Interaction of the Gaussian Pulse EM Wave with Suddenly Created Cold Magnetized Plasma

Zoran M. Trifković
Associate Professor
University of Belgrade
Faculty of Mechanical Engineering

The linear transformation of the source Gaussian pulse electromagnetic wave, with propagating in a free space along an external static magnetic field, due to the sudden creation of cold linear plasma, is studied. This transformation has been analyzed by using the first order perturbation theory in radio approximation. Spatial distribution of the new created static magnetic field mode is presented in corresponding diagram.

Keywords: *suddenly created plasma, perturbation technique, Fourier transform, Laplace transform, static magnetic mode generation.*

1. INTRODUCTION

Rapidly created plasmas appear practically in all pulse gas discharges, laser created plasmas, lightning, and plasmas created by nuclear explosions. If the rise time of plasma is much smaller than the decay time, it is possible to approximate the time variation of plasma parameters with the Heaviside step function. Linear transformation of the plane electromagnetic wave (EMW) in suddenly created plasma was investigated in [1]. It was shown that for $t < 0$, the source EMW propagating along the z-direction in free space with wave number k_0 and angular frequency ω_0 transforms in the suddenly created plasma having an infinite extent into two new modes propagating in opposite directions with identical upshifted frequencies $\omega_1 = \sqrt{\omega_0^2 + \omega_p^2}$, with electron plasma angular frequency $\omega_P = \sqrt{N_0 q^2 / \varepsilon_0 m} \approx 18\pi \cdot N_0^{1/2}$, where N_0 is electron plasma density. The basic results of the transformation of EMW in such time varying linear media have been summarized by Kalluri [2]. Some of these results have been verified by Particle In Cell simulation [3] and by experimentally created microwave plasma previously illuminated by stationary microwave source [4]. By the use of second-order perturbation technique, the transformation of the source EMW in the nonlinear media was done in [5-10]. The process of third harmonic generation (THG) [11,12] is caused by the coupling of the transverse first harmonic and longitudinal second harmonic modes through Lorentz force and convective term in the equation of electron fluid motion and could have respectable values for the specific values of source wave frequencies. In spite of THG being a weak process, it is dipole allowed and therefore occurs in all materials, including materials with inversion symmetry. When using focused high-intensity ultra-short laser pulses, this normally weak THG process becomes highly operative at a simple air-dielectric interface and is much stronger than in the bulk of most dielectric materials. In all previous papers the transformation of a plane or a circular polarized source EMW has been analyzed.

In this paper it is assumed that, for $t < 0$, the Gaussian pulse EMW, for the first time in the theory of interaction of a EMW with time-varying media , with angular frequency ω_0 and wave number $\mathbf{k}_0$, is propagating in free space along the static magnetic field $\mathbf{B}_0 = B_0 \cdot \mathbf{z}$, where $\mathbf{z}$ is unit vector in positive z-direction. At $t = 0$ the entire free space is ionized with an electron plasma density N_0. The transformation of source EMW into plasma stationary and traveling waves have been analyzed by the use of first order perturbation theory. The efficiency of excitation of static spatially-varying magnetic mode has been obtained in a closed form and studied for specific values of angular frequency of the source wave, electron plasma density and time duration of the plasma medium.

2. PROBLEM FORMULATION AND SOLUTION

Electric and magnetic fields of the source EMW propagating in free space for $t < 0$ are given by:

$$\mathbf{e}_0(z,t) = \mathbf{x} \cdot E_0 \exp\left[\left(\frac{\omega_0 t - k_0 z}{2\omega_0 T}\right)^2\right] \cos(\omega_0 t - k_0 z), \quad (1)$$

$$\mathbf{h}_0(z,t) = \mathbf{y} \cdot H_0 \exp\left[\left(\frac{\omega_0 t - k_0 z}{2\omega_0 T}\right)^2\right] \cos(\omega_0 t - k_0 z), \quad (2)$$

where $\mathbf{x}$ and $\mathbf{y}$ are unit vectors in positive direction of x and y-axis, with $H_0 = \sqrt{\varepsilon_0 / \mu_0} E_0$, where ε_0 and μ_0 are electric permittivity and magnetic permeability of the free space, respectively.

The EM and electron velocity fields $\mathbf{e}$ (z,t), $\mathbf{h}$ (z,t) and $\mathbf{u}$ (z,t) in magnetoplasma medium have to satisfy the following equations:

$$\nabla \times \mathbf{e}_1(z,t) = -\mu_0 \frac{\partial \mathbf{h}_1(z,t)}{\partial t}, \quad (3)$$

$$\nabla \times \mathbf{h}_1(z,t) = -N_0 q \mathbf{u}_1(z,t) + \varepsilon_0 \frac{\partial \mathbf{e}_1(z,t)}{\partial t}, \quad (4)$$

Received: March 2011, Accepted: May 2011
Correspondence to: Dr Zoran Trifković
Faculty of Mechanical Engineering,
Kraljice Marije 16, 11120 Belgrade 35, Serbia
E-mail: ztrifkovic@mas.bg.ac.rs

$$\frac{\partial \mathbf{u}_1(z,t)}{\partial t} = -\frac{q}{m}\mathbf{e}_1(z,t) . \quad (5)$$

In order to solve the system of partial differential equations (3) to (5) we have applied Laplace transform in time:

$$L(f(z,t)) = \int_0^{\infty} f(z,t)\exp(-st)\mathrm{d}t = F(z,s) . \quad (6)$$

and, as the plasma is unbound, Fourier transform in space:

$$F(f(z,s)) = \int_{-\infty}^{+\infty} f(z,s)\exp(-jk)\mathrm{d}z = F(k,s) . \quad (7)$$

In a domain of complex frequency $s = j\omega$ (collisionless plasma), $j = \sqrt{-1}$, and wave number k the EM and velocity fields are defined by the following system of linear algebraic equations:

$$jkE_1(k,s) - \mu_0 sH_1(k,s) = \mu_0 H_1(k,t=0) , \quad (8)$$

$$\begin{gathered} jkH_1(k,s) + \varepsilon_0 sE_1(k,s) - N_0 qU_1(k,s) = \\ = \varepsilon_0 E_1(k,t=0) , \end{gathered} \quad (9)$$

$$\frac{q}{m}E_1(k,s) + sU_1(k,s) = 0 , \quad (10)$$

where

$$E_1(k,t=0) = \frac{E_0\sqrt{\pi}}{K_0}\left\{\exp\left[-\left(\frac{k-k_0}{K_0}\right)^2\right] + \right.$$

$$\left. + \exp\left[-\left(\frac{k+k_0}{K_0}\right)^2\right]\right\}$$

$$H_1(k,t=0) = \sqrt{\varepsilon_0/\mu_0}\,E_1(k,t=0) ,$$

$$K_0 = \sqrt{\frac{k_0}{\omega_0 T}} . \quad (11)$$

Solving the algebraic equations (8) to (10) one obtains the transformed EM and electron velocity fields in the form:

$$E_1(k,s) = \frac{s - jkc}{s^2 + k^2c^2 + \omega_p^2}E_1(k,t=0) , \quad (12)$$

$$H_1(k,s) = \left(-\frac{jkc}{\mu_0 s}\frac{s - jkc}{s^2 + k^2c^2 + \omega_p^2} + \frac{1}{s}\sqrt{\frac{\varepsilon_0}{\mu_0}}\right)\cdot$$

$$\cdot E_1(k,t=0) , \quad (13)$$

$$U_1(k,s) = \frac{\varepsilon_0}{N_0 q}sE_1(k,s) . \quad (14)$$

After performing an inverse Laplace and Fourier transform to (12) and (13), the following EM components are obtained:

$$\mathbf{e}_1(z,t) = \mathbf{x}\cdot\Big[E_1^t\cos(\omega_1 t - k_0 z - \theta_1) +$$

$$+ E_1^r\cos(\omega_1 t + k_0 z - \theta_1)\Big] , \quad (15)$$

$$\mathbf{h}_1(z,t) = \mathbf{y}\cdot\Big[H_{10}\cos(k_0 z - \theta_2) + H_1^t\cos(\omega_1 t - k_0 z - \theta_3) +$$

$$+ H_1^r\cos(\omega_1 t + k_0 z - \theta_3)\Big] . \quad (16)$$

Amplitudes and phase shift of the electric field components have the following form:

$$E_1^{t,r} = \sqrt{(A_1^{t,r})^2 + (A_2^{t,r})^2} ,$$

$$\mathrm{tg}\theta_1 = \frac{A_2^{t,r}}{A_1^{t,r}} ,$$

$$A_1^{t,r} = E_0\left\{\frac{1}{2}\left(1 \pm \frac{\omega_0}{\omega_1}\right) + \frac{1}{4T^2}\left[\mp\frac{3\omega_0\omega_p^2}{\omega_1^5} - \right.\right.$$

$$\left.\left. - \frac{\omega_1 \pm \omega_0}{\omega_1}\left(\frac{\omega_0}{\omega_1}t \mp \frac{z}{c}\right)^2\right]\right\} ,$$

$$A_2^{t,r} = \frac{E_0}{4T^2}\left[\mp\frac{2\omega_p^2}{\omega_1^3}\left(\frac{\omega_0}{\omega_1}t \mp \frac{z}{c}\right) - \frac{\omega_p^2}{\omega_1^4}(\omega_1 \pm \omega_0)t\right] . \quad (17)$$

The new created EM wave in plasma has the angular frequency $\omega_1 = \sqrt{\omega_0^2 + \omega_p^2}$. Amplitudes and phase shifts of the magnetic field components have the following form:

$$H_{10} = \sqrt{B_1^2 + B_2^2} ,$$

$$\mathrm{tg}\theta_2 = \frac{B_2^{t,r}}{B_1^{t,r}} ,$$

$$B_1 = H_{01}\frac{\omega_0^2}{\omega_1^2} + \frac{\omega_p^2}{2T^2\omega_1^2}\left(4\frac{\omega_0^2}{\omega_1^2} - 1\right) - \left(\frac{z}{c}\right)^2 ,$$

$$B_2 = H_0\frac{\omega_p^2\omega_0}{T^2\omega_1^4}\frac{z}{c} ,$$

$$\mathrm{tg}\theta_3 = \frac{C_2^{t,r}}{C_1^{t,r}} ,$$

$$C_1^{t,r} = H_0\frac{\omega_1 \pm \omega_0}{2\omega_1^2}\left\{\pm\omega_0 + \frac{1}{T^2\omega_1^2}\left[(\omega_1 \pm \omega_0)\left(2 \mp 3\frac{\omega_0}{\omega_1}\right) \pm \right.\right.$$

$$\left.\left. \pm 2\omega_0\left(2 \pm \frac{\omega_0}{\omega_1} - 4\frac{\omega_0^2}{\omega_1^2}\right)\right] \mp \omega_0\left(\frac{\omega_0}{\omega_1}t \mp \frac{z}{c}\right)^2\right\} ,$$

$$C_2^{t,r} = H_0\frac{\omega_1 \pm \omega_0}{2T^2\omega_1^3}\left[2\left(\omega_1 \pm \omega_0 - 2\frac{\omega_0^2}{\omega_1}\right)\cdot\right.$$

$$\left.\cdot\left(\frac{\omega_0}{\omega_1}t \mp \frac{z}{c}\right) \mp \frac{\omega_0\omega_p^2}{\omega_1^2}t\right] . \quad (18)$$

In the above equations the superscript t and upper sign refer to transmitted and superscript r and lower sign to the reflected wave.

Spatial distribution of the new created static magnetic mode-the first term in (16), see Figure 1, has made with specific values of following parameters: $\omega_0 \sim 10^6$ Hz, the source wave is in a radio frequency range, $N_0 \sim 10^{22}$ m^{-3}, rapidly created plasma is generated by lightning and $T \sim 10^8/\omega_p$, plasma time duration is about 100 μs.

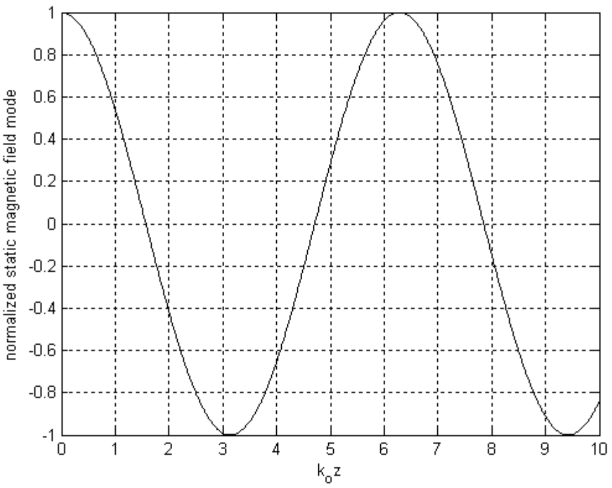

Figure 1. Spatial distribution of the new creted static magnetic field mode, normalized on the source wave magnetic field amplitude

3. CONCLUSION

For the first time in the theory of the interaction of a EMW with time-varying media, the initial value problem of interaction of the Gaussian pulse EM source wave with suddenly created cold magnetoplasma medium, in the particular case of longitudinal propagation, is solved in the closed form. The source wave, due to sudden generation of the cold plasma, splits into two traveling EM waves (one transmitted and one reflected with the same upshifted angular frequency) and one spatial-varying static magnetic mode with the peak value the same as the amplitude of the magnetic field of the source EM wave and spatial period $k_0z = 3.2$.

ACKNOWLEDGMENT

The Ministry of Science and Environmental Protection of Republic of Serbia supported this work, under Contract TR 37019.

REFERENCES

[1] Jiang, C.-L.: Wave propagation and dipole radiation in a suddenly created plasma, IEEE Transactions on Antennas and Propagation, Vol. 23, No. 1, pp. 83-90, 1975.

[2] Kalluri, D.K.: *Electromagnetics of Complex Media*, CRC Press, Boca Raton, 1999.

[3] Wilks, S.C., Dawson, J.M. and Mori, W.B.: Frequency up-conversion of electromagnetic radiation with use of an overdense plasma, Physical Review Letters, Vol. 61, No. 3, pp. 337-340, 1988.

[4] Joshi, C.J., Clayton, C.E., Marsh, K., Hopkins, D.B., Sessler, A. and Whittum, D.: Demonstration of the frequency upshifting of microwave radiation by rapid plasma creation, IEEE Transactions on Plasma Science, Vol. 18, No. 5, pp. 814-818, 1990.

[5] Stanic, B.V., Drljaca, P.M. and Boskovic, B.O.: Electron plasma waves generation in suddenly created isotropic plasma, IEEE Transactions on Plasma Science, Vol. 26, No. 5, pp. 1514-1519, 1998.

[6] Trifkovic, Z.M. and Stanic, B.V.: Nonlinear transformation of electromagnetic waves in suddenly created cold magnetized plasma: Longitudinal propagation, Journal of Applied Physics, Vol. 92, No. 7, pp. 3472-3479, 2002.

[7] Trifkovic, Z.M. and Stanic, B.V.: Nonlinear transformation of electromagnetic wave in time-varying plasma medium: Longitudinal propagation, Journal of Applied Physics, Vol. 100, No. 2, pp. 023303-1-023303-6, 2006.

[8] Trifkovic, Z.M. and Osmokrovic, P.V.: Resonant generation of the rectification electric mode in suddenly created cold lossless magnetized plasma. transversal propagation, Journal of Applied Physics, Vol. 105, No. 1, pp. 013310-1-013310-5, 2009.

[9] Trifković, Z.M. and Osmokrović, P.: Conversion of a left-hand circularly polarized whistler wave into the magnetic energy of a controllable helical wiggler wave, Plasma Devices and Operations, Vol. 17, No. 3, pp. 215-220, 2009.

[10] Trifković, Z.M., Cvetić, J.M. and Osmokrović, P.: Generation of degenerate modes in suddenly created cold weakly nonlinear magnetized plasma, Plasma Devices and Operations, Vol. 17, No. 4, pp. 301-308, 2009.

[11] Stanić, B.V. and Zhang, W.J.: The third harmonic generation in weakly nonlinear suddenly created plasma, in: *Proceedings of the 19th SPIG Contributed Papers*, 30.08-04.09.1998, Zlatibor, Serbia, pp. 717-720.

[12] Stanic, B.V. and Trifkovic, Z.M.: The third harmonic generation of traveling and stationary wave modes in a suddenly created nonlinear cold plasma, IEEE Transactions on Plasma Science, Vol. 35, No. 4, pp. 1041-1045, 2007.

ИНТЕРАКЦИЈА ГАУСНО-ИМПУЛСНОГ ЕЛЕКТРОМАГНЕТСКОГ ТАЛАСА СА НАГЛО СТВОРЕНОМ ХЛАДНОМ МАГНЕТИЗОВАНОМ ПЛАЗМОМ

Зоран М. Трифковић

По први пут у теорији интеракције електромагнетских таласа са временско промењљивим просторима пропагације анализирана је трансформација изворног таласа у облику Гаусовог импулса. Као временски промењљив простор узета је нагло створена магнетизована

плазма. Анализа је спроведена коришћењем пертубационе теорије првог реда у радио апроксимацији. Решења новонасталих поља у плазми су добијена у затвореној форми и приказана је просторна расподела статичког магнетског поља добијеног као последица ове интеракције.