The Transport Aircraft Minimum Pollution Climb Schedule

Petar Mirosavljević
Assistant Professor
University of Belgrade
The Faculty of Traffic and Transport Engineering

Slobodan Gvozdenović
Full Professor
University of Belgrade
The Faculty of Traffic and Transport Engineering

Ljubiša Vasov
Associate Professor
University of Belgrade
The Faculty of Traffic and Transport Engineering

Today ACARE ses the targets that drastically lower transport aircraft air pollution. Airline must find the strategy to satisfy a new pollution standard with the present aircraft performances and new flight techniques. The paper sets the analysis of airline direct operating cost increase in a climb flight phase, under the pollution reduction criterion. Presented analysis defines airline strategy, based on real aircraft performance and real environmental pollution constraints. In this investigation, obtained theoretical model of the optimal climb speed schedule with altitude is presented; under the criterion of minimum climb total cost and minimum climb pollution. Optimal climb speed schedule is controlled by CI. In this paper is proposed a new operative parameter PI for pollution minimization. The final result of paper is an operative climb schedule for PI and CI. This obtained climb schedule can be applied in any transport aircraft equipped with primary flight instruments: altimeter, air speed indicator and Mach meter, but also in modern turbofan transport aircrafts equipped with FMS.

Keywords: *turbofan aircraft, climb schedule, pollution, minimization, pollution index, cost index.*

1. INTRODUCTION

The impact of pollution is measured from pollutant directly (CO_2, SO_2, etc.) and non-directly (NO_x) related to fuel consumption. The analysis introduced in this paper, sets up the strategies of direct operating cost increase minimization for minimal pollution, for short and medium range routes.

The suggested flight technique, obtained from proposed strategy, must be prepared for real operational application in transport aircraft with on board FMS installed. The research [1], shows low level of real application of FMS for direct cost minimization. On one side, the goal of some air transport carriers is only to minimize fuel consumption without additional investment in airframe and/or engine, neglecting flight time consumption.

Real application of FMS for cost minimization is the application of *CI*, which is the ratio of unit time and unit fuel cost. At present, *CI* has been modified in the form of *DCI*, which comprises operative disturbances connected with aircraft delay and other planned flight time disturbances. In a climb flight phase for the set of *CI*, *FMS* calculates climb schedule in *FMS* MIN COST mode. Another option in *FMS* is to enter predefined climb mode as a combination of *CAS* speed and *M* and *TOC*. The result of this paper is a climb schedule in the form of *CAS/M* combination and determined *TOC*, which can be calculated within FMS, or can be used in any turbofan aircraft equipped with altimeter, air speed indicator and Mach meter.

The previous work published in many papers has been focused on the cruise flight phase analysis as the most dominant. The aim of this paper is to develop the importance of climb flight phase and introduce the impact of pollution on the climb phase. Contrary to the other analyses, this paper fixed cruise and descent phase and investigates the possibilities of cost and pollution minimization. The developed climb analysis shows fuel and time increase with pollution decrease. The major influence factors of fuel burned, discussed in this paper, are CO_2 and NO_x. The emission of CO_2 and NO_x depend on the type of fuel, amount of fuel burned and flight level at which fuel is burned.

We can set up direct relationship of fuel burned and CO_2 emission for transport aircraft. For kerosene Jet A1 fuel used in transport turbofan aircraft, 1kg of fuel burned produces 3.15kg of CO_2. Other potential climate impact of transport aircraft are forms of oxides of nitrogen, water vapour, oxides of sulphur, condensation trails and cirrus cloudiness. Emission of NO_x pollutant is not linearly related to fuel consumption and must be calculated using BM2. The emission related to airframe is connected with CO_2 emission, but engine emission is related to trade between CO_2 emission reduction and NO_x emission increase. During the climb flight phase the pollution of NO_x emission is evidently dominant compared to CO_2 emission, as a result of higher fuel flow. On short haul flights, where the climb phase reduces range in cruise flight phase, even higher pollution is generated in a climb compared to cruise flight phase. The paper provides comparison between minimum pollution, through costs generated by pollution and minimum climb cost, as a result of the transport aircraft climb phase.

The climb pollution cost is defined as a sum of pollution cost cleaning from pollutant directly and non-directly related to fuel consumption.

As a result, we must increase direct operating costs (increase fuel and time consumption) to reduce

Received: May 2012, Accepted: November 2012
Correspondence to: Dr Petar Mirosavljević.
Faculty of Traffic and Transport engineering,
Vojvode Stepe 305, 11000 Belgrade, Serbia
E-mail: perami@sf.bg.ac.rs

pollution (flight at lower *M* and lower *CAS* speed). This paper defines the ratio of increase of direct operating costs and decrease of the pollution. In other words, it defines the sum of direct operating costs we must increase in order to reduce the pollution. From comparison of transport aircrafts B737300 and B767300, we can find difference of climb pollution and direct operating climb costs and the difference in the ratio of direct operating climb cost increase and climb pollution reduction. In this paper, we present the comparison of climb schedule for a minimum cost climb schedule for B737300 with engines CFM56-3-B1 and B767300 with engines PW4056. Presented climb schedule is determined by PEM, which guarantees appliance not only for low speed, but also for high subsonic *M,* where effect of compressibility is present. In addition, a strategy is obtained on real engine characteristics such as variation of thrust and fuel flow with *M* and engine low-pressure rotor speed. Nowadays, the pollution emission caused by air traffic is regulated by the *LTO* cycle (Landing, Take-Off, or flight below 3000ft). On the other hand, the pollution emission for flights over 3000ft has not yet been regulated by ICAO. The transport turbofan aircraft usually operates on FL on the border of troposphere, where direct pollution is produced in the most vulnerable region of atmosphere.

As a well-known fact, ICAO sets up emission certified data for measuring pollution in *LTO* flight phase. This technique of pollution emission measurement is based on Emission Index, consumed fuel and throttle setting. ICAO emission data bank is independent of the pilot operations, aircraft mass, ISA deviation and possible engine date take off operations. For the reason of the limited application of ICAO emission data bank and for the need to achieve the results as close as possible to real aircraft operations, in this paper we applied BM2 [2]. BM2 is accepted emission calculation method and approved by ICAO Committee on Aviation Environmental Protection in March 2003. The Aircraft climb schedule comparison is based on real aircraft data extracted from [3-4] which are applied on BM2 for calculation of the pollution above 3000ft. The combination of high speed drag polar, installed engine data and BB2, offers possibilities for achievement the results which can be applied in actual airplane's operational use.

The parameter *COST* consists of time dependent costs and fuel depend costs. In a traditional approach, fuel cost is the function of only fuel price and consumed fuel. The time dependent costs are usually expresed in USD, and are the function of elapsed time and time price in [USD/Fhr]. Today, fuel cost is usually expressed in USD currency, and is a function of consumed fuel and fuel price [USD/kg], but they are also associated with additional costs generated by fuel consumption, such as environmental pollution, which is expressed in pollution cost. The elementary cost function relates to only cost of time and cost of fuel, spent during each flight phase.

This elementary cost function can be developed to also account the cost of pollution, which comprises the influence of two most important pollutants CO_2 and NO_x of the combustion process in turbofan engine. Since emission of CO_2 is linearly related to fuel consumption, we can calculate the cost of CO_2 pollution form of consumed fuel. The emission of NO_x can be expressed as the product of Emission Index, fuel flow and time in mode.

Now, we can introduce the modified *COST* (USD) function, which includes cost of pollution, $COST_p$ (USD). The cost of CO_2 pollution is expressed in USD per kg of CO_2 (mean value 0.028 USD/kg), published by [5] and cost of NO_x (middle value 3.4 [USD/kg]) pollution in USD per kg of NO_x. The emission of NO_x pollutant is not linearly related to fuel consumption and must be calculated using BM2. BM2 for the given engine and [6] data, build up relation with fuel flow and Reference Emission Index of NO_x emission $REINO_x$ [gNO_x/kg fuel], for ISA SL conditions. Then, $REINO_x$ must be adjusted for atmospheric and flight conditions to find $EINO_x$.

The results are optimal *M* or *CAS* speed distribution with a climb altitude, which results in minimum pollution cost and minimum direct operating cost. The condition for optimal *M* and *CAS* speed is obtained by logarithmic differential method, which is numerically solved by the Newton method. The final result is adopted for the application in on board *FMS*, in which the climb schedule from minimum pollution can be entered. Once distribution of optimal *M* number values, and altitude *h* is defined, for minimum costs climb technique, it is necessary to enable operative use of achieved result, so that it can be applicable in *FMS*. It brings about a new optimization problem, that is: adjustment of theoretic results to practical use in the way that optimization results are approximated in the form of constant speed *CAS* and constant *M* number, (climb schedule), so to be entered before take-off in *FMS*. In a dramatic new world changing, the aviation sector always experiences change in leadership. The aviation must set up an example of efforts to minimize pollution as a part of global environmental concerns. This paper links direct operating costs and pollution from fuel consumption process in climb a phase for the existing turbofan aircraft that belongs to older generation. The research offers solutions for operation application. The other side of pollution minimization which can be further developed, is on aircraft engine manufacturers and anew jet fuel application. The research can be developed in the application of new engines on old airframes and old engine modification for the achievement of lower pollution, with as low increase as possible of direct operating costs.

2. THE RECENT POLLUTION CONTROL ACTIVITIES

As we all are aware of recent economic crisis which entered all segments of society, we must also be prepared for future developments and post crisis events. The world air transport system is changing in a rapid way, also as a consequence of economic crisis and escalating environmental concerns. Concerns over global warming are now also focused on air carriers and general aviation. All of these issues need to be addressed for future air traffic systems, and new technology needs to be applied to the basic aircraft configuration, engines, and subsystems and the airspace in which they operate. All of these stages of development must be completed, while maintaining or improving flight safety as stated in [7].

Air traffic participates with 2-3% in global fossil fuel consumption. As a comparison, the whole transportation sector currently accounts for 20-25% of all fossil fuel consumption. Thus, the aviation sector consumes 13% of the fossil fuel used in transportation; it is the second biggest sector after road transportation, which consumes 80% [8]. The air transport industry will experience strong growth, during the next 15 years and forecasts indicate that this demand will continue to increase in the forthcoming years. The average increase of global air transport is predicted to be around 5.3% per year, till [9].

It is essential that these air transport industry developments are addressed in an appropriate way. The society will approve the aviation industry growth as a sustainable development; otherwise environmental impacts will act as a constraint to the growth. It is clear that air transport industry potentials must be explored and developed, but within the frame of global standards of engine emissions, operational emission mitigation procedures and market based economic instruments to encourage the use of aircraft with lower emission.

This trend of world changing in a way of lower pollution is moved by introducing new standards such as ACARE, [10] and new international documents such as Kyoto protocol [11]. The ACARE has set up targets for the year 2020 in order to reduce NO_x and CO_2 emission per passenger per nautical mile. This reduction is significant, 50% in case of CO_2 emissions per passenger kilometre and 80% in case of NO_x[7]. From the perspective of global pollution, it also affects aircraft and engine manufacturers. For example, the latest ICAO Oxides of Nitrogen (NO_x) Emission Standards became applicable in November 2005 and apply to engines manufactured after 31 December 2007.

The air transport is only a kind of transport that emitted pollution directly to the upper layers of atmosphere. This paper, using a climb phase example, develops the strategy with one aim: how to satisfy two contradictory goals: to reduce pollution in the upper layer of troposphere (in order to satisfy ACARE standards) and to reduce increase of fuel consumption (contradictory decrease of pollution production and increase of fuel consumption). Present, the economy and ecology aspects are equally important, but in the future standards and protocols, charges and air regulations will give advantage to pollution reduction. Two major groups of direct operating costs reduction and pollution reduction can be separated. The first case are actions which result in fuel saving, but require investment such as new generation of engines, new generation of combustion chambers, winglet installation, re-engine, better software for *FMS* improvement, etc.

All of them require investments in the forms of cash or credit, which are additional ballast on air carrier finance in long term, and sometimes with unexpected results. The second group are the cases where air carrier, in search for fuel saving, uses assets which are already available, without additional investments in aircraft. This paper is a tribute to the second group of fuel saving methods. The place where we can find the improvements in fuel savings is the aircraft performances. The aircraft performances play a major role in pollution reduction of today's commercial airlines. Many papers were published dealing with all kinds of transportations as a topic, with an aim to determine the reduction of fuel consumption and emission such as [12]. One also very interesting investigation has been done in the area of alternative fuels such as [13-15].

3. THE POLLUTION AND FUEL CONSUMPTION ON SHORT-HAUL FLIGHTS

In order to make comparison of fuel consumption and emission of pollutants, we make comparison on short-haul flight profile [5], published document for the use of different cost-benefit analysis, in which it uses average air route with a distance of 926km (500 Nm) for IFR type of flights as an example. The most used aircraft type in EU is B737, in different series. B737300 aircrafts make up to 90 percent of low-cost airlines fleet, which in the year 2007 covered 40 percent of EU air traffic market. According to [5] data, in order to make cost-benefit analysis, statistics of different types of aircrafts were used in European airlines fleet. According to the statistics, B737 makes up to 10.1 percent (1422 aircrafts) of total aircraft number, which makes it the most widely used type of carrier on a singular basis.

The second place in twin turbofan engine aircraft category for Atlantic route, which connect North America and European continent belongs to the type 767300, fitted with PW4056 engines. The twin turbofan aircraft B767300 can be fitted with three types of turbofan engines: PW4060, PW4056 and CF6-80A, [16], [17]. Both B767300 and 737300 (fitted with engines CFM56-3-B1) will be the subject of primary interest in further analysis.

4. FUEL CONSUMPTION AND POLLUTION IN CLIMB PHASE

The previous work, published in papers, has had the focus on cruise flight phase analysis as most dominant. The aim of this paper is to develop importance of climb flight phase and introduce climb phase impact on pollution. Contrary to other analysis, in this paper we assumed constant cruise (const. cruise M) and descent flight technique (const. descent M and CAS speed) and investigate possibilities of cost and pollution minimization by the application of different climb flight techniques. The results of the analysis show increase of fuel and time consumption together with pollution decrease.

The two major impact factors of fuel burned, discussed in this paper, are CO_2 and NO_x. The emission of CO_2 and NO_x depend of the type of fuel, amount of fuel burned and flight level where fuel is burned. We can set up direct relationship of fuel burned and CO_2 emission for transport aircraft. Other potential climate impact factors of transport aircraft are: oxides of nitrogen, water vapour, oxides of sulphur, condensation trails and cirrus cloudiness. Emission of NO_x pollutant is not linearly related to fuel consumption and must be calculated by using BM2 published in [2]. The emission related to airframe is connected to CO_2 emission, but engine emission is related to trade between CO_2 emission reduction and NO_x emission increase. During the climb flight phase, dominant pollution of NO_x emission is evident, compared to CO_2 emission, as a result of higher fuel flow. The paper provides comparison between minimum pollution, through cost of cleaning pollution and minimum climbs direct operating costs, as a result of transport aircraft climb phase, which is a part of *en route* flight. The climb pollution costs are defined as a sum of pollution cost cleaning, from pollutant directly related and non-directly related to fuel consumption. As a result, we must increase direct operating costs (increase fuel and time consumption) in order to reduce pollution (flight at lower *M* and lower *CAS* speed). This paper defines the ratio between the increase of direct operating costs and the decrease of pollution. In other words, it calculates how much we must increase direct operating costs for the reduction of pollution. From comparison which was done for B737300 type of carrier with engines model CFM56-3-B1 and B767300 type of carrier with engines model PW4056, we can find the difference of climb pollution and direct operating climb costs and difference in the ratio of direct operating climb cost increase and climb pollution reduction. The pollution emission over flight altitude 3000ft, which is connected with flight in the troposphere and the stratosphere, are not subject to the *LTO* regulation. The aircraft flight is single case in transport of direct pollution in most vulnerable region of atmosphere in troposphere and stratosphere. As a well known fact, ICAO [6] has set up emission certified data for measuring pollution in *LTO* flight phase. This technique of pollution emission measurement is based on the Emission Index, consumed fuel and throttle setting. ICAO emission data bank is independent of pilot operations, aircraft mass, ISA deviation and possible engine derate take off operations. For the reason of limited application of ICAO emission data bank and for the need to achieve results as close as possible to real aircraft operations, in the paper we apply BM2. BM2 represents emission calculation method accepted and approved by ICAO Committee on Aviation Environmental Protection in March 2003.The most important element of aircraft performance study is the development of aerodynamic data and installed aircraft engine data, realistically as much as possible. This is difficult process and many authors developed their researches, but with the lack of realistic data. The example of the lack of real aircraft data is given in work [18-19]. For this reason, authors use aircraft performance model, published by EUROCONTROL Experimental Centre called BADA (Base of Aircraft Data), published in [21]. As BADA aircraft performance model uses simple aircraft drag model, users of model develop various modifications. Such an example is [22] which developed modification for BADA aircraft drag model introducing modification to account compressibility effects (i.e., transonic drag rise effects). The approach developed in the paper is based on real aircraft data (real aircraft data is known as "book level of aircraft performance"), published for B767300 in [3] and engine data [20]. The real aircraft data are published for B737300 in [4]

The combination of real drag polar and installed engine data and BM2 offer possibilities to achieve the results that can be applied in actual airplane's operational use. The flight *COST* consists of time dependent costs and fuel depend costs. In a traditional approach, fuel cost is the function of only fuel price and amount of fuel consumed. The time dependent cost is usually expressed in USD, and is the function of time consumed and time price [USD/Fhr]. Today, fuel cost is usually expressed in USD, and is the function of consumed fuel and fuel price [USD/kg]. This elementary cost function can be used for the development of pollution cost, which is comprise of influence of cleaning the two most important pollutants: CO_2 and $NO_{x,}$ in combustion process of transport aircraft turbofan engine. Since emission of CO_2 is linearly related to fuel consumption, we can calculate the cost of CO_2 pollution from consumed fuel amount. The emission of NO_x can be expressed, as stated by BM2, as a product of *EINOx*, fuel flow and time in flight mode.

5. AIRCRAFT CLIMB AND EN ROUTE FLIGHT

The presented research is based on *en route* flight phases which include: climb, acceleration, cruise and descent flight phase. All examined flight phases are segmented in small increments which is a common method of step by step calculation in the point-mass performance model as shown in [23]. *En route* phase starts with climbing from 3000ft QNH to *TOC* based on optimal climb law, for minimum climb pollution costs or minimum direct operating costs, acceleration phase on cruising *h* to cruising *Mach* number M_{cr}=0.74, cruising with constant *h*. From *TOD* start descent flight phase, with constant descent law 074/250kt to 3000ft QNH.

This cruise and descent regime was taken from FPPM, published by [24]. Such cruising and descending phase model was chosen to display the usefulness of minimum pollution costs climb. For climbing, we used maximum climb thrust setting, whereas in cruising, the program of continual thrust setting decrease is used as cruise progresses, because the cruise is done under *const. h* and *const.* $M_{cr.}$ In descending, the regime of minimum power is used (low idle thrust). Special care is directed to climb flight phase. In order to set climb parameters, we applied basic climb equations, in which *h* range between 3000ft QNH and *TOC* altitude is divided into segments, denotes as "*i*" (*i=1,..,j)*. Limitations applied in the calculations of climb, cruise and descent flight phases are based on [25].

Limitations applied in calculations of the climb phase are:

- available thrust is equal to the maximum climb thrust

$$T_{\max cl} = T_{n_{cl}} \quad \forall h_i, i = 1, ..., j \tag{1}$$

- fuel flow is function of h, M and climb thrust
- climb can be considered up to h which represents operative top of the flight, and which is defined by $ROC_{max}=500$ [ft/min]
- flight is straight, without turns or changes of flight direction
- the change of climb angle is small $\dot{\gamma}=0$
- the equations which describes climb flight, in each segment of climb are calculated for small climb angle accepted, $\gamma<13$, which result in $\cos\gamma \approx 1$, $\sin\gamma \approx \gamma$.
- c.g. position do not have influence on drag value obtained from high speed polar

The aircraft mass in the first climb segment is $m_{cl\ 1}$. For each climb segment i, we calculated used time $t_{cl\ i}$, fuel $g_{cl\ i}$ and range $X_{cl\ i}$. Equations which describe climb flight are:

- rate of climb or rate of h change

$$\frac{dh}{dt} \approx M \cdot a \cdot \gamma \tag{2}$$

- where a is denotes speed of sound, at h
- flight speed in horizontal plane

$$\frac{dX}{dt} \approx M \cdot a \tag{3}$$

- change of aircraft weight m_{cl} during climbing

$$\frac{dm_{cl}}{dt} = -F_{f\,cl} \tag{4}$$

- balance of forces perpendicular to flight direction

$$R_Z = g \cdot m_{cl} \tag{5}$$

- balance of forces in true air speed directions

$$\frac{d(M \cdot a)}{dt} \cdot m_{cl} = Tn_{cl} - R_X - g \cdot m_{cl} \cdot \gamma \tag{6}$$

Parameters needed to define climb parameters for flight in i-th segment h intervals, with constant M number in troposphere are: rate of climb ROC_i, for used fuel, $(ROC_i/F_{f\,i})$ and for range in climb, it is necessary to first define climb angle γ_i.

$$m_{cl\ i} = m_{cl\,i-1} - g_{cl\ i-1},\ (i=2, j) \tag{7}$$

$$ROC_i = \frac{M_i \cdot a_{sl} \cdot \sqrt{\theta_i} \cdot \dfrac{Tn_{cl\,i} - R_{Xi}}{g \cdot m_{cl\ i}}}{K_i},\ (i=1,..j) \tag{8}$$

$$\left(\frac{ROC_i}{F_{f\,i}}\right) = \frac{\dfrac{M_i \cdot a_{sl} \cdot \sqrt{\theta_i} \cdot \dfrac{T_{n_{cl\,i}} - R_{Xi}}{g \cdot m_{cl\ i}}}{K_i}}{F_{f\,i}},\ (i=1,..,j) \tag{9}$$

$$\gamma_i = ArcSin\left(\frac{\dfrac{T_{n_{cl\,i}} - R_{Xi}}{g \cdot m_{cl\ i}}}{K_i}\right),\ (i=1,..,j) \tag{10}$$

$$K_i = -0.133 \cdot M_i^2 + \left(1+0.2\cdot M_i^2\right) - \left(1+0.2\cdot M_i^2\right)^{-2.5} \tag{11}$$

$$M_i = \sqrt{5} \cdot \sqrt{\left[\left(\frac{1}{\delta_i} \cdot \left\{\left[1+0.2\cdot\left(\frac{CAS_i}{a_o}\right)^2\right]^{\frac{1}{3.5}} - 1\right\} + 1\right)^{\frac{1}{3.5}} - 1\right]} \tag{12}$$

For basic segment i of climb, it is possible to define time, fuel and horizontal range needed in order for aircraft to climb from h_{i-1} to h_i.

$$t_{cl\ i} = \int_{h_{i-1}}^{h_i} \frac{dh}{ROC_i},\ (i=1,..,j) \tag{13}$$

$$g_{cl\ i} = \int_{h_{i-1}}^{h_i} \frac{F_{f\,i}}{ROC_i} dh,\ (i=1,..,j) \tag{14}$$

$$X_{cl\ i} = t_{cl\ i} \cdot (\cos\gamma_i) \cdot M_i \cdot a_{sl} \cdot \sqrt{\theta_i},\ (i=1,..,j) \tag{15}$$

Climbing parameters, from $i=1$, which fits to segment of h at the beginning of climbing to $i=j$, which fits to segment of TOC are: total amount of NO_x emission during climbing, total amount of CO_2 emission during climbing, total amount of fuel consumed during climbing, total time spent during climbing, total path distance covered during climbing.

$$m_{NOx\ cl} = \sum_{i=1}^{j} g_{cl\,i} \cdot \frac{EINOx_{cl\ i}}{1000} \tag{16}$$

$$m_{co_2\ cl} = \sum_{i=1}^{j} 3.15 \cdot g_{cl\ i} \tag{17}$$

$$g_{cl} = \sum_{i=1}^{j} g_{cl\ i} \tag{18}$$

$$t_{cl} = \sum_{i=1}^{j} t_{cl\,i} \tag{19}$$

$$R_{cl} = \frac{\sum_{i=1}^{j} X_{cl\ i}}{1000} \tag{20}$$

Climbing parameters are determined for each segment of climb flight, from $i=1$, which fits the segment of h at the beginning of climbing to $i=j$, which fits the segment of height TOC. The calculation was determined as a total amount of NO_x emission during climbing $m_{NOx\ cl}$, total amount of CO_2 emission during climbing $m_{co_2 cl}$, total amount of fuel consumed during climbing g_{cl} and total time spent during climbing during climbing t_{cl}.

$$m_{NOx\ cl} = \sum_{i=1}^{j} g_{cl\,i} \cdot \frac{EINO_{x_{cl\ i}}}{1000} \tag{21}$$

$$m_{co_2 cl} = \sum_{i=1}^{j} 3.15 \cdot g_{cl\ i} \tag{22}$$

$$g_{cl} = \sum_{i=1}^{j} g_{cli} \tag{23}$$

$$t_{cl} = \sum_{i=1}^{j} t_{cl\ i} \tag{24}$$

After aircraft reaches TOC in ft, we calculate fuel, time and distance needed to speed up to Mach number in cruise, Mcr. After cruse flight phase, we determined parameters in a descent flight phase. As a final result, we can obtain total fuel g_t, time t_t, mass CO2 is $m_{co_2 t}$ emission and mass of NOx emission $m_{NOx\ t}$

6. THE OPTIMIZATION MODEL

The time dependant cost, *TC* usually expressed in USD currency, is the function of time consumed and time cost *ct* [USD/Fhr]. Today fuel cost, *FC* [USD], is function of consumed amount of fuel and fuel price *cf* [USD/kg]. The elementary cost the function relates to the cost of time and cost of fuel, spent during each flight phase.

$$COST = TC + FC \tag{25}$$

Each segment of climbing can be defined by the method of total differential, optimal M_i for minimum climb costs. Total operation climb costs on *i*-th climb segment $COST_{cl\ i}$ [USD], consist of climb fuel cost and climb time cost. We can develop function of climb costs as presented by (26).

$$COST_{cl\ i} = (ct \cdot t_{cli}) / 3600 + cf \cdot g_{cl\ i} \tag{26}$$

Minimizing the climb cost $COST_{cl\ i}$ is possible, if we minimize the sum of costs of fuel spent and time spent on *i*-th climb segment.

$$COST_{cl\ i} = \left(ct \cdot \frac{dh}{ROC_i} \right) / 3600 + cf \cdot \frac{F_{f\,i}}{ROC_i}\, dh \tag{27}$$

After we arrange this redefined condition for minimum climb costs on *i*-th climb segment, we calculate costs change, with altitude in form of parameter $RCC_i \left[USD / m \right]$ (*Rate of Climb Cost*), published by [26].

$$RCC_i = \frac{ct}{3600} \cdot \frac{1}{ROC_i} + cf \cdot \frac{F_{f\,i}}{ROC_i} \tag{28}$$

The aim of optimizing is minimizing of RCC_i. The aim of optimizing can be achieved if we define optimal distribution of *M* with climb altitude. The echnique used to find the condition of optimal M_i for each *i*-th climb segment is total differential. Let's define total differential of RCC_i on *i*-th climb segment. The condition for minimizing RCC_i is achieved by total differential RCC_i by M_i. The result of differential is, then, equalized with 0 and solved by M_i. The condition for minimum climbs costs, in case we optimize by M_i, for the representative engine performance, is shown in (29).

$$dRCC_{M\,i} = \left[\frac{\partial RCC_i}{\partial M_i} \right] d\ln M_i = 0,\ h_i = const \tag{29}$$

$$dRCC_{hi} = \left[\frac{\partial RCC_i}{\partial h_i} \right] d\ln h_i = 0,\ M_i = const \tag{30}$$

As we can see from (28), the dominant parameter is the ratio of unit time and unit fuel cost, *ct/cf* in dimension [USD/Fhr/cent/lb$_{\text{fuel}}$]. The first optimization process is to find optimal M_i on a given h_i. The development of (29) is shown in (32). The solutions of condition shown in (32), for different values of *ct/cf*, are shown in Figure 1 by solid line for aircraft B767300, [26]. The one parameter which connects *ct* and *cf* is a well-known operational parameter in air transport industry, *CI* [27]. The second optimization process is to find optimal h_i on given M_i. The development of (30) is shown in (33). The solutions of condition shown in (33), for different values of *CI*, was shown in Figure 1 with dashed line for aircraft B767300. Taking into account both optimization processes, we obtain global optimization result, which determines optimal *h* and optimal *M* for given *CI*, aircraft mass as shown in Figure 1 by the intersection of solid and dashed line. The series of global optimal solutions is given in Figure 2 for B767300 and in Figure 3 for B737300, for *CI* values: *CI=0* (minimum flight fuel consumption), *CI=30* (*Long Range Climb*) and *CI=999* (minimum flight time).

$$CI = \frac{ct}{cf \cdot 100} \tag{31}$$

$$1 = \frac{M_i}{K_i} \cdot \frac{\partial K_i}{\partial M_i} - \frac{M_i}{(T_{n_{cli}} - Rx_i)} \cdot \frac{\partial (T_{n_{cli}} - Rx_i)}{\partial M_i} + \frac{M_i}{F_{f\,i}} \cdot \frac{\partial F_{fi}}{\partial M_i} + \frac{M_i}{\frac{CI}{36 \cdot F_{f\,i}} + 1} \cdot \frac{\partial \left(\frac{CI}{36 \cdot F_{f\,i}} + 1 \right)}{\partial M_i},\ h_i = const. \tag{32}$$

$$1 = \frac{h_i}{K_i} \cdot \frac{\partial K_i}{\partial h_i} - \frac{h_i}{(T_{n_{cli}} - Rx_i)} \cdot \frac{\partial (T_{n_{cli}} - Rx_i)}{\partial h_i} + \frac{h_i}{F_{f\,i}} \cdot \frac{\partial F_{fi}}{\partial h_i} + \frac{h_i}{\frac{CI}{36 \cdot F_{f\,i}} + 1} \cdot \frac{\partial \left(\frac{CI}{36 \cdot F_{f\,i}} + 1 \right)}{\partial h_i},\ M_i = const. \tag{33}$$

For the total climb flight phase, we can determine total operating climb cost as $COST_{cl}$ (function of consumed fuel and spent time in climb). In case that we examine *en route* flight (climb from 3000ft QNH, cruise, descent to 3000ft QNH), we can determine total operating pollution climb cost as $COST_{en\ route}$ (function of consumed fuel and spent time in total en route flight). Once we know how much important pollution is, we can develop the cost of cleaning the pollution, from the products of consumed fuel in climb flight phase. In the following research, we can use the costs as a measure of pollution. For each segment of climbing, it is possible to define by the method of total differential, optimal M_i, which is *Mach* number for the minimum climb pollution costs $COST_{pcli}$. Total operation pollution climb costs on

i-th climb segment $COST_{pcli}$, consist of climb CO_2 cleaning costs and climb NO_x cleaning cost.

We can define the function of climb pollution costs as follows in (34)

$$COST_{pcl\,i} = \frac{F_{f\,cl\,i} \cdot t_{cl\,i} \cdot EINOx_{cl\,i} \cdot c_{pNOx}}{1000} + \frac{g_{cl\,i} \cdot 3.15 \cdot c_{pCO_2}}{1000} \quad (34)$$

where $COST_{pcli}$, represents climb pollution cost in USD. Minimizing climb pollution cost $COST_{pcli}$ is possible if we minimize the sum of costs of CO_2 cleaning and cost of NO_x cleaning on i-th climb segment. Emission of NO_x pollutant is not linearly related to fuel consumption and must be calculated by using BM2 [2]. BM2 for given engine and ICAO Engine Exhaust Emission Data Bank builds up the relation with fuel flow and Reference Emission Index of NO_x emission, $REINOx$ [gNO_x/kg fuel], for *ISA SL* conditions. Reference Emission Index of NO_x emission, $REINOx$ [gNO_x/kg fuel] is a function of corrected fuel flow FF_{cor} as shown in Figure 4

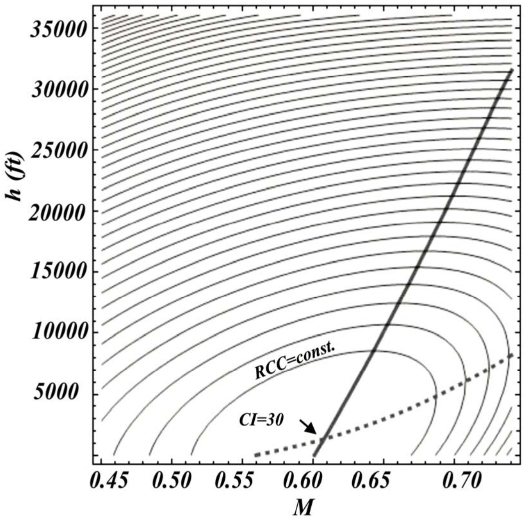

Figure 1. The contours of constant *RCC* and function of optimal *M* distribution for minimum *RCC*, for minimum direct operating costs, for B767300 with PW4056 engines, for CI=*30*

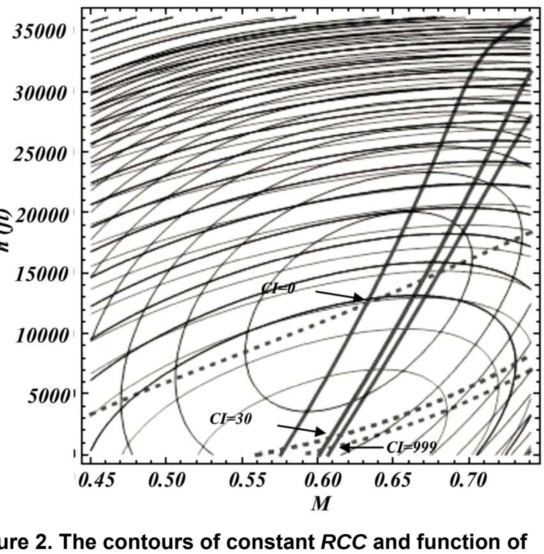

Figure 2. The contours of constant *RCC* and function of optimal *M* distribution for minimum *RCC*, with *h*, for minimum direct operating costs, for B767300 with PW4056 engines, for *CI*=0,*CI*=30 and *CI*=999

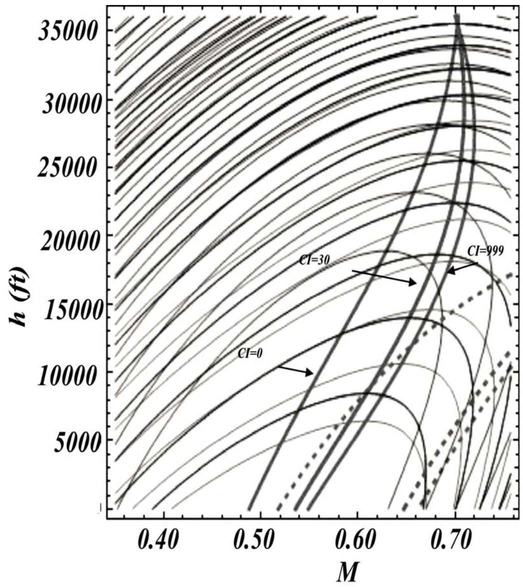

Figure 3. The contours of constant RCC and function of optimal M distribution for minimum *RCC*, with *h*, for minimum direct operating costs, for B737300 with CFM56 engines, for *CI*=0,*CI*=30 and *CI*=999

$$FF_{cor} = \frac{F_f}{\delta} \cdot \theta^{3.8} \cdot e^{0.2M^2} \quad (35)$$

where θ denotes relative temperature, δ denotes relative pressure. Then, the emission index $EINOx$, must be adjasted for atmospheric and flight condition by (37).

$$EINOx = REINOx \cdot e^H \cdot \sqrt{\frac{\delta^{1.02}}{\theta^{3.3}}} \quad (36)$$

Elements for calibration on real atmospheric condition are defined by the coefficient H in (37)

$$H = -19 \cdot (\varpi - 0.0063)$$
$$\varpi = \frac{0.62198 \cdot \phi \cdot Pv}{P_{amb} - \phi \cdot Pv} \quad (37)$$
$$Pv = 0.014504 \cdot 10^{\beta}$$

where ϕ denotes relative humidity (standard value 0.6), ϖ is denotes specific humidity, Pv[psi] denotes saturation vapour pressure, P_{amb}[psi] is denotes ambient pressure, T_{amb} [^{o}C] ambient temperature. Exponent β is the function of ambient temperature T_{amb} in ^{o}C and detail computation can be found in [2]. Analysis of BM2 shows that $EINO_x$ are the functions of flight altitude and $REINO_x$. For a given engine $REINO_x$ increase with corrected fuel flow (at *SL*, *ISA* condition) increase as shown in Figure 4. Other elements of (34) are related to flight altitude or ambient pressure and ambient temperature. Using the standard value of pollution cleaning cost [5], we can calculate cost associated with air pollution. As our aim is to achieve operational application results of this analysis, we should adapt them for the application in FMS, as suggested by [1]. After rearranging conditions for the minimum pollution cost, we can develop new parameter, rate of climb pollution cost, *RCPC* in $[USD/m]$ (*Rate of Climb Pollution Cost*) shown in (39), derived form (38).

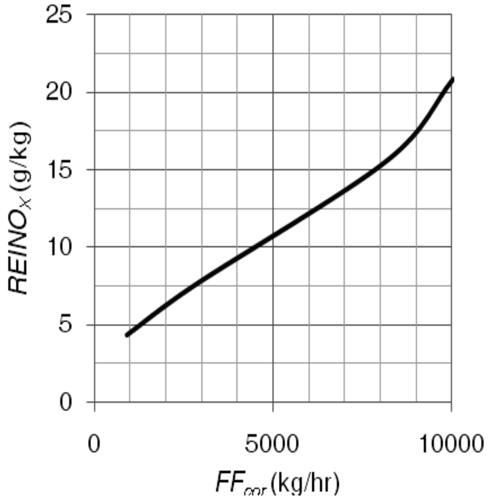

Figure 4. Relation between *$REINO_x$* and *FF_{cor}*, ICAO data bank for CFM 56 turbofan engine

$$COST_{pcl\ i} = \int_{h_i}^{h_{i+1}} \left(F_{f\,cl\ i} \cdot \frac{dh}{ROC_i} \cdot \frac{EINOx_{cl\ i}}{1000} \cdot c_{pNOx} \right) dh + \\ +3.15 \cdot \int_{h_i}^{h_{i+1}} \left(\frac{c_{pCO_2}}{1000} \cdot \frac{F_{f\,cl\ i}}{ROC_i} \right) dh \quad (38)$$

$$RCPC_i = \frac{F_{f\,cl\ i}}{ROC_i} \cdot \left(\frac{EINOx_{cl\ i}}{1000} \cdot c_{pNOx} + \frac{c_{pCO_2}}{1000} \cdot 3.15 \right) \quad (39)$$

The aim of optimizing $COST_{pcl\ i}$ is to minimize $RCPC_i$. The aim of optimization can be achieved if we define optimal distribution of *M*, with the climb altitude. The Technique used to find condition of optimal M_i for each *i*-th climb segment is total differential.

$$RCPC_i = \left[\frac{F_{fi}}{\dfrac{M_i \cdot a_{sl} \cdot \sqrt{\theta_i} \cdot (\dfrac{T_{ni} - Rx_i}{G})}{K_i}} \right] \cdot \\ \cdot \left(\frac{EINOx_{cl\ i} \cdot c_{pNOx}}{1000} + \frac{c_{pCO_2}.3.15}{1000} \right) \quad (40)$$

Let's develop total differential of $RCPC_i$ on *i*-th climb segment. The first optimization is to find optimal M_i for a given altitude h_i. The condition for minimizing $RCPC_i$ is achieved by total differential of $RCPC_i$ by M_i. The result of differential is then equalized with 0 and solved by M_i, which is numerically solved by the Newton method. The condition for minimum climb pollution costs, in case we optimize by M_i, for the representative engine and aircraft flight performance model, can be presented as shown in (41).

$$dRCPC_{M\,i} = \left[\frac{\partial RCPC_i}{\partial M_i} \right] dM_i = 0,\ h_i = const \quad (41)$$

The second optimization is to find optimal h_i for a given M_i. The condition for minimizing $RCPC_i$ is achieved by total differential of $RCPC_i$ by h_i. The result of differential is then equalized with 0 and solved by h_i, which is numerically solved by the Newton method.

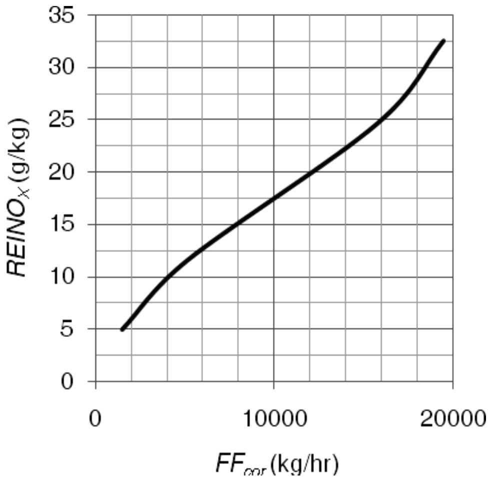

Figure 5. Relation between *$REINO_x$* and *FF_{cor}*, ICAO data bank for PW4056 turbofan engine

The condition for minimum climb pollution costs, in case we optimize by h_i, for a given *Mach* number, for the representative engine and aircraft flight performance model, can be presented as shown in (42).

$$dRCPC_{hi} = \left[\frac{\partial RCPC_i}{\partial h_i} \right] dh_i = 0,\ M_i = const \quad (42)$$

For flight in the troposphere, in case for the climb with constant M_i, we can enter the substitution of acceleration factor developed in (43) in $K_i = 1 + AF_i$ [23].

$$AF_i = -0.133 \cdot M_i^2 \quad (43)$$

In case of stratospheric flight *AF* is equal to zero (speed of sound is constant in stratosphere), [23], which relaes the optimization of $RCPC_i$.

$$AF_i = 0 . \quad (44)$$

This leads us to different optimal solutions for troposphere and stratospheric flight. We can conclude that for flight over FL360 we have different optimal M_i distributions with *h* compare to M_i distributions with *h* below FL360. The condition for calculation of M_i on *i*-th climb segment for minimum climb pollution costs $RCPC_i$ is defined by (41) and condition for calculation of h_i at a given climb M_i, for minimum climb pollution costs $RCPC_i$ is defined by (42). With the cost of CO_2 pollution C_{pCO_2} [USD/$tone_{CO2}$], published in [5], and the cost of pollution C_{pNOx} in [USD/kg_{NOx}], published in [5]. The solutions of conditions (42) and (43) for different values of C_{pNOx}/C_{pCO_2} are shown in Figure 9. As we can see from (40), the dominant parameter is the ratio of unit cost of NO_x pollution and unit cost of CO_2 pollution, so we can develop C_{pNOx}/C_{pCO_2} [$tone_{CO_2}/kg_{Nox}$]. This parameter can be developed as the Pollution Index or *PI,* as shown in (45). In case of flight in the troposphere the condition can be developed for minimum pollution costs as a function of *PI*. The parameter *PI* can be the parameter connected with pollution cost in one country, determined by local impact of pollution on the community. For total climb flight phase we can determine total operating pollution climb cost as $COST_{pcl}$. In case when we examine *en route* flight (climb from 3000ft QNH, cruise, descent to 3000ft QNH), we can determine total operating pollution climb cost as $COST_p$. Also, for the minimum total operating pollution climb cost we can determine $COST_{pCOST}$, which represents direct operating cost from *en route* flight with the application of minimum pollution climb technique.

$$PI=\frac{c_{pNOx}}{c_{pCO_2}}. \qquad (45)$$

The first optimization process is to find the optimal *Mi* on given h_i. The development of condition shown in (41) is shown in (46), for different values of *PI*. The solution of (46) is shown in Figure 6 by a solid line for aircraft B767300. The second optimization process is to find optimal h_i for a given M_i. The development of condition shown in (42) is shown in (47), for different values of *PI*.

The solution of (47) is shown in Figure 6 with dashed line for aircraft B767300. Taking into account both optimization processes we obtain global optimization result, which determines optimal *h* and optimal *M* for a given *PI* and aircraft mass, as shown in Figure 6 by the intersection of solid and dashed line.

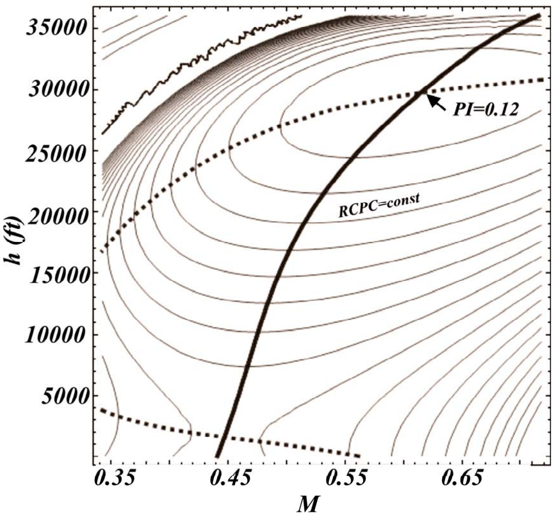

Figure 6. The contours of constant *RCPC* and function of optimal *M* distribution for minimum *RCPC*, with *h*, for minimum pollution for B767300 with PW4056 engines for *PI*=0.121

$$0=\frac{M_i}{F_{fi}}\cdot\frac{\partial F_{fi}}{\partial M_i}+1-\frac{M_i}{K_i}\cdot\frac{\partial K_i}{\partial M_i}+\frac{M_i}{(T_{ni}-R_{xi})}\cdot\frac{\partial (T_{ni}-R_{xi})}{\partial M_i}+$$
$$+\frac{M_i}{\left(EINOx_{cl\ i}\cdot PI+3.15\right)}\cdot\frac{\partial\left(EINOx_{cl\ i}\cdot PI+3.15\right)}{\partial M_i}, \qquad (46)$$
$$h_i=const$$

$$0=\frac{h_i}{F_{fi}}\cdot\frac{\partial F_{fi}}{\partial h_i}+1-\frac{h_i}{K_i}\cdot\frac{\partial K_i}{\partial h_i}+\frac{h_i}{(T_{ni}-R_{xi})}\cdot\frac{\partial (T_{ni}-R_{xi})}{\partial h_i}+$$
$$+\frac{h_i}{\left(EINOx_{cl\ i}\cdot PI+3.15\right)}\cdot\frac{\partial\left(EINOx_{cl\ i}\cdot PI+3.15\right)}{\partial h_i}, \qquad (47)$$
$$M_i=const$$

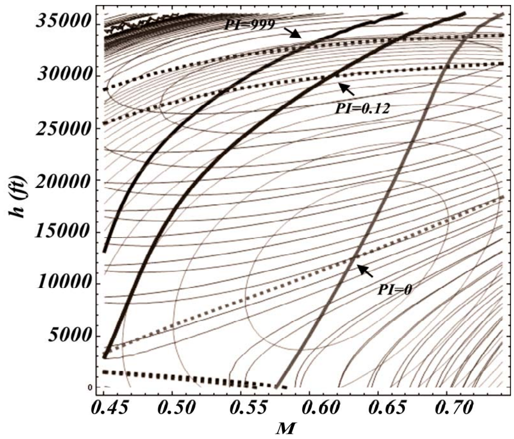

Figure 7. The contours of constant *RCPC* and function of optimal *M* distribution for minimum *RCPC*, with *h*, for minimum pollution for B767300 with PW4056 engines for *PI*=999, *PI*=0.121, *PI*=0

The series of global optimal solutions is given on Figure 7 for B767300 and on Figure 8 for B737300, for PI values: PI =0 (minimum CO2 pollution or minimum fuel consumption CI=0), PI=0.12 and PI=999 (minimum NOx pollution).

7. CLIMB LAW DETERMINATION

Once the distribution of optimal *M* (for all *i*-th flight increments), with altitude *h* is defined, for the minimum costs climb technique and minimum climb pollution costs, it is necessary to enable the operative use of achieved result, so that it can be applicable in *FMS*. That presents the new optimization problem that is: adjustment of theoretic results, to practical use in a way that optimization results are approximated in form of constant speed *CAS* and constant *M* number, (climb schedule), so to be entered before take-off in *FMS*. Post-optimal results adjustment to operative use requires solving of new a optimization task.

The aim of post-optimal adjustment for application in *FMS is*:

- to define constant speed *CAS,*
- to define h_{co}, up to which the constant *CAS* is applied and from which we begin to apply climb with const *M* number.
- to find *M* number that is constant during climb,
- constant *CAS* and constant *M* represent optimum combination (climb schedule).

It should take optimal result, to get as close as possible to operative use, which means Air Traffic Control restrictions must be obeyed. The transport aircraft in flight operations use climb schedule in the form of *CAS/M* as stated in [27]. Sub-optimal schedule of climbing has minimum exception margin from the optimal speed distribution in climbing as shown in Figure 10.

Sub-optimal climb schedule consists of *CAS* speed and *M* number, which makes it useful in operative exploitation obeying the system limitations.

Speed limits in all flight phases are in the form of maximum speed, CAS_{MO} and maximum Mach number, M_{MO}. The methodology of theoretical result transformation to operative climb speed schedule was taken from [26].

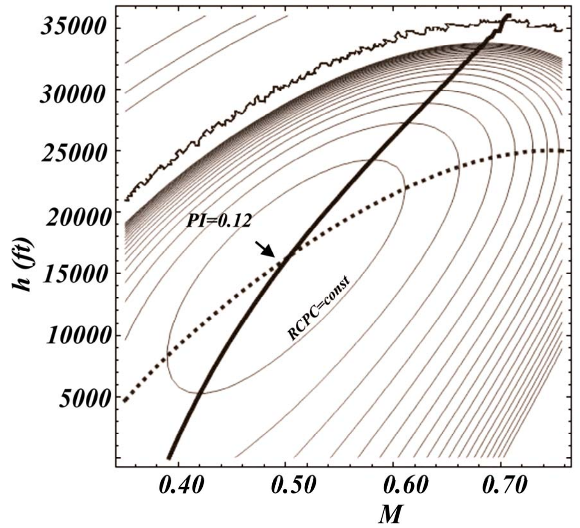

Figure 8. The contours of constant *RCPC* and function of optimal *M* distribution and optimal *h* distribution, for minimum *RCPC*, with *h* and *M*, for minimum pollution for B737300 with CFM56 engines in case of *PI*=0.121

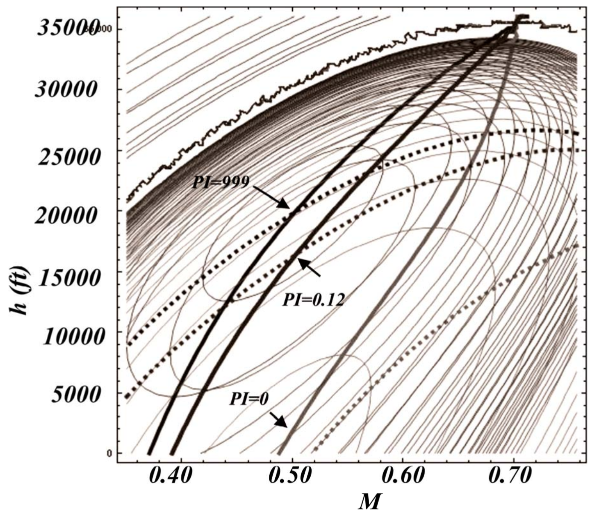

Figure 9. The contours of constant *RCPC* and function of optimal *M* distribution for minimum *RCPC*, with *h*, for minimum pollution for B737300 with CFM56 engines for *PI*=999, *PI*=0.121 and *PI=0*

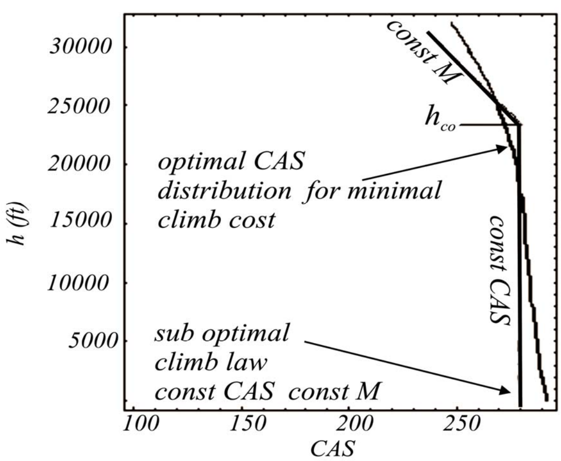

Figure 10. Adjustment of optimal *CAS* distribution with *h* with climb schedule: const *CAS*, const *M* and h_{co}

8. METHOD OF VALIDATION SOFTWARE AND TEST FACILITY

For the shown model of flight profile, in which standard cruise and standard descend are presented, the influence of different climb techniques has been analyzed. Climb techniques which are taken into consideration are: climb for minimum pollution costs and climb for minimum direct operating costs. Such problem assumption offers possibility to isolate the influence of optimum climb for minimum costs and minimum pollution on other flight phase.

There have been several experiments on presented model in order to notice the advantages and disadvantages of use of each of the two climb techniques during *en route* flight profile. Aircraft flight model and presented model of aerodynamic and engine characteristics are used for making software *OPTIMALCLIMBPOLL v.2.1*. based on platform *Mathematica®7.0*. Results which are acquired by software are *en route* flight profile with special climb flight profile presentation, diagram of change of *M* with altitude. In order to research influence of climb technique, we took into the consideration following two factors, which influence NO_x emission, CO_2 emission, flight time, and flight fuel of total flight: aircraft mass at the beginning of climb $m_{cl\ 1}$ and altitude *TOC*. We analyzed the influence of *FL* (Flight Level, e.g. *FL*100 is equal to altitude 10000ft) change from *FL200* to *FL320* at const mass m_{cl1}=50000kg (B737300) and m_{cl1}=143000kg (B767300).

For the cruise, we made the analysis with the adoption of *Mach* number in cruise M_{cr} equal to 0.74. All analyses are done for the range *R*=960km. The results obtained from software *OPTIMALCLIMBPOLL v.2.1.*, which are prepared for operational use, were validated at Flight Simulator at The Faculty of Traffic and Transport Engineering, University of Belgrade as shown in Figure 11. The Flight Simulator was designed and built by the authors of this article, in period 2005 to 2010.

Figure 11. Flight simulator at The Faculty of Traffic and Transport Engineering, University of Belgrade

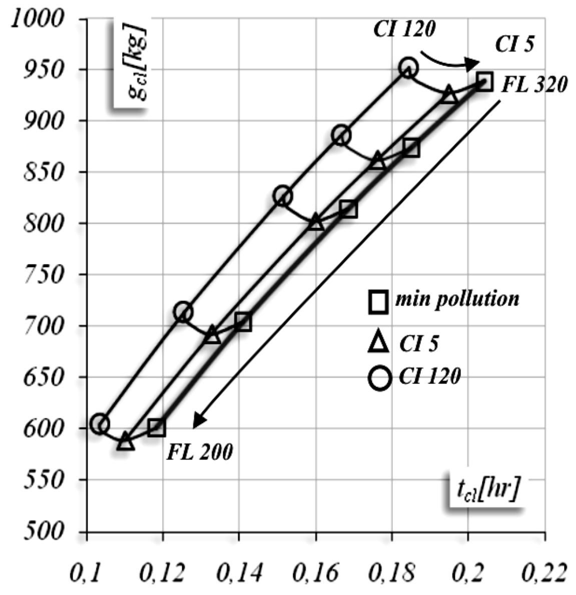

Figure 12. Mass of fuel and elapsed time in climb flight phase for different climb techniques

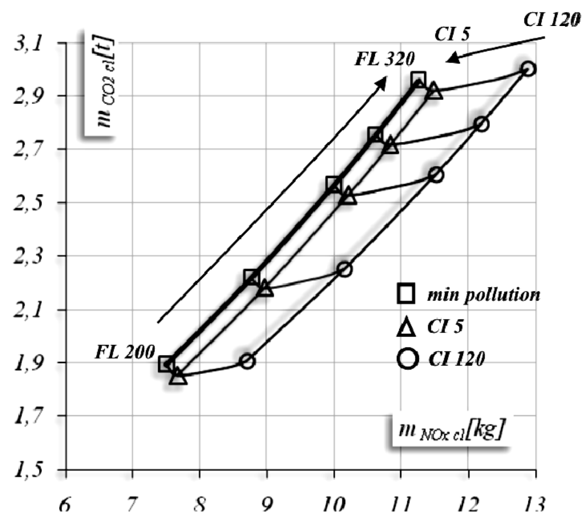

Figure 13. Mass of CO_2 and NO_x emission for climb flight complete with different climb techniques

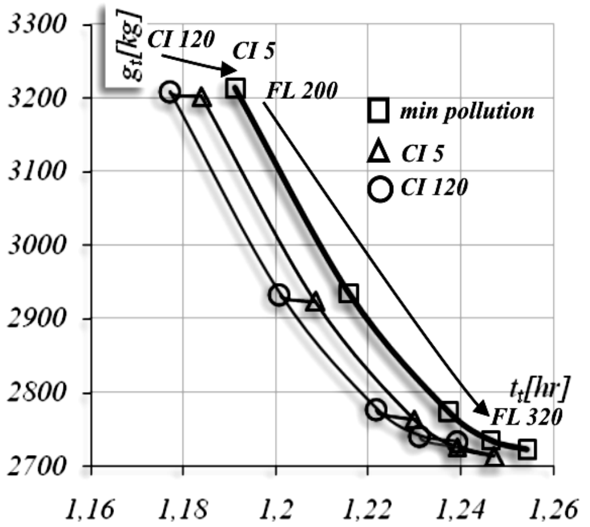

Figure 14. Mass of fuel and elapsed time for *en route* flight complete with different climb techniques

9. RESULTS OF ANALYSIS

Analysis of results achieved for the given variation of entering data enabled us to determine the climb technique for minimum costs and climb technique for minimum pollution.

The next step in the analysis is to determine of *en route* flight pollution cost (climb schedule defined by *PI*) change with *FL*, $COST_p$ and direct operating cost of *en route* flight $COST_{enroute}$ (climb schedule defined by *CI*), change with *FL*.

$$\Delta COST = \frac{100 \cdot COST_{enroute\ j}}{min\left\{COST_{enroute\ j;\, j=FL200,..,FL320}\right\}} - 100 \quad (48)$$

$$\Delta COST = \frac{100 \cdot COST_{enroute\ j}}{min\left\{COST_{enroute\ j;\, j=FL200,..,FL320}\right\}} - 100 \quad (48)$$

$$\Delta COST_p = \frac{100 \cdot COST_{p\ j}}{min\left\{COST_{p\ j;\, j=FL200,..,FL320}\right\}} - 100 \quad (50)$$

From Figure 12 we can conclude that optimal *FL* for higher value of *CI* is lower *FL* for the given research method, aircraft model and optimization approximation. Contrary to this conclusion optimal *FL* for lower pollution is at maximum *FL,* which can be obtained.

As an example we find that in the case of *CI*=90, minimum cost can be achieved at *FL240*, but for minimum pollution cost can be achieved at *FL320.* As we can conclude from Figure 12, climb phase flight fuel and time increase with *FL*, but also increase fuel consumption with increase *CI* and decrease of time spent in the climb phase.

The extreme two values are: *CI=120* can be named climb for minimum climb time and *CI=5* that can be named climb for a minimum climb fuel. The climb schedule names follow from this nomenclature are minimum climb time technique and minimum climb fuel technique, respectively.

The lowest pollution level for a given *PI***,** requires lower speeds and results in lower fuel consumption and higher climb time. These climb techniques, for minimum pollution, imply higher direct operating costs, compared to minimal direct operating costs of climb techniques. If we compare, produced mass of CO_2 and NO_x emission during climb flight phase, as shown on Figure 13, we can notice the increase of emission with *FL* increase. Also, climb technique for minimum time produces the biggest pollution of NO_x emission. Contrary to previous conclusion, if we examine total *en route* flight, with the application of variation of climb techniques, we can find new results, as shown in Figure 14 and Figure 15. The new results are totally opposite; the lowest pollution of total *en route* flight was obtained at maximum *FL320*. Higher *FL* results in lower fuel consumption and higher total flight time. Lower *FL* results in lower total time and higher fuel consumption. Again, *en route* with climb techniques for minimum time, produce the biggest NO_x pollution. In Figure 14 is shown change of fuel and time for *en route* flight for different climb techniques.

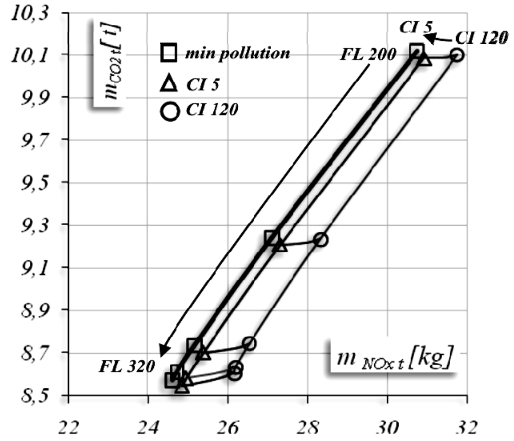

Figure 15. Mass of CO_2 and NO_x emission in *en route* flight phase for different climb techniques

The highest total *en route* time and fuel is obtained in case when we apply climb for minimum pollution defined by *PI*. Finally, we determine climb flight techniques, which result in pollution minimization, but with increase of direct operation costs. In other words, we determine the answer to the question: how much environmental protection costs?

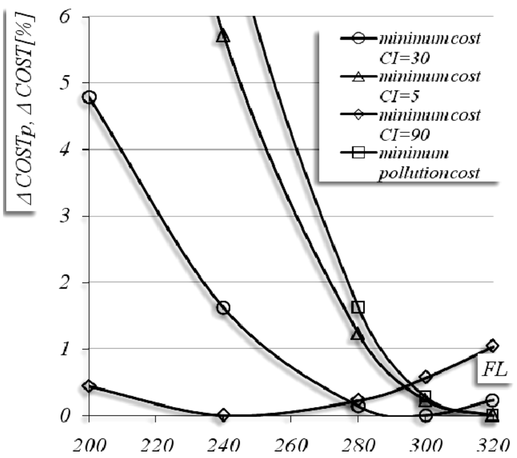

Figure 16. The percentage change of direct operating cost and pollution costs, in climb, with *FL*, for total e*n route* flight

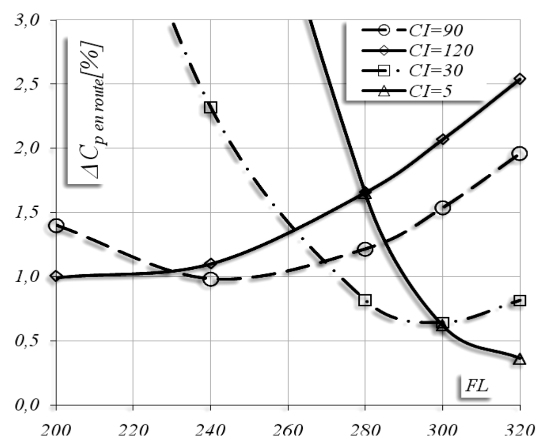

Figure 17. The percentage increase of direct operating cost for minimum pollution for *en route* flight

We can then define the percentage of increase of direct operating costs $\Delta C_{p\ enroute}$ (from consumed fuel and elapsed time), as stated in (50) and shown in Figure 17.

$$\Delta C_{penroute} = \frac{COST_p - \min\left\{COST_{enroute\ y;y=FL200,...,FL320}\right\}}{\min\left\{COST_{enroute\ y;y=FL200,...,FL320}\right\}} \cdot 100 \quad (49)$$

The aim is to minimize $\Delta C_{p\ enroute}$, by applying adequate *TOC*, at the end of climb phase.

$$\Delta C_{p\ enroute} \underset{TOC}{\rightarrow} \min \quad (50)$$

The solution of (51) is obvious, from Figure 17: as *CI* increases, the *FL* or *TOC* for minimal value of $\Delta C_{p\ enroute}$, decreases. In case of *CI*=90, the minimal value of $\Delta C_{p\ enroute}$ is achieved on *FL*240, in case of *CI*=30, the minimal value of $\Delta C_{p\ enroute}$ is achieved on *FL* 300. As shown from Figure 17, we can find, that increase of direct operating cost for achieving minimum pollution $\Delta C_{p\ enroute}$ can be reduced from 3.0% to below 0.7% (in case of *CI*=5) for suitable choice of *FL* or *TOC* (from *FL270* to *FL320*), and climb techniques for minimum climb pollution cost. By presented method, with acceptable level of increase $COST_{enroute}$, we can achieve minimum $COST_p$

10. CONCLUSION

This paper connects direct operating costs and pollution from fuel consumption process in climb phase and *en route* flight for existing turbofan transport aircrafts. As a support in flight planning process, we develop conditions for climb direct operating costs and climb pollution, determination and minimization.

Also, we developed methodology for air pollution quantification and minimization during the climb phase. Both results are tools form a measurement of ACARE standard satisfaction and cost of this satisfaction.

The result of this paper is climb schedule for minimum pollution that can be entered to the existing aircraft on board FMS and on aircraft equipped only with altimeter, air speed indicator and Mach meter.

In the paper, a new approach for defining conditions for minimum pollution cost climb technique was defined, and we also defined a new parameter of *Pollution Index* for achieving optimization goal, minimization of *Rate of Climb Pollution Cost.* The optimal solution for climb was then standardized in form of climb schedule and prepared for operational use for all kinds of turbofan transport aircrafts. As an extension of climb flight phase, we examined total *en route* flight and detected optimal *FL* for *TOC* for minimum direct operating costs and minimum pollution.

Only full understanding and wide approach will allow minimization of total turbofan aircraft pollution. Recalling assumptions of constant cruise *Mach* number and constant descent schedule, which isolated climb influence on total pollution costs, the existence of further development of pollution cost optimization in cruise and descent phase is very obvious.

The pollution assessment gives an opportunity for air operators to determine and minimize air pollution in the air transport sector with minimum increase of direct operating costs. In a dramatic new world changing, the leadership in change is the only possible way in transport aviation section.

The air transport industry must set up an example of efforts to minimize pollution as a part of global environmental concerns.

REFERENCES

[1] IATA, Guidance Material and Best Practices for Fuel and Environmental Management: Ref. No: 9093-01, ISBN 92-9195-444-6, Montreal, Geneva, pp. 23-24, 2004.

[2] Baughcum, S. L., et al.: Scheduled Civil Aircraft Emissions Inventories for 1992: Database Development and Analysis, Appendix D: Boeing Method 2 Fuel Flow Methodology Description, Report NASA CR 4700, 1996.

[3] The Boeing Company: Performance Engineers Manual B767, The Boeing Commercial Airplane Group, 6-L89A, December 01, Seattle, Washington, USA, 1988.

[4] The Boeing Company: Performance Manual for 737-300 CFM56-3-B1 (Estimated Performance), Document No. D6-37042-3, Boeing Commercial Airplane Company, Seattle, Washington, Nov. 1981.

[5] EUROCONTROL: Standard Inputs for Eurocontrol Cost benefit Analyses, Edition, 2010.

[6] ICAO: Engine Exhaust Emissions Data Bank.

[7] Report of the Group of Personalities Meeting society's needs and winning global leadership, European Aeronauticas: A Vision for 2020, 2001.

[8] Houghton J.T., Ding, Y., Griggs, D.J., Noguer, M., van der Linden, P.J., Dai, X., Maskell, K., and Johnson, C.A. (eds.): *IPCC. Climate Change 2001: The Scientific Basis*, Cambridge University Press, Cambridge, UK and New York, USA, 2001.

[9] Rogers, H.L., Lee, D.S.; Raper, D.W., Foster, P.M., Wilson, C.W. and Newton, P.J.: The impacts of aviation on the atmosphere, The Aeronautical Journal, Vol. 106, No. 1064, pp. 521-546, 2002,.

[10] The European Commission: The challenge of the environment. Strategic research agenda, Vol.2., Brussels, pp.60-110, 2002.

[11] IEA/OECD: Saving oil and reducing CO2 emission in transport, options and strategies, Paris, France, 2001.

[12] Mickūnaitis, V., Pikūnas, A. and Mackoit, I.: Reducing fuel consumption and CO2 Emission in motor car, Transport, Vol. 22, No 3, pp. 160–163, 2007.

[13] Lingaitis, P.L. and Pukalska, S.: The economic effect of using biological diesel oil on railway transport, Transport, Vol. 23, No. 4, pp. 287-290, 2008.

[14] Ibrahim Al-Hasa, M. and Al-Momany, M.: The effect of iso-butanol-diesel blends on engine performance, Transport, Vol. 23, No. 4, pp. 306-310, 2008.

[15] Boychenko, S., Shkilnuk, I. and Turchak, V.,: The problems of biopollution with jet fuels and the way of achieving solution, transport, Vol. 23, No. 4, pp. 253-257, 2008.

[16] EASA: Easa type-certificate data sheet, TCDS IM.A.035, Vol. 2, No. 3 , pp. 1-19,2005.

[17] EASA: Aircraft default engine assignments in EDMS 3.0, Attachment C.

[18] Filippone, A.: Comprehensive analysis of transport aircraft flight performance, Progress in Aerospace Sciences, Vol. 44, No. 3, pp.192-236, 2008.

[19] Filippone, A.: Data and performances of selected aircraft and rotorcraft, Progress in Aerospace Sciences, Vol. 36, Vol. 8, pp. 629-654, 2000.

[20] McCormick, B.W.: *Aerodynamics, Aeronautics, and Flight Mechanic*, Second Edition, John Wiley and Sons, Inc., 1995.

[21] EEC: Eurocontrol experimental centre, BADA.

[22] SAGE: Technical manual, Version 1.5, FAA-EE-2005-01, Federal Aviation Administration, Office of Environment and Energy, 2005.

[23] Padilla, C.E.: *Optimizing Jet Transport Efficiency: Performance, Operations and Economics*, McGraw-Hill, 1996.

[24] The Boeing Commercial Airplane Group: Flight Planning and Performance Manual, Flight Operations Engineering, NO.D632T003-XXX, August 25, Seattle, Washington, U.S.A., 2000.

[25] Hull, D.: *Fundamentals of Airplane Flight Mechanics*, Springer, 2007.

[26] Mirosavljevic, P., Gvozdenovic, S. and Cokorilo, O.: The turbofan aircraft minimum cost climb technique, Aircraft Engineering and Aerospace Technology, Vol. 81, No. 4, pp. 334-342, 2009,.

[27] McLean, D.; The operational efficiency of passenger aircraft, Aircraft Engineering and Aerospace Technology, Vo. 78, No. 1; pp. 32-38 2006.

NOMENCLATURE

ICAO	International Civil Aviation Organization
ACARE	Advisory Council of Aeronautical Research in Europe
AFM	Aircraft Flight Manual
ISA	International Standard Atmosphere
MCL	Maximum Climb Thrust
PEM	Performance Engineers Manual
EASA	European Aviation Safety Agency
EINOx	Emission index, [g NO_x / kg fuel]
FAA	Federal Aviation Authority

FMS	Flight Management System
FF	fuel flow, in kg/s
PI	Pollution Index
CI	Cost Index
CAS	Calibrated Air Speed, [kt]
COST	Direct Operations Costs
TOC	Top of Climb
TOD	Top of Descent
DCI	Dynamic Cost Index
$COST_p$	direct pollution operating costs for *en route* flight
$COST_{enroute}$	direct operating costs for *en route* flight
G	aircraft weight, [N]
g_f	fuel consumed during flight phase[kg]
S	reference wing area, [m^2]
h_{co}	cross-over altitude
h	pressure altitude
M	*Mach* number
SL	Sea Level
T_n	total available net thrust, [N[
t	time spent during flight phase, [sec]

МИНИМИЗАЦИЈА ЕМИСИЈЕ ПОЛУТАНАТА ТРАНСПОРТНОГ АВИОНА У ФАЗИ ЛЕТА ПЕЊАЊЕ

Петар Миросављевић, Слободан Гвозденовић, Љубиша Васов

ACARE је успоставио низ циљева за драстично смањење емисије штетних гасова које производи транспортни авион током оперативне експлоатације. Нови захтеви минимизације емисије полутаната постављени пред авио-компаније захтевају развој нових техника летења на авионима са постојећим летним перформансама. У овом раду је представљена анализа повећања оперативних трошкова експлоатације авиона у фази лета пењање, под критеријумом минимизације емисије полутаната. Представљена анализа дефинише стратегију авио-компанија на задовољењу нових стандарда загађења, засновану на ограничињама стварних перформанси авиона и реалном загађењу животне средине. У овом истраживању је добијени теоријски модел оптималног закана пењања под критеријумом минималних укупних трошкова пењања и минималне емисије полутаната. Оптимални закон пењања је уобичајно дефинисан путем CI. У овом раду је предложен нови оперативни параметара PI за минимизацију загађења. Крајњи резултат рада је оперативни закон пењања за задати PI и CI. Добијене вредности закона пењања могу се применити у било ком транспортном авиону, који је опремљен основним инструментима за управљање летом: висиномер, брзиномер и Mach метар, али и у модерним транспортним турбо-фенским авионима опремљеним са FMS системом.