

Influence of the Geometry Parameters of Cardan Joint Rolling Parts on the Load Distribution

Zoran Stamenić

Teaching Assistant
University of Belgrade
Faculty of Mechanical Engineering

Mileta Ristivojević

Full time Professor
University of Belgrade
Faculty of Mechanical Engineering

Milan Tasić

Professor
Tehnikum Taurunum
College of Applied Sciences
Belgrade

Radivoje Mitrović

Full time Professor
University of Belgrade
Faculty of Mechanical Engineering

Cardan joints have a wide range of applications in mechanical engineering because of the capability of transferring the load and motion from one shaft to another with different joint angle. The vital parts of these joints have a limited life due to insufficient surface strength in the area of rolling parts contact. Load distribution of Cardan joints composite parts have a major impact on their capacity and service life. Therefore, a detailed analysis of load distribution along the line of contact between Cardan Cross pins and rolling parts – needles, was carried out in this paper. Appropriate analytical and numerical models are developed. Based on the developed models, it is possible to optimize the geometrical parameters in terms of uniformity of load distribution of Cardan joints component in contact.

Keywords: *Geometry of Cardan joints, linear load distribution, analytical and numerical model.*

1. INTRODUCTION

The increasing complexity of mechanical systems requires an increasing cost of their development, production and exploitation. At the same time, demands for energy and environmental efficiency, reliability, service life and load capacity are growing. Today, in an era of high competition, it is important that all machinery works reliably. Hence, the increase in functional ability of mechanical systems and structures are imperative for all participants in the chain - from product development, its production and control, to maintenance. This applies not only to the final, finished product, but also to their component parts, assemblies, subassemblies, and all the elements involved in the integration of the product. Improving the structural characteristics of each of them contributes to the overall mechanical system functional ability.

In general, all mechanical systems consist of driving and operational groups, or driving and operational (driven) machines. The connection of these machines is achieved by special joints, machine elements - couplings. Their main function is to transfer loads and movement from the drive to driven machine. It is known that these joints are often the weakest link in any mechanical system. They significantly affect the quality of machine design, not only in terms of load capacity and reliability, but also in terms of operational performance, energy and environmental efficiency.

According to this function, a large number of different coupling types were developed [1,2]. Most of these joints (couplings) have been developed to connect the drive and working machine whose position does not change each other during exploitation. However, in

agricultural and other transport vehicles and facilities, exploitation conditions require different mutual positions of the axes of driving and operating machinery – angular displacement. Due to the specific work conditions, a much smaller number of joints that connect this type of driving and operating machinery were developed. Pioneers in the design of these couplings are Gerolamo Cardano (1501-1576), and Robert Hooke (1635-1703). And this is the origin of the names of these joints: Cardan joint (continental Europe) and Hook joint (English speaking world).

Due to specific conditions, the vital parts of these joints have a limited service life. The complex geometry, kinematics and dynamics of these joints, contributed to the certain number of research and papers.

The greater use of Cardan joints was introduced in the 1930s with front wheel drive automobiles design. Premature joint failures were attributed to the rocking torque which were investigated by Dodge [3] and Evernden [4]. The joints failed prematurely due to the rocking torque. In the 1960s Yang and Freudenstein [5] applied dual number algebra to the spherical four-bar spatial mechanism. This approach provided a convenient method for kinematic analysis. In recent years, dual numbers have been applied to the universal joint with manufacturing tolerances by Freudenstein and Fischer [6]. The tolerances were modeled as a series of angular and translational perturbations. General universal joint design guidelines have been written by Wagner and Cooney [7], Shigley and Mischke [8], Lee [9], Machine Design [10] and Lingaiah [11]. All of these articles assume small joint angles and do not consider manufacturing tolerances. In article [12], Scott Randall Hummel and Constantin Chassapis described configuration design and optimization of universal joints introducing a manufacturing tolerances.

In an initial effort to optimize the geometry of the universal joint, Hummel and Chassapis focused on developing a design methodology that optimizes the

Received: April 2012, Accepted: May 2012

Correspondence to: MrSci Zoran Stamenić
Faculty of Mechanical Engineering,
Kraljice Marije 16, 11120 Belgrade 35, Serbia
E-mail: zstamenic@mas.bg.ac.rs

strength of the joint while checking for interference between the various components, but ignored the effects that manufacturing tolerances may have on the performance of the optimized joint. However, design criteria are not available in any of the current design guides to address tolerance related issues in Cardan joints [12].

Service life of Cardan coupling primarily depends on the surface strength of the area of rolling elements contact surfaces, i.e., their ability to oppose to different types of failure.

On the contact surfaces of gear pairs ISO standard [13] registered over 20 different forms of fracture and damage. Almost the same types of damage are present at the contact surface of roller bearings and Cardan couplings. Such a large number of failure forms and contact damage of rolling elements surfaces is the result of a large number of influencing factors on their surface hardness: conditions of formation of contact surfaces, shape tolerances of rolling parts, roughness, load distribution on the simultaneous parts in contact; load distribution along the contact line, surface hardness, lubrication conditions, a burn conditions.

The greatest influence on the strength of the contact surface has a load distribution along the line of contact and load distribution of rolling parts simultaneously [14]. With increasing a degree of uneven distribution of the line load strength, decreases the strength of contact surface, which reflects the increasing intensity of various forms of failure. It should be noted that one aspect of the failure and produced particles of material, accelerate the development of other types of damage on contact surfaces. Compared with other machine elements, roller bearings and gear pairs, Cardan coupling components have the most unfavorable geometric and kinematics conditions in terms of uniformity of load distribution along the line of contact. Even in ideal conditions, the geometric accuracy of parts in contact of Cardan coupling, on the contact surfaces can not be achieved uniform load distribution. The main reason is the extremely unfavorable position of the contact line of rolling parts in relation to the direction of the angular velocity vector. The points of contact that are farthest from the axis of rotation, first come into contact [14]. These are the points on the top of the sleeve on Cardan cross pin.

The degree of involvement of other points of contact line depends on the geometric characteristics of bearing rolling parts, the contact deformation and the intensity of the service load. Theoretical and experimental studies have shown that parts of the pin sleeve near its root almost always remain intact, because they do not participate in the load transfer. Thus, the central problem in Cardan coupling is to reduce the uneven load distribution along the line of contact of rolling parts, or to unload parts on the top of a cross pin sleeve. According to this, based on previous researches, the two design solutions were formed.

The first solution is based on the correction of rolling parts contact line - the needles. This correction involves the correction of the geometry of needles ends. Another solution is based on the use of two rolling parts in line - shorter needles instead of one long needle.

In order to study the load distribution in Cardan joints assembly, an appropriate geometric model has developed in this paper. According to this model, the influence of geometrical parameters of rolling parts on the linear load distribution was analyzed. The analyses of obtained results should identify the optimal geometry of rolling parts in terms of uniformity of linear load distribution. With developed numerical model (FEM), the influence of load on the contact stress state of rolling components in contact was analyzed.

2. APPLIED LOAD

Analytical and experimental studies dedicated to determine the service load of machine parts, components and assemblies are always current. The complexity of this research is particularly essential in Cardan coupling, due to the strong influence of kinematics and geometric requirements of manufacturing accuracy - manufacturing tolerances. The main characteristics of the service load are: the direction, intensity, character of load cycle fluctuation and number of cycles. Service load determined on the basis of installed power and motion speed is the nominal load:

$$F_{nom} = f_1(P; \omega) \text{ or } F_{nom} = f_2(P; v).$$

The accurate determination of the characteristics of the service load - the real load is very complex in the most cases. In economic terms it is often unjustified, because besides theoretical research, a complex experimental research is necessary. Therefore, most analysis of the functional ability of machine parts and assemblies are implemented on the basis of main load for calculation (F_{calc}).

$$F_{calc} = F_{nom} \cdot K, K \geq 1.$$

This factor K does not cover the whole range of exploitation conditions, therefore for certain class of condition, a detailed theoretical and experimental research were carried out. Results are presented in tables and/or diagrams for certain machine components. Typical examples of this procedure are the factors of exploitation conditions in calculation of gears, belt and chain pairs, as well as rolling bearings [15-19].

The factor of exploitation conditions for gear pairs is determined as:

$$K = K_A \cdot K_v \cdot K_\beta \cdot K_\alpha,$$

where:

K_A – application factor,

K_v – dynamic factor,

K_β – load distribution factor along the current line of contact,

K_α – load distribution factor of the simultaneously conjugate gear teeth pairs.

In the scientific and specialized literature these factors are not fully described, so there is not enough reliable quantitative and qualitative information for Cardan joints too. In some researches [20] the influence of geometric characteristics on stress distribution in

critical areas of shaft and fork were investigated. The researches were conducted not from surface damages but the fracture and total destruction through all cross section point of view. On the contrary, the influence of geometric characteristics of Cardan joints rolling elements on the linear load distribution – on the factor K_{β} , will be carried out in this paper. In Fig.1 are presented nominal load and calculation load in relation with real service load. It is necessary to underline that in random exploitation conditions; very often real service load is less than nominal one.

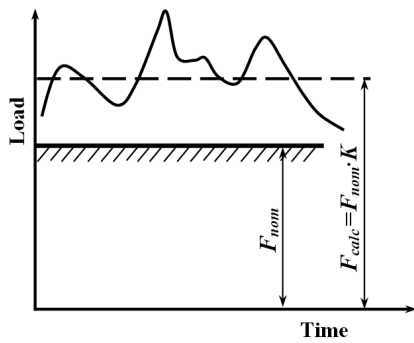


Fig. 1. Nominal, calculation and real load

2.1 Load distribution

Cardan coupling (Fig.2a) load is in the form of tangential force F_t which is caused by torque transmitted by coupling. This force is orthogonal directed at Cardan cross axes but distributed not evenly along the line of contact at rolling components. The biggest intensity of nominal load is at points which belongs to the sleeve root, diagram in Fig. 2b.

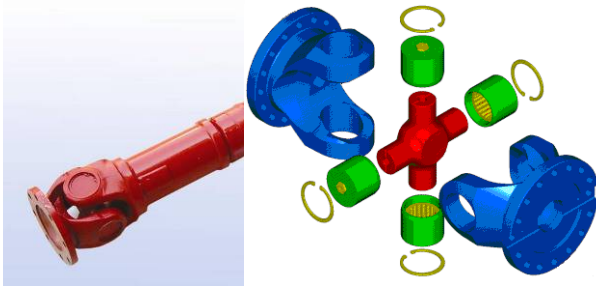


Fig. 2a. Assembled and exploded view of Cardan coupling

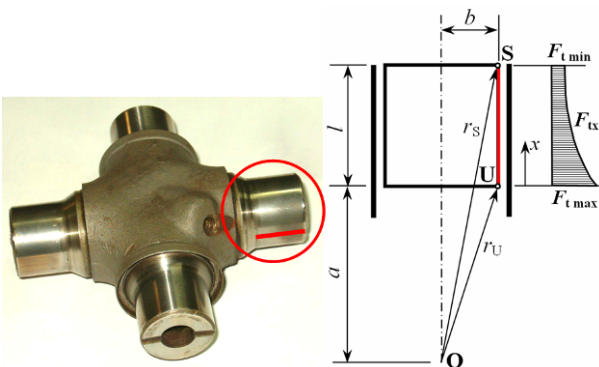


Fig. 2b. Load distribution along Cardan cross sleeve lateral line

The lowest intensity of load is at the points which belongs to the top of the Cardan cross sleeve. The

difference between these boundary of load values depends on Cardan cross geometry. For the purpose to define the influence of geometrical parameters on the degree of uneven load distribution along sleeve lateral line, the appropriate analytical expression is formed. The load ratio in the form of tangential forces in a random point (X) on sleeve lateral line is observed according to its maximum value:

$$\frac{F_{tx}}{F_{txmax}} = \frac{1}{1 + k \cdot \frac{l}{a}}$$

where: $k = x/l$.

The correlation between load ratio and geometrical characteristics of sleeve is shown in the diagram in Fig. 3.

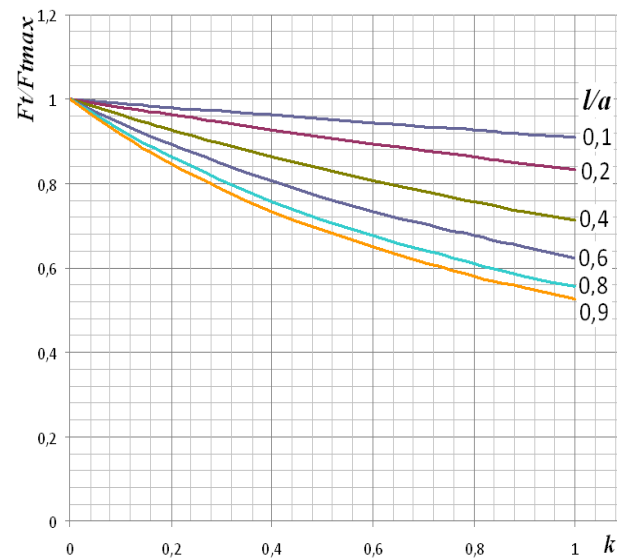


Fig. 3. Influence of Cardan cross geometry on load distribution along Cardan cross lateral line

A higher degree of uneven load distribution corresponds to larger values of parameter k and to the ratio l/a . Lower gradient of load decrease from the root to the top of a Cardan cross sleeve corresponds to small values of ratio l/a . This means that the degree of uneven load distribution on a sleeve will be smaller if a sleeve is more distant from the bearing axis.

3. GEOMETRY FACTOR OF LINEAR LOAD DISTRIBUTION

In Cardan joints rolling components in contact, the length of contact lines and degree of engagement of some of its points, are significant function of sleeve geometrical parameters, i.e. distance between points on contact line from the axis of rotation. During load transfer, points of the sleeve top first make contact. The degree of engagement of further sleeve points depends on geometric characteristics of rolling elements in contact, stiffness and value of load. At the same time, points of the sleeve root almost always remain unloaded. Because of their unfavorable geometry and mutual position of elements in contact, a point of the sleeve root fails to form a line of contact. Thus, the dominant condition to achieve more even linear load

distribution is the formation of a longer contact line along a lateral line of sleeve in the load free state.

In order to find the geometric characteristics which will generate the longer contact line, an appropriate geometric model is formed. The correlation between the displacement of the sleeve lateral line points and geometric characteristics of the sleeve, diameter and sleeve length as well as a sleeve distance from its axis of rotation, is established. To form this correlation, a geometrical model was observed as shown in Fig. 4.

In the presence of radial clearance Z_1 , point S on the top of the sleeve, to establish contact with the appropriate point on the opposite part in contact - bushing, must be rotated around the axis O for elementary angle $\Delta\psi_s$. Corresponding to this Cardan cross turning angle, is elementary path - elementary arch movement of point S for the value SS' , Fig. 4. At the same time, the turning angle $\Delta\psi_u$, is equal to elementary path - elementary arch movement of point U for the value UU' .

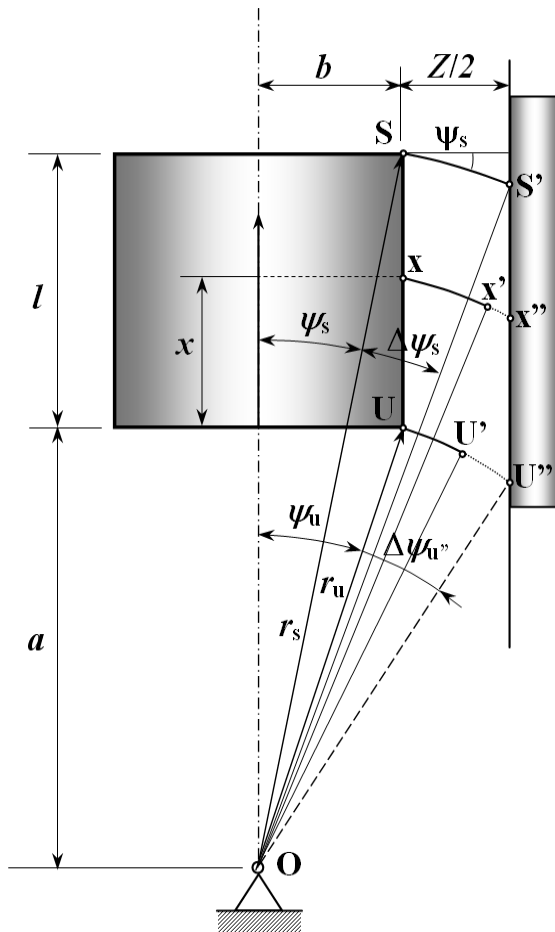


Fig. 4. Geometrical model of Cardan cross parts in contact

Point U in sleeve root, has to rotate around the axis O for elementary angle $\Delta\psi_u$ for establishing a contact with appropriate point of the opposite rolling part in contact - needle. To this angle of rotation, corresponding is elementary path - elementary arch movement of point U for the value UU' .

Displacement of point U in a sleeve root for the value of UU' , and point S on the sleeve top for the value of SS' , is the moment of rolling parts contact. This

primary contact of rolling parts in point S' was achieved without load influence. In order to achieve contact of other points on the sleeve lateral line with the corresponding points on the rolling part (e.g. a random point X' with point X''), an additional turning for appropriate angle is needed. For point U it is angle $\Delta\psi_u$, and for point X is angle $\Delta\psi_x$. The values of these rotation angles, in fact their angular displacements $U'U''$ and $X'X''$, depend on the geometrical parameters of rolling parts in contact. These additional angular displacements of sleeve points are provided with elastic deformation of rolling parts in contact, under the load. The values of these deformations are small and limited by the elastic properties of parts in contact, with their geometry and load intensity. Accordingly to this, the values of angular displacements $U'U''$ and $X'X''$ should be as small as possible. The amount of those displacements is dictated by the geometry of rolling parts. Therefore, the engaging of certain sleeve points in the load transfer primarily depends on the geometric characteristics of rolling parts in contact. To analyze the influence of geometrical characteristics of the rolling parts on the linear load distribution, the angular displacements of points on the sleeve lateral line are observed.

Based on the geometric model in Fig. 4, for the angular displacement of point S on the sleeve top, and some point X on sleeve lateral line, the following expressions are:

$$SS_1 = r_s \cdot \Delta\psi_s, \quad (1)$$

$$xx_1 = r_x \cdot \Delta\psi_x \quad (2)$$

where,

$$r_x = \sqrt{(a+x)^2 + b^2}, \quad (3)$$

$$r_s = \sqrt{(a+l)^2 + b^2}. \quad (4)$$

Point S , on the top of the sleeve, first makes contact with the composite part - needle, when the sleeve rotates for the angle $\Delta\psi_s$. Simultaneously, some point X on the sleeve lateral line will rotate for the same angle. According, these follows the equality:

$$\Delta\psi_x = \Delta\psi_s. \quad (5)$$

Substituting (1) in (2) and (5) it is obtained:

$$\frac{XX_1}{SS_1} = \frac{r_x}{r_s}. \quad (6)$$

Substituting (3) and (4) into (6), we get a general expression for analyzing the geometry values of the rolling parts influence on load distribution of rolling parts in contact:

$$G_r = \frac{XX_1}{SS_1} = \sqrt{\frac{1 + 2k\left(\frac{l}{a}\right) + k^2\left(\frac{l}{a}\right)^2 + \left(\frac{b}{a}\right)^2}{1 + 2\cdot\left(\frac{l}{a}\right) + \left(\frac{l}{a}\right)^2 + \left(\frac{b}{a}\right)^2}} \quad (7)$$

where:

d – sleeve diameter, $b=d/2$, $\overline{OA} = a$

l – length of sleeve, $k = x/l$

With the ratio, of angular displacements of the farthest sleeve point from the axis of rotation – the point S to some point X with sleeve rotation for angle $\Delta\psi_s$, the geometric factor of linear load distribution G_r is defined.

Based on the geometric factor G_r a qualitative analysis of load distribution along the line of contact of rolling parts can be performed. This factor represents the geometric potential of rolling parts in contact, in the form of corresponding load distribution along the line of contact. In accordance with the definition of G_r factor, its value is less than, or equal to the boundary case – number 1:

$$G_r \leq 1$$

By varying the values of the parameter k in the interval $k \in [0..1]$, according to expression (7), the influence of geometry on the sleeve linear load distribution can be analyzed. For example, for $k=1$, factor G_r reaches its maximum value $G_{r1} = 1$, Fig. 6a.

For $k=0$, refer to (7), the expression for the geometric factor of the linear load distribution at point U closest to the axis of rotation O , is:

$$G_{r0} = \frac{UU_1}{SS_1} = \sqrt{\frac{1 + \left(\frac{b}{a}\right)^2}{1 + 2 \cdot \left(\frac{l}{a}\right) + \left(\frac{l}{a}\right)^2 + \left(\frac{b}{a}\right)^2}} \quad (8)$$

Small values of factor G_{r0} , generates long arc length $U'U''$ requiring extensive additional sleeve rotation, which is provided from elastic deformation of rolling parts. Since these deformations are relatively small, additional sleeve rotation will not be enough to generate adequate length of rolling parts contact line. As a result, the load linear distribution will be very uneven, Fig. 5a.

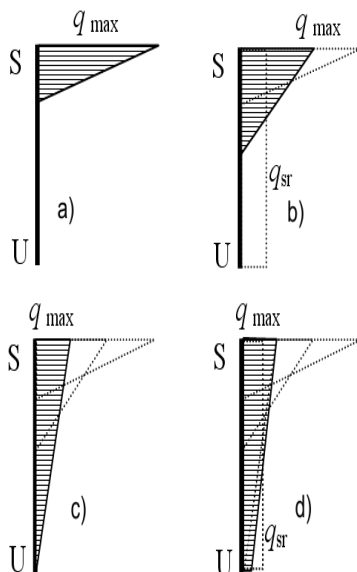


Fig. 5. The theoretical linear load distribution for values of factor $G_r \leq 1$

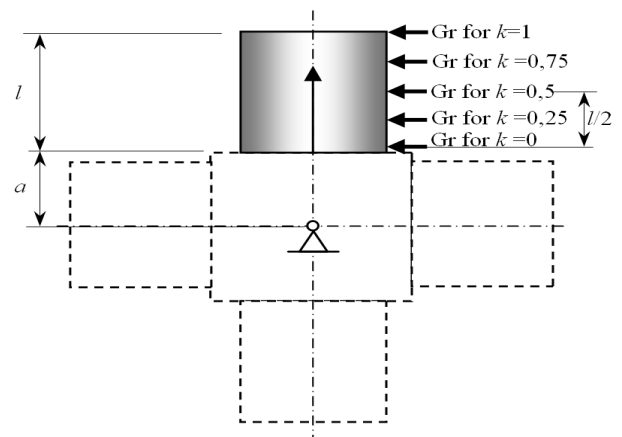
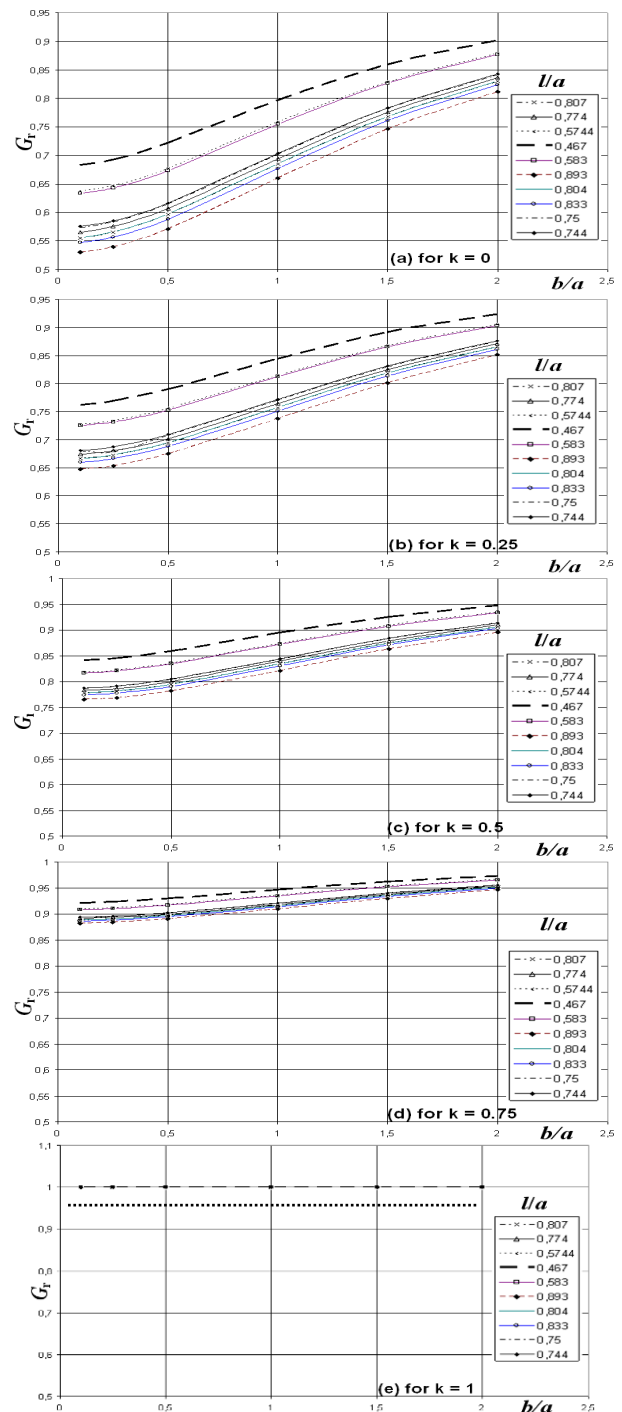


Fig. 6. The dependence of factor G_r of the geometrical values of rolling elements in contact

Increasing values of factor G_r , the length of the arc $U'U''$ is reduced. This reflects in increasing of the length of the theoretical lines of contact and reduces the degree of uneven load distribution (Fig. 5b). When the geometric factor of load reaches the value $G_r = 1$, then the theoretical length of the contact line reaches its maximum value, equal to the length of the lateral lines of sleeve. At the same time, the degree of uneven loading (Fig. 5c) is reduced. Distribution of linear – continuous load, has a triangular shape, Figs. 5 a-c. With load increasing, due to contact deformations, triangular load distribution is transformed into trapezoidal (Fig. 5d). To obtain a better load distribution along the line of contact, it is necessary to ensure proper compatibility between geometric values of rolling parts and the contact deformation due to external load.

In continuation of these researches it is needed to examine the real value of G_r factor, which can be achieved based on the geometric characteristics of rolling parts (l, b, a). For this purpose, by varying the ratio of geometric values of rolling parts ($l/a, b/a$), the values of factor G_r were analyzed. The results of these studies are presented by diagrams in Fig. 6.

On the basis of the functional dependences shown in Fig. 6, the following conclusions can be drawn:

The course of G_r factor variation in all points of lateral line ($k \in [0..1]$), has its linear and nonlinear part. The linear part is in the interval of variation of geometry values a and b , $b/a \in [0,5..1,5]$. In addition, the line direction of linear part of G_r factor variations depends on the point position along the sleeve lateral line. The coefficient of line direction is the largest, i.e. is the steepest, at the points belonging to the sleeve root (Fig. 6). At the points on the sleeve top, the values of coefficient of direction are much smaller. In addition, at a point on the sleeve top, the coefficient of direction is the same for all curves and G_r factor reaches its maximum value, for $k = 1$, Fig. 6e.

The effect of relation (l/a) on the value of G_r factor are is the biggest in points in the area of sleeve root $k \in [0..0,25]$, Fig.6, and substantially smaller in area $k \in [0,5..0,75]$. At the same time, values of G_r factor is inversely proportional to the values of the l/a ratio.

3.1 Load distribution factor along the line of contact

The distribution of the linear load depends primarily on the degree of parallelism of the lateral lines of rolling parts in contact, then on the shape and dimension deviations of rolling parts and their mutual position, stiffness of rolling parts and the intensity of load that is transmitted.

If the parts in contact of the Cardan joints were made absolutely accurate and assembled without deviation to mutual position of rolling parts, the linear load distribution would depend only on geometry and would be uneven. Uneven distribution is covered by the factor G_r . To achieve a contact of rolling parts, generated in one part or along the whole length of the theoretical line of contact, it is necessary to provide appropriate elastic deformation. Since the elastic deformation is proportional to the load, the law of linear

load variation will have the same trend of variation, as the trend of deformation.

To determine the load, essential for testing the functional ability of the contact surfaces of rolling parts (sleeve, needle and bushing), the line load distribution is taken into account with the factor K_β . It is defined as the ratio of maximum linear load (q_{max}) and fictive average linear load (q_{sr}) evenly distributed [15], Fig. 5b.

$$K_\beta = \frac{q_{max}}{q_{sr}} \quad (9)$$

In the area $0 < k \leq 1$, an analytical expression for the determination of K_β factor, follows from the equality of areas under the line of linear load distribution and fictive ideal load distribution line, Fig. 5b.

$$\frac{1}{2} q_{max} \cdot (l - x) = l \cdot q_{sr} \quad (10)$$

From this equality there follows the expression

$$K_\beta = \frac{q_{max}}{q_{sr}} = \frac{2}{1 - k} \quad (11)$$

In accordance with this expression, in Fig. 7, line (a) shows the factor K_β changing as a function of the position of contact points on the sleeve lateral line, i.e. from the parameter k . In theory, when coupled parts are absolutely rigid and inaccurate in terms of shape, dimensions and mutual position, the factor K_β intensity tends to infinitely large values, Fig. 7. In this special case, when the parameter $k=1$ total load is transferred by only one point of lateral line, this is point S on the sleeve top, Fig. 4. When the parameter $k=0$, and factor $K_\beta=2$, all points of the theoretical lines of contact are involved in the load transmission, except the last point, which is on the sleeve root – point U.

When the parameter $k = 0$, an analytical expression for the determination of K_β follows from the equality of the area of real law of linear load distribution – trapezoidal shape and area of a rectangle, as an imaginary linear evenly distributed load, shown by dashed line (q_{sr}) in Fig. 5d.

$$K_\beta = \frac{q_{max}}{q_{sr}} = \frac{2}{1 + \frac{q_{min}}{q_{max}}} = \frac{2}{1 + G_{r0}} \quad (12)$$

In accordance with this expression, Fig. 7, line (b) shows the change of K_β factor as a function of the geometric factor of distribution G_{r0} .

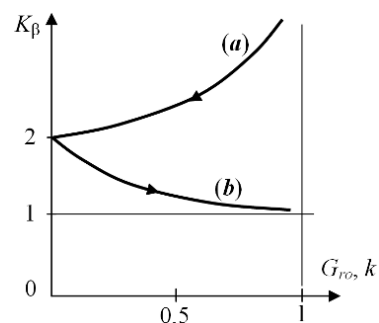


Fig. 7. Dependence of load distribution factor on the parameter k and the geometry factor G_{r0}

In the case where all points on the sleeve lateral line participated in the formation of contact line of rolling parts in load free state ($G_{ro}=1$), the linear load distribution would be ideally equal ($K_{\beta}=1$), Fig. 7. In real conditions, the values of G_{ro} factor are less than the number 1, the diagrams in Fig. 6. Accordingly as the value of factor G_r decreases, the uneven linear load distribution increases, and a value of factor K_{β} approaches the number of 2, Fig. 7.

4. NUMERIC MODEL

In order to analyze the influence of the linear load distribution on the operational capacity of Cardan joint rolling elements in terms of surface load, the numerical simulations of the contact load on the contact surfaces of journal sleeves and bushings of the Cardan cross is carried out.

A plane model of pins and bushings of Cardan cross assembly is considered, and only one of four pins (Fig. 8), because the same load image is repeated on each of them. Mathematical model is shown on Fig. 8. In the same figure (down left corner) a model of assembly is presented as defined by 30381 space tetrahedral finite elements and 49925 nodes with 149775 degrees of freedom. With green arrows, along the bushing model contour, boundary conditions are presented. Thus, the moving of nodes on the outer surface of the sleeve is completely prevented. In the central area of the Cardan cross body model all movements are limited, except rotation around the axis passing through the center of the body perpendicular to its frontal area. In the circumference of the model body, the pink arrows indicate the effect of torque, fed from the shaft.

Magnified is shown the net of finite elements that is specifically fine-divided in zone in which the stresses on the rolling contact surfaces of the Cardan cross sleeves and bushing were analyzed. In Fig. 8 (down right corner), is a magnified view that shows the fitting between the sleeve and the bushing surface. The emphasis is placed on the measure of bevel angle between the contact surfaces of pins and bushings originating from rotation necessary to annul the gap at the top of the sleeve. In Fig. 8 (in the middle) is shown the boundary (contact) condition that defines the point at the leading surface of the sleeve, which can not penetrate through the bushing contact surface. By gradually increasing the load, with constant checking whether a moved element node 1 (sleeve) penetrates the elementary surface of the element 2 (bushing), an increasing of contact surface is defined.

Calculation process is nonlinear, because the size of their contact surfaces is changing as a function of load and elasticity of rolling parts. This change fundamentally alters the stiffness matrix of the system.

Figure 9 shows the inclined plane model of Cardan cross sleeve. The elementary surface of the sleeve, which is in contact with a corresponding surface of the bushing is especially emphasized.

In the contact surface upper zone the contact of rolling elements is achieved, therefore, this is the zone with the greatest intensity of stress. Because of the iterative calculation procedure, contact surface is not

continuous but consists of a number of local points where stresses are concentrated. The values of the equivalent (von Mises) stress are shown, or strains along the contact surface of pins and bushings.

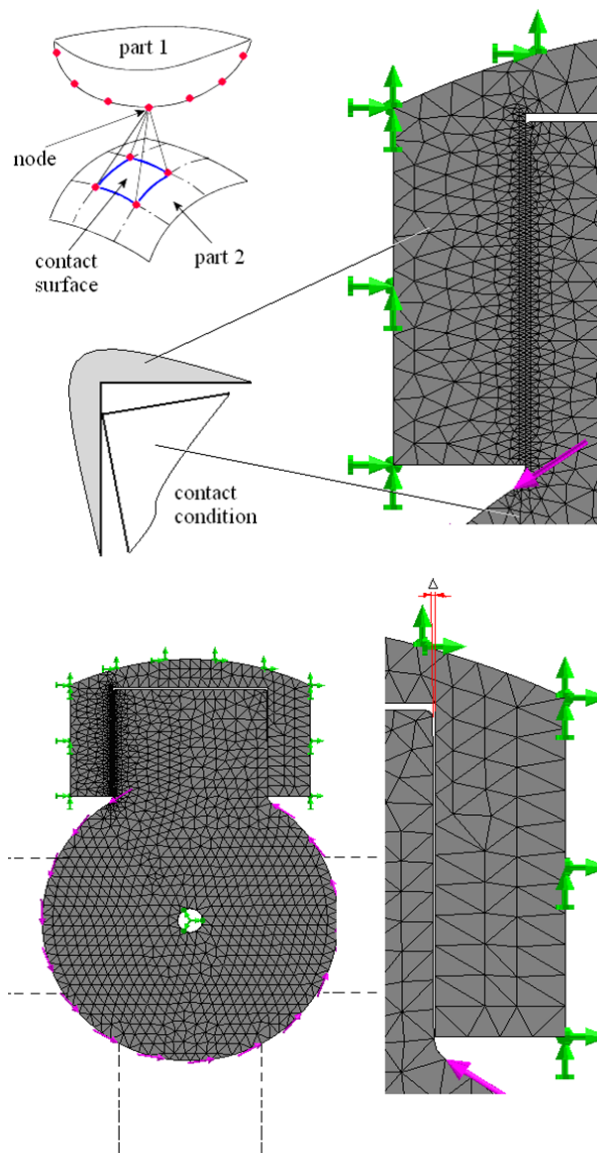


Fig. 8. Nonlinear mathematical model of the Cardan cross sleeves and bearing bushing

Fig. 9 shows the diagram of stress value variation along the center line on the sleeve contact surface. Presented stresses are the result of the pressure in the contact surface, as well as the effect of sleeve bending. Therefore, in the root of a sleeve are stress values are increased. The values of surface pressure can not be separated from the stress matrix, because they are defined in the centroid of each of the tetrahedral elements. Surface pressure can be determined indirectly by the reaction in the nodes that are in contact, and surface of the corresponding finite elements.

Based on the analysis it can be concluded that the highest stresses (200...240 N/mm²) are generated in the points that belong to the top of the sleeve ($k = 0.75...1.0$). The minimum stress values should be at the base – the sleeve root. However, because of the influence of stress due to sleeve bending, the smallest stresses are generated in the middle area of the sleeve, $k = 0.5$.

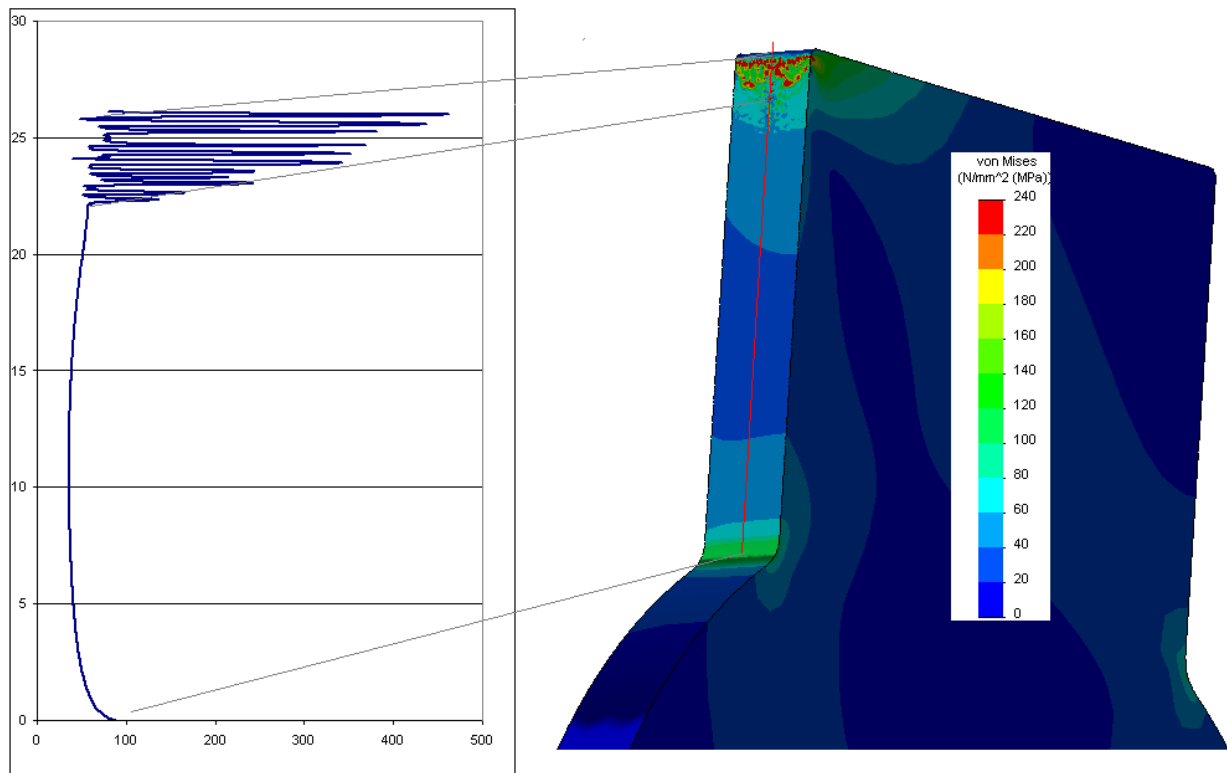


Fig. 9. The stress states on the contact surface of the Cardan cross drive pin and bushing surface

5. CONCLUSIONS

The main indicators of functional ability of Cardan couplings are their service life and reliability. The greatest influence on these indicators of functional ability has the character of the linear load distribution on Cardan joints rolling parts. In order to shed some more light to the phenomena of linear load distribution and correlation between the geometric characteristics of rolling elements and the distribution of the linear load, analytical and numerical models under appropriate assumptions are established in this paper.

The obtained results show that with the standard (the existing) geometry of rolling parts, the engagement of points in the root of Cardan cross journal in the load transfer is negligible due to very uneven linear load distribution. This unfavorable distribution of the linear load can be reduced with an appropriate combination of geometrical parameters of rolling parts in contact. Greater improvement of linear load distribution requires the appropriate shape reconstruction of rolling parts.

Based on the developed numerical model, the correlation between the stress state in the contact surfaces of rolling parts and the distribution of the linear load is analyzed. In accordance with the uneven load distribution, the stress image on contact surfaces is uneven too. Greater improvement of linear load distribution and stress state in the contact surfaces needs an adequate shape reconstruction of the rolling parts geometry, which is the subject of continuation of these researches, and there are part of a larger research programme. In [21] it is noted „Consequently, the opinion that the theory of Cardan drives has already

been developed and nothing could be added to it, is inconsistent...”.

ACKNOWLEDGEMENT

This research has been developed within the activity of the Project TR35029, financed by the Ministry of Education and Science of the Republic of Serbia

REFERENCES

- [1] Mitrović, R.: *Couplings, Scope Mechanical Engineering*, Zavod za udžbenike Beograd, 2005, (in Serbian).
- [2] Ognjanović, M.: *Machine Elements*, University of Belgrade, Faculty of Mechanical Engineering, 2011, (in Serbian).
- [3] Dodge, A.Y.: Bearing loads due to universal joint action, *Automotive Industries*, Vol. 83, pp.636-639, 1940.
- [4] Evernden, H.I.F.: The Propeller Shaft or Hooke's Coupling and the Cardan Joint, in: *Proceedings of the Institute of Mechanical Engineers, Automotive Division, Part III*, pp. 100–110, 1948.
- [5] Yang, A.T. and Freudenstein, F.: Application of dual-number quaternion algebra to the analysis of spatial mechanisms, *Journal of Applied Mechanics*, Transactions of ASME, Vol. 86, No. 2, pp. 300–309, 1964.
- [6] Fischer, I., and Freudenstein, F.: Internal force and moment transmission in a cardan joint with manufacturing tolerances, *ASME Journal of*

- Mechanisms, Transmissions and Automation in Design, Vol.106,,pp.301-311, 1984.
- [7] Wagner, E.R. and Cooney, C.E.: *Universal Joint And Driveshaft Design Manual*, Advances in engineering series, No. 7, pp. 39-75, Society of Automotive Engineers, Warrendale, PA, 1979.
- [8] Shigley, J.E. and Mischke, C.R.: *Standard Handbook Of Machine Design*, pp. 19–28, McGraw-Hill, New York, 1986.
- [9] Paros, J. M. and Weisbord, L.: How to design flexure hinges, *Machine Design*, Vol. 37, pp. 151–156, 1965.
- [10] *Machine Design* 11, pp. 47–481. 1939.
- [11] Lingaiah, K.: *Machine Design Data Handbook*, pp. 25–90, McGraw-Hill, New York, 1965.
- [12] Hummel, S. R. and Chassapis, C: Configuration design and optimization of universal joints with manufacturing tolerances, *Mechanism and Machine Theory*, Vol. 35, No. 3, pp. 463-476, 2000.
- [13] ISO – 10825 – 1995, Wear and damage to gear teeth, <http://www.iso.org>.
- [14] Savić, Z. and Puljezević, M.: Addition to the research of Cardan shaft load capability, in: *Proceedings of Power Transmission Symposium*, Jugoslovensko društvo za motore i motorna vozila, Beograd, 1976., pp. 49-69.
- [15] Ristivojević, M. and Mitrović, R: *Load Distribution – Gear Pairs and Rolling Bearings*, Faculty of Mechanical Engineering, University of Belgrade, 2002, (in Serbian)..
- [16] Mitrović, R.: *Research Of Design and Tribology Parameter Influences on Ball Rolling Bearing Under a High Revolutions*, PhD Thesis, Faculty of Mechanical Engineering, University of Belgrade, 1992, (in Serbian).
- [17] Lazović, T., Ristivojević, M. and Mitrović, R.: Mathematical model of load distribution in rolling bearing, *FME Transactions*, Vol. 36, No. 4, pp. 189-196, 2008.
- [18] Lazović, T.: Influence of rolling bearing geometry influences on load distribution on rolling bodies and stiffness, M.A.Sc. thesis, Faculty of Mechanical Engineering, University of Belgrade, 2000, (in Serbian).
- [19] Tomović, R.: Calculation of the boundary values of rolling bearing deflection in relation to the number of active rolling elements, *Mechanism and Machine Theory*, Vol. 47., pp 74-88, 2012.
- [20] Živković, K., Ivanović, L. and Stojanović, B.: The effect of geometry on the stress distribution of cross shaft, 7th Int. Conf. IRMES, Zlatibor 2011, pp. 245-252.
- [21] Daskalov, A.Y.: Kinematic analysis of cardan drives, *Mechanism and Machine Theory*, Vol. 25, No. 5, pp 479-486, 1990.

**ИСТРАЖИВАЊЕ УТИЦАЈА
ГЕОМЕТРИЈСКИХ ВЕЛИЧИНА СПРЕГНУТИХ
ДЕЛОВА КАРДАНОВЕ СПОЈНИЦЕ НА
РАСПОДЕЛУ ОПТЕРЕЋЕЊА**

**Зоран Стаменић, Милета Ристивојевић, Милан
Тасић, Радивоје Митровић**

Карданове спојнице се широко примењују у машинству због могућности преношења оптерећења и обртног кретања са једног вратила на друго, чије се осе међусобно секу под променљивим углом. Витални делови ових спојница имају ограничени радни век због недовољне површинске чврстоће контактних површина спрегнутих делова. Расподела оптерећења спрегнутих делова Карданове спојнице има велики утицај на носивост и њихов радни век. У том циљу, у овом раду, детаљно је анализирана расподела оптерећења дуж линије додира рукавца Кардановог крста и котрљајних тела – иглица. Развијени су одговарајући аналитички и нумерички модели. На основу развијених модела, могуће је извршити оптимизацију геометријских величина са аспекта равномерности расподеле оптерећења спрегнутих делова Карданове спојнице.

