the calculation of the flow parameters inside the high pressure line is a very complex task.

In this paper, the hydrodynamical model of the flow processes in the high pressure line and the method of its solving by applying finite differences method with the separation of the flux vector will be presented.

2. THE HYDRODYNAMICAL PROCESSES IN THE HIGH PRESSURE LINE

In the fuel injection system of the "pump – high pressure line – injector" type, Fig. 1, fuel is injected under variable flow conditions and propagation impulses between the high pressure pump and the injector. The volumes which are fuelled are finite and the tube length is limited. The pressures in the high pressure line go up to several hundred bars, and in new systems up to 1200 bars.

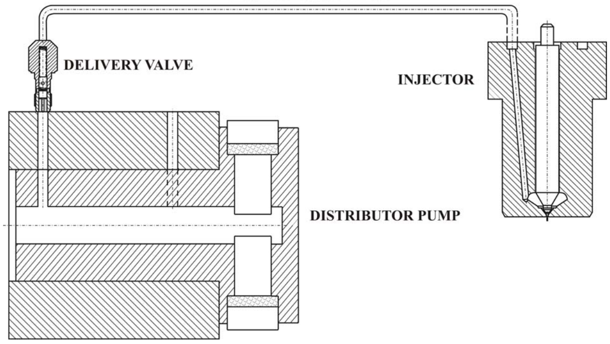

Figure 1. Fuel injection system with the piston-radial distribution pump

Pressure waves are propagated from the high pressure pump, with finite velocity – sound velocity. This flow is described by Navier-Stokes equations and the continuity equation of a complex form [1]. Considering the fact that this system of non-homogeneous partial differential equations is very complex in terms of integration, it is necessary to make some specific assumptions, which simplify and ease its solving.

The flow in the high pressure line is regarded as one-dimensional, i.e. the flow parameters depend only on one coordinate in direction of the high pressure line axis and the flow velocity vector direction coincides with it. Since the tube diameter is very small, the tube cross-section is considered constant and the bend of tube is negligible. The pressure wave is normal on the axis of the tube. It is considered that there is friction on interior surface of the tube wall, while the viscosity friction between fluid layers is negligible. The processes in high pressure line are regarded as isentropic.

Thus, Navier-Stokes equations and continuity equation are very simplified and obtain the following form:

Received: October 2008, Accepted: November 2008
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$$\frac{\partial \rho}{\partial t}+v\frac{\partial \rho}{\partial x}+\rho\frac{\partial v}{\partial x}=0\,,$$
$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial p_{tr}}{\partial x}=0\,. \qquad (1)$$

This system of two equations has three unknown variables and one more condition is necessary for solving it. In this case, it is best to use the equation of state. In general, the fluid density depends on pressure and temperature, but if the change of the working fluid state is isentropic and if Laplace equation for sound velocity is used, system of equations is reduced to the following:

$$\frac{\partial \rho}{\partial t}+v\frac{\partial \rho}{\partial x}+\rho\frac{\partial v}{\partial x}=0\,,$$
$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}+\lambda\frac{v|v|}{2d}=0\,. \qquad (2)$$

Non-linear partial differential equations system (2) has complex boundary conditions, and the exact analytical solution cannot be obtained. Because of that, various numerical and graphical methods are applied.

The process which is often used for solving this system consists of differential equations linearization. If we assume low fluid velocity in relation to sound velocity, we can neglect the convection component. Also, if we neglect the friction as a function of velocity square, the system of equations will be linear:

$$\frac{\partial p}{\partial t}+a^2\rho\frac{\partial v}{\partial x}=0\,,$$
$$\frac{\partial v}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0\,. \qquad (3)$$

3. SOLVING EQUATIONS WHICH DESCRIBE HYDRODYNAMICAL PROCESSES IN THE HIGH PRESSURE LINE

For solving the partial differential equations (3) which describe the hydrodynamical process of the flow in the high pressure line, we will use the method of finite differences with central pattern at space coordinate in combination with the separation of flux vector [2,3]. We will solve the one-dimensional equations systems in the following form:

$$\frac{\partial}{\partial t}\{v\}+\frac{\partial}{\partial x}\{E\}=0\,, \qquad (4)$$

i.e. the quasi-linear form:

$$\frac{\partial}{\partial t}\{v\}+\left[J(v)\right]\frac{\partial}{\partial x}\{v\}=0 \qquad (5)$$

where $\{E\}$ is the flux vector and $[J(v)]$ Jacobian, obtained by derivation of vector $\{E\}$ from vector $\{v\}$.

Only the central pattern for approximation of the derivative along x-axis gives stable calculation for positive and negative pressure waves. The application of non-symmetrical operators can increase stability, reduce the problems to the two-diagonal system of equations instead of the tree-diagonal one in implicit formulations, and provide better dispersion and dissipation characteristics. By approximating the partial derivative in space by a backward pattern, it can be concluded that asymmetrical approximation of the derivative cannot be constructed at space coordinate which will be simultaneously stable for both its positive and negative eigenvalues.

According to Euler theorem for homogeneous function, it follows that:

$$E=[J]\{v\}\,. \qquad (6)$$

As far as vector $\{E\}$ meets the necessary level of homogeneity and as far as $[J]$ has the appropriate number of linearly independent vectors, vector $\{E\}$ can be split in two parts, each suitable for its eigenvectors. One part will correspond only to positive own values, and the other to the negative ones, i.e.:

$$\{E\}=\{E^+\}+\{E^-\} \qquad (7)$$

where $\{E^+\}$ corresponds to positive eigenvectors of the matrix $[J]$ and $\{E^-\}$ to negative eigenvectors of the matrix $[J]$. The equation (7) can be made as follows:

$$\{E\}=([J^+]+[J^-])\{v\}=\{E^+\}+\{E^-\} \qquad (8)$$

where:

$$\{J\}=[J^+]+[J^-]\,, \qquad (9)$$

$$\{E^+\}=[J^+]\{v\}\,, \qquad (10)$$

$$\{E^-\}=[J^-]\{v\}\,. \qquad (11)$$

By writing (3) into a vector form, we get:

$$\frac{\partial}{\partial t}\begin{Bmatrix} v \\ p \end{Bmatrix}+\begin{bmatrix} 0 & 1/\rho \\ \rho\cdot a^2 & 0 \end{bmatrix}\frac{\partial}{\partial x}\begin{Bmatrix} v \\ p \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (12)$$

where:

$$[J]=\begin{bmatrix} 0 & 1/\rho \\ \rho\cdot a^2 & 0 \end{bmatrix}. \qquad (13)$$

Not considering the obtainment method of matrix $[S]$, it can be confirmed by multiplication that matrix $[S]$ and matrix $[S]^{-1}$ transform Jacobian into diagonal form:

$$[S][J][S]^{-1}=\begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix}. \qquad (14)$$

The part of Jacobian, which corresponds to positive "a" and negative "$-a$" eigenvalue is calculated easily:

$$[J^+]=[S]^{-1}[\Lambda^+][S]=\begin{bmatrix} a/2 & 1/2\cdot\rho \\ \rho\cdot a^2/2 & a/2 \end{bmatrix}, \qquad (15)$$

$$[J^-]=[S]^{-1}[\Lambda^-][S]=\begin{bmatrix} -a/2 & 1/2\cdot\rho \\ \rho\cdot a^2/2 & -a/2 \end{bmatrix}. \qquad (16)$$

Matrix $[\Lambda]$ is a diagonal matrix and eigenvalues λ_i are real. Composite variable $\{v\}$ is given by:

$$\{v\}=\begin{Bmatrix} v+p/\rho\cdot a \\ v-p/\rho\cdot a \end{Bmatrix}. \qquad (17)$$

We can use (17) for the analytical solution. Along the characteristic $x = x_0 + at$, the first component of a composite vector is a constant, while the second component of the vector is a constant along the characteristic $x = x_0 - at$. The flux vector is split:

$$\left\{E^{+}\right\} = [J^{+}]\{v\} = \left\{\begin{matrix} \dfrac{p}{2\cdot\rho} + \dfrac{v\cdot a}{2} \\ \dfrac{v\cdot\rho\cdot a^2}{2} + \dfrac{p\cdot a}{2} \end{matrix}\right\}, \quad (18)$$

$$\left\{E^{-}\right\} = [J^{-}]\{v\} = \left\{\begin{matrix} \dfrac{p}{2\cdot\rho} - \dfrac{v\cdot a}{2} \\ \dfrac{v\cdot\rho\cdot a^2}{2} - \dfrac{p\cdot a}{2} \end{matrix}\right\}. \quad (19)$$

The split flux vector is transformed into expanded form for individual points of the domain (i,j) along the high pressure line:

$$\left\{E_v^{+}\right\}_{\mathrm{j}}^{\mathrm{i}} = \frac{p(\mathrm{i}-1,\mathrm{j})}{2\cdot\rho} + v(\mathrm{i}-1,\mathrm{j})\cdot\frac{a}{2}, \quad (20)$$

$$\left\{E_v^{+}\right\}_{\mathrm{j}-1}^{\mathrm{i}} = \frac{p(\mathrm{i}-1,\mathrm{j}-1)}{2\cdot\rho} + v(\mathrm{i}-1,\mathrm{j}-1)\cdot\frac{a}{2}, \quad (21)$$

$$\left\{E_v^{-}\right\}_{\mathrm{j}+1}^{\mathrm{i}} = \frac{p(\mathrm{i}-1,\mathrm{j}+1)}{2\cdot\rho} - v(\mathrm{i}-1,\mathrm{j}+1)\cdot\frac{a}{2}, \quad (22)$$

$$\left\{E_v^{-}\right\}_{\mathrm{j}}^{\mathrm{i}} = \frac{p(\mathrm{i}-1,\mathrm{j})}{2\cdot\rho} - v(\mathrm{i}-1,\mathrm{j})\cdot\frac{a}{2}, \quad (23)$$

$$A(\mathrm{i},\mathrm{j}) = \left\{E_v^{+}\right\}_{\mathrm{j}}^{\mathrm{i}} - \left\{E_v^{+}\right\}_{\mathrm{j}-1}^{\mathrm{i}} + \left\{E_v^{-}\right\}_{\mathrm{j}+1}^{\mathrm{i}} - \left\{E_v^{-}\right\}_{\mathrm{j}}^{\mathrm{i}}, \quad (24)$$

$$\left\{E_p^{+}\right\}_{\mathrm{j}}^{\mathrm{i}} = v(\mathrm{i}-1,\mathrm{j})\cdot\rho\cdot\frac{a^2}{2} + p(\mathrm{i}-1,\mathrm{j})\cdot\frac{a}{2}, \quad (25)$$

$$\left\{E_p^{+}\right\}_{\mathrm{j}-1}^{\mathrm{i}} = v(\mathrm{i}-1,\mathrm{j}-1)\cdot\rho\cdot\frac{a^2}{2} + p(\mathrm{i}-1,\mathrm{j}-1)\cdot\frac{a}{2}, \quad (26)$$

$$\left\{E_p^{-}\right\}_{\mathrm{j}+1}^{\mathrm{i}} = v(\mathrm{i}-1,\mathrm{j}+1)\cdot\rho\cdot\frac{a^2}{2} - p(\mathrm{i}-1,\mathrm{j}+1)\cdot\frac{a}{2}, \quad (27)$$

$$\left\{E_p^{-}\right\}_{\mathrm{j}}^{\mathrm{i}} = v(\mathrm{i}-1,\mathrm{j})\cdot\rho\cdot\frac{a^2}{2} + p(\mathrm{i}-1,\mathrm{j})\cdot\frac{a}{2}, \quad (28)$$

$$B(\mathrm{i},\mathrm{j}) = \left\{E_p^{+}\right\}_{\mathrm{j}}^{\mathrm{i}} - \left\{E_p^{+}\right\}_{\mathrm{j}-1}^{\mathrm{i}} + \left\{E_p^{-}\right\}_{\mathrm{j}+1}^{\mathrm{i}} - \left\{E_p^{-}\right\}_{\mathrm{j}}^{\mathrm{i}}. \quad (29)$$

The components derived previously from the split flux vector in the expanded form are used in the equations for calculating velocity and fluid pressure at individual points (i,j) along the high pressure line.

$$v^{*}(\mathrm{i},\mathrm{j}) = v(\mathrm{i}-1,\mathrm{j}) - \frac{\Delta t}{\Delta x}A(\mathrm{i},\mathrm{j}) \quad (30)$$

$$p^{*}(\mathrm{i},\mathrm{j}) = p(\mathrm{i}-1,\mathrm{j}) - \frac{\Delta t}{\Delta x}B(\mathrm{i},\mathrm{j}) \quad (31)$$

The values v^{*}(i,j) and p^{*}(i,j) present velocities and fluid pressures along the high pressure line at points (i,j), which were obtained in the first step of the calculation.

$$A^{*}(\mathrm{i},\mathrm{j}) = \left\{E_v^{+}\right\}_{\mathrm{j}+1}^{\mathrm{i}*} - \left\{E_v^{+}\right\}_{\mathrm{j}}^{\mathrm{i}*} + \left\{E_v^{-}\right\}_{\mathrm{j}}^{\mathrm{i}*} - \left\{E_v^{-}\right\}_{\mathrm{j}-1}^{\mathrm{i}*} \quad (32)$$

$$B^{*}(\mathrm{i},\mathrm{j}) = \left\{E_p^{+}\right\}_{\mathrm{j}+1}^{\mathrm{i}*} - \left\{E_p^{+}\right\}_{\mathrm{j}}^{\mathrm{i}*} + \left\{E_p^{-}\right\}_{\mathrm{j}}^{\mathrm{i}*} - \left\{E_p^{-}\right\}_{\mathrm{j}-1}^{\mathrm{i}*} \quad (33)$$

Values which were marked with * in previous two equations were obtained by using values for velocity v^{*}(i,j) and pressure p^{*}(i,j) in terms used for their calculation. Finally, the equations for calculating flow velocity and fluid pressure along the high pressure line in the second step are:

$$v(\mathrm{i},\mathrm{j}) = \frac{v(\mathrm{i}-1,\mathrm{j}) + v^{*}(\mathrm{i},\mathrm{j})}{2} - \frac{\Delta t}{2\cdot\Delta x}A^{*}(\mathrm{i},\mathrm{j}), \quad (34)$$

$$p(\mathrm{i},\mathrm{j}) = \frac{p(\mathrm{i}-1,\mathrm{j}) + p^{*}(\mathrm{i},\mathrm{j})}{2} - \frac{\Delta t}{2\cdot\Delta x}B^{*}(\mathrm{i},\mathrm{j}). \quad (35)$$

The calculated values v(i,j) and p(i,j) present final values of the fluid velocity and pressure along the high pressure line, for the given interval Δt i.e. given integration level, Fig. 2,

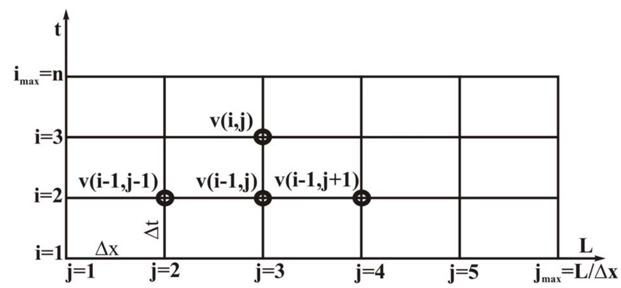

Figure 2. Calculation scheme for fluid velocity along the high pressure line

where j is the number of segments along x-axis, i.e. the tube,

$$\mathrm{j} = 2,3,4.....(L/\Delta x) - 1, \quad (36)$$

and i is the number of levels along time axis,

$$\mathrm{i} = 2,3,4.....n. \quad (37)$$

The condition for convergence, i.e. the condition for stable solution is:

$$\left(a + |v|\right)_{\max}\Delta t < \Delta x \quad (38)$$

The state of working fluid in the space from the delivery valve to the injector at the beginning of the injection process is not known, so the initial conditions of the integration are assumed. The time between two injections is longer than the injection itself, so it can be assumed that the working fluid at the beginning of the process does not move, i.e. the fluid velocity equals zero in each cross-section of the tube. Also, it can be considered that the fluid pressure is constant in each cross section of the tube at the beginning of the process.

At $L = 0$, the fluid pressure at the beginning of the high pressure line is obtained from boundary conditions by a mathematical model of the process in the delivery valve:

$$p(\mathrm{i},1) = p_V(\mathrm{i})\ . \quad (39)$$

At $L = L_{max}$, the pressure at the end of the high pressure line is obtained from boundary conditions by a mathematical model of the process in the injector:

$$p(\mathrm{i},L/\Delta x) = p_k(\mathrm{i})\ . \quad (40)$$

Mathematical models for boundary conditions at delivery valve and injector can be found in [4].

4. THE RESULTS OF FLUID FLOW PARAMETERS CALCULATION

The fuel injection system scheme whose working process was modeled is shown in Figure 1. The system is composed of piston-radial distributor pump (IPM 099.33.50), the tube (length 340 mm, inside diameter 2 mm) and the injector (IPM type 303.71.00, YDN O SD 293). The calculation and measurement of individual parameters were made under maximum fuel delivery and the pump shaft speed of 1500 rpm.

The results shown in Figures 3 and 4 were obtained by solving the model of the working process in the fuel injection system in diesel engines [4]. The previously described method of finite differences with separation of the flux vector was used for solving the equations which describe the hydrodynamical processes in the high pressure line. Figure 3 shows a comparison of calculated and measured fluid pressure and Figure 4 shows the change in fluid velocity at the end of the high pressure line, both as the function of the pump shaft angle.

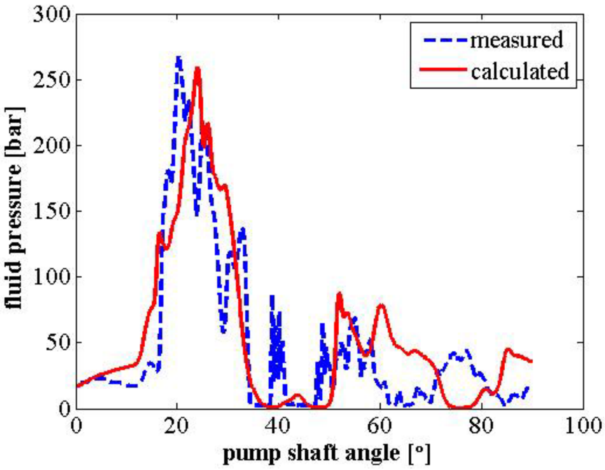

Figure 3. The comparison of measured and calculated fluid pressure at the end of the high pressure line

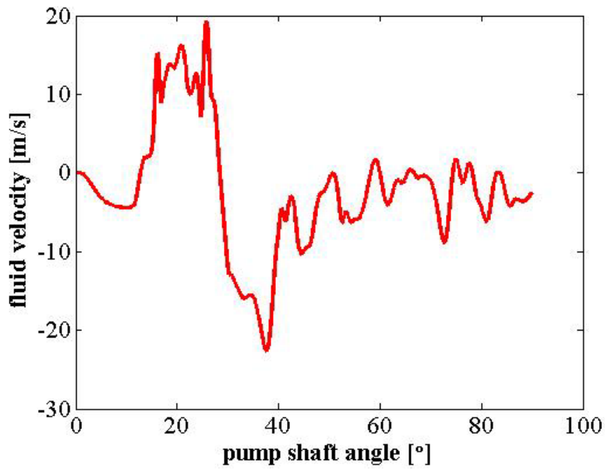

Figure 4. The change in fluid velocity at the end of the high pressure line

There is a moderately good agreement between simulation and experimental data for pressure at injector, Fig. 3. Model prediction of pressure peak value is very good, but some phase shift between measured and calculated lines can be observed.

5. CONCLUSION

The numerical method of finite differences with separation of the flux vector can be easily applied in solving the hyperbolic partial equations (3) which describe the hydrodynamical processes in the high pressure line of the fuel injection system of diesel engines. Selection of the integration step along both the time and space coordinates must satisfy the condition for stable solution (38). Also, this integration time step must be compatible with time step used in integration of differential equations for boundary conditions in the delivery valve and the injector.

The advantage of splitting the vector $\{E\}$ into two parts is in avoiding the transformation of the equations at each point in the calculation domain into a series of disconnected partial differential equations. Instead of transforming the equations each time, only vector $\{E\}$ components are calculated.

By analyzing accessible papers which consider the process of solving the equations which describe the hydrodynamical processes in the high pressure line of diesel injection system, it can be concluded that the method presented in this paper was used only in one paper for calculation of flow parameters in the high pressure line [5]. However, in the mentioned paper, only the results of flow parameters are given without analyzing the application of the method. In most papers published so far and concerning this field, the finite differences method combined with characteristic method has been used.

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NOMENCLATURE

a	sound velocity in the high pressure line
n	maximum number of levels along t axis
p	fluid pressure in the high pressure line
p_v	fluid pressure in the delivery valve
p_k	fluid pressure in the injector
v	fluid flow velocity in the high pressure line
t	time
x	coordinate along the tube
d	tube diameter
L	tube length
i	number of levels along time axis
j	number of segments along the tube
E	flux vector
J	Jacobian

Greek symbols

Λ	diagonal matrix
λ	hydraulic friction coefficient
ρ	fluid density in the high pressure line

ПРОРАЧУН ПАРАМЕТАРА СТРУЈАЊА ДУЖ ЦЕВОВОДА ВИСОКОГ ПРИТИСКА СИСТЕМА ЗА УБРИЗГАВАЊЕ ГОРИВА КОД ДИЗЕЛ-МОТОРА

Славко Ракић, Милош Цветић

Хидродинамички процеси у цевоводу високог притиска система за убризгавање горива код дизел-мотора су описани хиперболичким парцијалним диференцијалним једначинама. Да би решили поменуте једначине, неопходно је њихово свођење на један од карактеристичних облика, а затим се применом одговарајуће нумеричке методе прорачунавају вредности притиска и брзине флуида дуж цевовода високог притиска. У овом раду је за решавање карактеристичног облика једначина коришћена метода коначних разлика са раздвајањем флукс вектора. Поменута метода омогућује прорачун параметара струјања дуж цевовода на веома једноставан и ефикасан начин, избегавајући трансформацију једначина за сваку тачку прорачунске области.