Thickness Variation Parameter in a Thin Rotating Disc by Finite Deformation

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Seth's transition theory is applied to the problems of thickness variation parameter in a thin rotating disc by finite deformation. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield condition. It has been observed that effect of thickness for incompressible material of the rotating disc required higher percentage increased in angular speed to become fully plastic as compared to rotating disc made of compressible materials. For flat disc compressible materials required higher percentage increased in angular speed to become fully plastic as compared to disc made of incompressible material. With effect of thickness circumferential stresses are maximum at the external surface for compressible materials as compared to incompressible materials whereas for flats disc circumferential stresses are maximum at the internal surface for incompressible material as compared to compressible materials.

Keywords *Stresses, displacement, disc, angular speed, thickness, deformation.*

1. INTRODUCTION

Disc plays an important role in machine design. Stress analysis of rotating discs has an important role in engineering design. Rotating discs are the most critical part of rotors, turbines motor, compressors, high speed gears, flywheel, sink fits, turbo jet engines and computer's disc drive etc.

The analysis of thin rotating discs made of isotropic material has been discussed extensively by Timoshenko and Goodier [1] in the elastic range and by Chakrabarty [2] and Heyman [3] for the plastic range. Their solution for the problem of fully plastic state does not involve the plane stress condition, that is to say, we can obtain the same stresses and angular velocity required by the disc to become fully plastic without using the plane stress condition (i.e. T_{zz}=0).

Seth's transition theory [4] does not require any assumptions like an yield condition, incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out.

This theory utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deformed field and has been successfully applied to a large number of problems [5, 6-9, 11-28].

Seth [5] has defined the generalized principal strain measure as,

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$$e_{ii} = \int_0^{\overset{A}{e}_{ii}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)^{n/2}\right], \; i=(1,2,3) \quad (1)$$

where n is the measure and $\overset{A}{e}_{ii}$ is the Almansi finite strain components. For n =-2, -1,0, 1, 2 it gives Cauchy, Green Hencky, Swainger and Almansi measures, respectively.

Here we investigate thickness variation parameter in a thin Rotating Disc by finite deformation by using Seth's transition theory. The thickness of disc is assumed to vary along the radius in the form:

$$h = h_0 \left(r / b\right)^{-k} \quad (2)$$

where h_0 is the thickness at $r = b$ and k is the thickness parameter. Results obtained have been numerically analyzed and depicted graphically.

2. GOVERNING EQUATIONS

Consider a thin disc of variable thickness with inner radius a and outer radius b respectively. The disc is rotating with angular speed ω of gradually increasing magnitude about an axis perpendicular to its plane and passed through the center of the disc as shown in figure 1. The disc is thin and is effectively in a state of plane stress, that is, the axial stress T_{zz}=0 is zero. The displacement components in cylindrical polar co-ordinate are given by [5]:

$$u = r\left(1-\beta\right), \; v = 0, \; w = dz \quad (3)$$

where β is function of $r = \sqrt{x^2 + y^2}$ only and d is a constant.

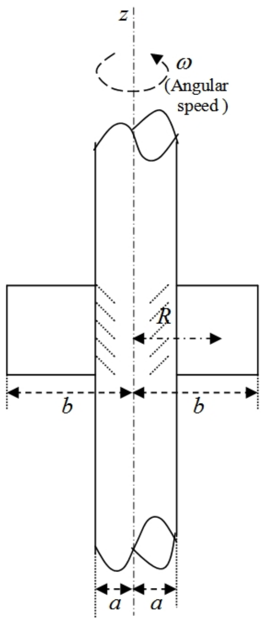

Figure 1. Geometry of Rotating Disc

The finite strain components are given by [5] as:

$$\left.\begin{aligned}
&\overset{A}{e}_{rr} \equiv \frac{\partial u}{\partial r}-\frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2=\frac{1}{2}\left[1-\left(r\beta'+\beta\right)^2\right];\\
&\overset{A}{e}_{\theta\theta} \equiv \frac{u}{r}-\frac{u^2}{2r^2}=\frac{1}{2}\left[1-\beta^2\right]\\
&\overset{A}{e}_{zz} \equiv \frac{\partial w}{\partial z}-\frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2=\frac{1}{2}\left[1-(1-d)^2\right];\\
&\overset{A}{e}_{r\theta}=\overset{A}{e}_{\theta z}=\overset{A}{e}_{zr}=0.
\end{aligned}\right\} \qquad (4)$$

where $\beta' = d\beta/dr$.

Substituting equation (4) in equation (1), the generalized components of strain are:

$$\left.\begin{aligned}
&e_{rr}=\frac{1}{n}\left[1-\left(r\beta'+\beta\right)^n\right];e_{\theta\theta}=\frac{1}{n}\left[1-\beta^n\right]\\
&e_{zz}=\frac{1}{n}\left[1-(1-d)^n\right];e_{r\theta}=e_{\theta z}=e_{zr}=0
\end{aligned}\right\} \qquad (5)$$

The stress –strain relations for isotropic material are given by [10]:

$$T_{ij}=\lambda\delta_{ij}I_1+2\mu e_{ij} \qquad (6)$$

where T_{ij} and eij are the stresses and strain components, λ and μ are lame's constants and $I_1=e_{kk}$ is the first strain invariant, δ_{ij} is the Kroncecker's delta.

Equations (6) for this problem become,

$$\left.\begin{aligned}
&T_{rr}=\frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2\mu e_{rr};\\
&T_{\theta\theta}=\frac{2\lambda\mu}{\lambda+2\mu}\left[e_{rr}+e_{\theta\theta}\right]+2e_{\theta\theta};\\
&T_{r\theta}=T_{\theta z}=T_{zr}=T_{zz}=0
\end{aligned}\right] \qquad (7)$$

Using equation (4) in equation (7), the strain components in terms of stresses are obtained as [11]:

$$\begin{aligned}
&e_{rr}\equiv\frac{\partial u}{\partial r}-\frac{1}{2}\left(\frac{\partial u}{\partial r}\right)^2=\frac{1}{2}\left[1-\left(r\beta'+\beta\right)^2\right]=\\
&=\frac{1}{E}\left[T_{rr}-\left(\frac{1-C}{2-C}\right)T_{\theta\theta}\right],\\
&e_{\theta\theta}\equiv\frac{u}{r}-\frac{u^2}{2r^2}=\frac{1}{2}\left[1-\beta^2\right]=\\
&=\frac{1}{E}\left[T_{\theta\theta}-\left(\frac{1-C}{2-C}\right)T_{rr}\right],\\
&e_{zz}\equiv\frac{\partial w}{\partial z}-\frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2=\frac{1}{2}\left[1-\left(1-d\right)^2\right]=\\
&=-\frac{(1-C)}{E(2-C)}\left[T_{rr}-T_{\theta\theta}\right],e_{r\theta}=e_{\theta z}=e_{zr}=0.
\end{aligned} \qquad (8)$$

where E is the Young's modulus and C is compressibility factor of the material in terms of Lame's constant, there are given by $E=\mu(3\lambda+2\mu)/(\lambda+\mu)$ and $C=2\mu/(\lambda+2\mu)$ Substituting equation (5) in equation (7), we get the stress as:

$$\begin{aligned}
&T_{rr}=\frac{2\mu}{n}\left[3-2C-\beta^n\left\{1-C+(2-C)(P+1)^n\right\}\right];\\
&T_{\theta\theta}=\frac{2\mu}{n}\left[3-2C-\beta^n\left\{2-C+(1-C)(P+1)^n\right\}\right],
\end{aligned} \qquad (9)$$

and $T_{r\theta}=T_{\theta z}=T_{zr}=T_{zz}=0,$ where $r\beta'=\beta P$.

Equations of equilibrium are all satisfied except:

$$\frac{d}{dr}\left(rhT_{rr}\right)-hT_{\theta\theta}+\rho\omega^2r^2h=0 \qquad (10)$$

where ρ is the density of the material of the rotating disc. Using equation (9) and (10), we get a non- linear differential equation in β as:

$$\begin{aligned}
&(2-C)n\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta}=\\
&=\left[\begin{aligned}
&\frac{rh'}{h}\left[3-2C-\beta^n\left\{1-C+(2-C)(P+1)^n\right\}\right]+\\
&+\frac{n\rho\omega^2r^2}{2\mu}\\
&+\beta^n\left[1-(P+1)^n-np\left\{1-C+(2-C)(P+1)^n\right\}\right]
\end{aligned}\right]
\end{aligned} \qquad (11)$$

where $\beta'=d\beta/dr$ (P is function of β and β is function of r only). From equation (11), the transition points of β are P=-1 and $\pm\infty$.

Boundry conditions: The boundary conditions are:

$$T_{rr}=0 \text{ at } r=a;\ T_{rr}=0 \text{ at } r=b \qquad (12)$$

3. SOLUTION THROUGH THE PROBLEMS

It has been shown [5,6-9,11-28] that the asymptotic solution through the principal stress leads from elastic state to plastic state at transition point, we define the transition function R as:

$$R = nT_{\theta\theta}/2\mu = \Big[\left(3-2C\right) - \\ -\beta^n\left\{2-C+\left(1-C\right)\left(P+1\right)^n\right\}\Big] \tag{13}$$

By taking the logarithmic differentiation of equation (13) with respect to r and using equation (11), we get:

$$\frac{d\left(\log R\right)}{dr} = -\frac{n\beta^n P}{r} \bullet \\ \bullet\left[\frac{\left(2-C\right)+\left(1-C\right)\left(P+1\right)^{n-1}\left(P+1+\beta\dfrac{dP}{d\beta}\right)}{r\left[3-2C-\beta^n\left\{2-C+\left(1-C\right)\left(P+1\right)^n\right\}\right]}\right] \tag{14}$$

Taking the asymptotic value of equation (14) at $P \to \pm\infty$ and integrating, we get:

$$R = \frac{D_1 r^{\nu-1}}{h} \tag{15}$$

where $\nu=(1-c)/(2-c)$ is Poisson's ratio in terms compressibility factor and D_1 is a constant of integration and can be determined by the given boundary condition.

From equation (13) and (15) and using equation (2), we get:

$$T_{\theta\theta} = \left(\frac{2\mu}{n}\right)\frac{D_1 r^{\nu+k-1} b^{-k}}{h_0} \tag{16}$$

Substituting equations (16), (2) in equation (10) and integrating, we get:

$$T_{rr} = \left(\frac{2\mu b^{-k}}{n\nu h_0}\right)D_1 r^{\nu+k-1} - \frac{\rho\omega^2 r^2}{\left(3-k\right)} + \frac{D_2 b^{-k}}{r h_0 r^{-k}} \tag{17}$$

where D_2 is a constant of integration and can be determined by the given boundary condition.

Using boundary condition from equation (12) in equation (17), we get:

$$D_1 = \frac{\rho\omega^2 n\nu h_0\left(b^{3-k}-a^{3-k}\right)b^k}{2\mu\left(3-k\right)\left(b^\nu - a^\nu\right)};$$

$$D_2 = \frac{\rho\omega^2 h_0}{\left(3-k\right)}b^k\left[a^{3-k} - \frac{\left(b^{3-k}-a^{3-k}\right)}{\left(b^\nu-a^\nu\right)}a^\nu\right].$$

Substituting the values of $D_1$and D_2 in equations (16) and (17), we get the transitional stresses and displacement as:

$$T_{\theta\theta} = \frac{\rho\omega^2\nu\left(b^{3-k}-a^{3-k}\right)}{\left(3-k\right)\left(b^\nu-a^\nu\right)}r^{\nu+k-1} \tag{18}$$

$$T_{rr} = \frac{\rho\omega^2 r^k}{\left(3-k\right)r}\left[\frac{\left(b^{3-k}-a^{3-k}\right)}{\left(b^\nu-a^\nu\right)}\left(r^\nu-a^\nu\right) - r^{3-k} + a^{3-k}\right] \tag{19}$$

Substituting equation (18) and (19) in second equation of (8), we get:

$$\beta = \sqrt{1-\frac{2\nu\rho\omega^2 r^{k-1}}{E\left(3-k\right)}\left[\frac{\left(b^{3-k}-a^{3-k}\right)}{\left(b^\nu-a^\nu\right)}a^\nu + r^{3-k} - a^{3-k}\right]}$$

where $E=2\mu(3-2C)/(2-C)$ is the Young's modulus in term of compressibility factor can be expresses as. Substituting the value β equation (3), we get displacement components as:

$$u = r - \\ -r\sqrt{1-\frac{2\nu\rho\omega^2 r^{k-1}}{E\left(3-k\right)}\left[\frac{\left(b^{3-k}-a^{3-k}\right)}{\left(b^\nu-a^\nu\right)}a^\nu + r^{3-k} - a^{3-k}\right]} \tag{20}$$

4. INITIAL YIELDING:

From equation (18), it is seen that $|T_{\theta\theta}|$ is maximum at the external surface (that is at r = b) for $k \geq 0.7$, therefore yielding of the disc takes place at the internal surface of the disc and equation (18) can be written as:

$$\left|T_{\theta\theta}\right|_{r=b} = \left|\frac{\rho\omega^2\nu\left(b^{3-k}-a^{3-k}\right)}{\left(3-k\right)\left(b^\nu-a^\nu\right)}b^{\nu+k-1}\right| = Y(say)$$

The angular speed necessary for initial yielding is given by:

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{Y} = \frac{\left(3-k\right)\left(b^\nu-a^\nu\right)b^2}{\nu b^{\nu+k-1}\left(b^{3-k}-a^{3-k}\right)} \tag{21}$$

and $\omega_i = \left(\Omega_i/b\right)\sqrt{Y/\rho}$.

5. FULLY - PLASTIC STATE

The disc becomes fully plastic ($C\to 0$ or $\nu\to 1/2$) at the internal surface (that is at r = a) and equation (18) become:

$$\left|T_{\theta\theta}\right|_{r=a} = \left|\frac{\rho\omega^2\left(b^{3-k}-a^{3-k}\right)}{2\left(3-k\right)\left(\sqrt{b}-\sqrt{a}\right)}a^{k-\frac{1}{2}}\right| = Y^*(say)$$

The angular speed required for fully plastic state is given by:

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y^*} = \frac{2\left(3-k\right)\left(\sqrt{b}-\sqrt{a}\right)b^2}{a^{k-\frac{1}{2}}\left(b^{3-k}-a^{3-k}\right)} \tag{22}$$

where $\omega_f = \left(\Omega_f/b\right)\sqrt{Y^*/\rho}$.

We introduce the following non-dimensional components $R=r/b$, $R_0=a/b$, $\sigma_r=T_{rr}/Y$, $\sigma_\Theta= T_{\Theta\Theta}/Y$, $U=u/b$, $\Omega^2=\rho\omega^2 b^2/Y$ and $H=Y/E$. Elastic-plastic transitional stresses, angular speed and displacement from equations (18), (19), (21) and (20) in non-dimensional form become:

$$\left.\begin{aligned}&\sigma_\theta=\left(\frac{\Omega_i^2\nu\left(1-R_0^{3-k}\right)R^{\nu+k-1}}{(3-k)\left(1-R_0^{\nu}\right)}\right);\sigma_r=\frac{\Omega_i^2R^{k-1}}{(3-k)}\left[\frac{\left(1-R_0^{3-k}\right)}{\left(1-R_0^{\nu}\right)}\left(R^{\nu}-R_0^{\nu}\right)-R^{3-k}+R_0^{3-k}\right]\\&\Omega_i^2=\frac{(3-k)\left(1-R_0^{\nu}\right)}{\nu\left(1-R_0^{3-k}\right)};U=R-R\sqrt{1-\frac{2\nu H\Omega_i^2R^{k-1}}{(3-k)}\left[\frac{\left(1-R_0{}^{3-k}\right)}{\left(1-R_0{}^{\nu}\right)}\mathrm{R}_0^{\nu}+\mathrm{R}^{3\text{-k}}-R_0{}^{3-k}\right]}\end{aligned}\right\}\quad(23)$$

Stresses, displacement and angular speed for fully plastic state ($C\to0$ or $\nu\to1/2$), are obtained from equations (18) - (21)in non-dimensional form as:

$$\left.\begin{aligned}&\sigma_\theta=\frac{\Omega_f^2\left(1-R_0^{3-k}\right)R^{k-\frac{1}{2}}}{2(3-k)\left(1-\sqrt{R_0}\right)};\sigma_r=\frac{\Omega_f^2R^{k-1}}{(3-k)}\left[\frac{\left(1-R_0^{3-k}\right)}{\left(1-\sqrt{R_0}\right)}\left(\sqrt{R}-\sqrt{R_0}\right)-R^{3-k}+R_0^{3-k}\right]\\&U=R-R\sqrt{1-\frac{H\Omega_f^2R^{k-1}}{(3-k)}\left[\frac{\left(1-R_0{}^{3-k}\right)}{\left(1-\sqrt{R_0}\right)}\sqrt{R_0}+\mathrm{R}^{3\text{-k}}-R_0{}^{3-k}\right]};\Omega_f^2=\frac{2(3-k)\left(1-\sqrt{R_0}\right)}{R_0^{k-\frac{1}{2}}\left(1-R_0^{3-k}\right)}\end{aligned}\right\}\quad(24)$$

6. PARTICULAR CASE

For a flat disc (k = 0) elastic-plastic transitional stresses and displacement from equation (18), (19) and (20) become:

$$T_{\theta\theta}=\frac{\rho\omega^2\nu\left(b^3-a^3\right)}{3\left(b^{\nu}-a^{\nu}\right)}r^{\nu-1}\quad(25)$$

$$T_{rr}=\frac{\rho\omega^2}{3r}\left[\frac{\left(b^3-a^3\right)}{\left(b^{\nu}-a^{\nu}\right)}\left(r^{\nu}-a^{\nu}\right)-r^3+a^3\right]\quad(26)$$

$$u=r-\\-r\sqrt{1-\frac{2\nu\rho\omega^2}{Er(3-k)}\left[\frac{\left(b^3-a^3\right)}{\left(b^{\nu}-a^{\nu}\right)}a^{\nu}+r^3-a^3\right]}\quad(27$$

From equation (25), it is seen that $|T_{\Theta\Theta}|$ is maximum at the internal surface and yielding take place at the bore, we have

$$\left|T_{\theta\theta}\right|_{r=a}=\frac{\rho\omega^2\nu\left(b^3-a^3\right)}{3\left(b^{\nu}-a^{\nu}\right)}a^{\nu-1}\equiv Y_1\quad(say)$$

The angular speed necessary for initial yielding is given by:

$$\Omega_i^2=\frac{\rho\omega_i^2b^2}{Y_1}=\frac{3\left(b^{\nu}-a^{\nu}\right)b^2}{\nu\left(b^3-a^3\right)a^{\nu-1}}\quad(28)$$

Elastic-plastic transitional stresses, displacement and angular speed from equations (25) –(28) in non-dimensional form become:

$$\left.\begin{aligned}&\sigma_\theta=\left(\frac{\Omega_i^2\nu\left(1-R_0^3\right)R^{\nu-1}}{3\left(1-R_0^{\nu}\right)}\right);\\&\sigma_r=\frac{\Omega_i^2}{3R}\left[\frac{\left(1-R_0^3\right)}{\left(1-R_0^{\nu}\right)}\left(R^{\nu}-R_0^{\nu}\right)-R^3+R_0^3\right]\\&\Omega_i^2=\frac{3\left(1-R_0^{\nu}\right)}{\nu\left(1-R_0^3\right)R_0{}^{\nu-1}};\\&U=R-\\&-R\sqrt{1-\frac{2\nu H\Omega_i^2}{3R}\left[\frac{\left(1-R_0{}^3\right)}{\left(1-R_0{}^{\nu}\right)}\mathrm{R}_0^{\nu}+\mathrm{R}^3-R_0{}^3\right]}\end{aligned}\right\}\quad(29)$$

For fully-plastic state ($C\to0$ or $\nu\to1/2$) at the external surface ($r=b$) and equation (25) becomes:

$$\left|T_{\theta\theta}\right|_{r=b}=\frac{\rho\omega^2\left(b^3-a^3\right)}{6\left(\sqrt{b}-\sqrt{a}\right)\sqrt{b}}=Y_1^*$$

The angular speed required for fully plastic state is given by:

$$\Omega_f^2=\frac{\rho\omega_f^2b^2}{Y_1^*}=\frac{6\sqrt{b}\left(\sqrt{b}-\sqrt{a}\right)b^2}{\left(b^3-a^3\right)}\quad(30)$$

Stresses, displacement and angular speed for fully plastic state ($C\to0$ or $\nu\to1/2$), are obtained from equations (25) - (28) and (29) in non-dimensional form as:

$$\left.\begin{array}{l}\sigma_\theta=\left(\dfrac{\Omega_f^2\left(1-R_0^3\right)}{6\sqrt{R}\left(1-\sqrt{R_0}\right)}\right);\sigma_r=\dfrac{\Omega_f^2}{3R}\left[\dfrac{\left(1-R_0^3\right)}{\left(1-\sqrt{R_0}\right)}\left(\sqrt{R}-\sqrt{R_0}\right)-R^3+R_0^3\right]\\ \Omega_f^2=\dfrac{6\left(1-\sqrt{R_0}\right)}{\left(1-R_0^3\right)};U=R-R\sqrt{1-\dfrac{H\Omega_f^2}{3R}\left[\dfrac{\left(1-R_0{}^3\right)}{\left(1-\sqrt{R_0}\right)}\sqrt{R_0}+\mathrm{R}^3-R_0{}^3\right]}\end{array}\right\} \quad (31)$$

Table 1. Angular speed required for initial yielding and fully plastic state

	Variable thickness k		Compressibility of material C	Angular speed required for initial yielding Ω_i^2	Angular speed required for fully-plastic state Ω_f^2	Percentage increase in angular speed $\left(\sqrt{\dfrac{\Omega_f^2}{\Omega_i^2}}-1\right)\times 100$
$0.5 < R < 1.0$	Flat Disc	0	0	1.420161	2.008410643	18.9207025 %
	Flat Disc	0	0.25	1.383601	2.008410643	20.4816269 %
	Flat Disc	0	0.5	1.336737	2.008410643	22.5753868 %
	1		0	1.562097	2.209138999	18.9207178 %
	1		0.25	1.599129	2.209138999	17.5356928 %
	1		0.5	1.650396	2.209138999	15.6957591 %
	2		0	1.171573	3.313708499	68.1792741 %
	2		0.25	1.199347	3.313708499	66.2205535 %
	2		0.5	1.237797	3.313708499	63.6185117 %

7. NUMERICAL RESULT AND DISCUSSION

For calculating the stresses, angular speed and displacement based on the above analysis, the following values have been taken as C = 0.00, 0.25, 0.5; $E/Y = H$ = 0.15, k = 0 (Flat Disc), 0.25, and 0.5 respectively. It can also be seen from Table I that with effect of thickness for incompressible material of the rotating disc required higher percentage increased in angular speed to become fully plastic as compared to rotating disc made of compressible materials.

For flat disc (say k=0) compressible materials required higher percentage increased in angular speed to become fully plastic as compared to disc made ofincompressible material. From figure 2 and 3, curves have been drawn between stress and displacement at the elastic-plastic transition state and fully plastic state of rotating disc having variable thickness (k = 1, 2) and flat disc (k = 0).

From figure 2 it has been seen that with the effect of thickness variation parameter circumferential stresses is maximum at the external surface for compressible materials as compared to incompressible materials, whereas for flat disc circumferential stresses are maximum at the internal surface for incompressible material as compared to compressible materials.

From figure 3, it can be seen that effect of thickness variation increases the values of circumferential stress at the external surface for fully plastic state.

8. CONCLUSION

It has been observed that effect of thickness for incompressible material of the rotating disc required higher percentage increased in angular speed to become fully plastic as compared to rotating disc made of compressible materials.

For flat disc compressible materials required higher percentage increased in angular speed to become fully plastic as compared to disc made incompressible material. With effect of thickness circumferential stresses is maximum at the external surface for compressible materials as compare to incompressible materials whereas for flats disc circumferential stresses is maximum at the internal surface for incompressible material as compared to compressible materials. Effect of thickness variation increases the values of circumferential stress at the external surface for fully plastic state.

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ВАРИЈАЦИЈА ПАРАМЕТРА ДЕБЉИНЕ ТАНКОГ РОТАЦИОНОГ ДИСКА ПОМОЋУ КОНАЧНИХ ДЕФОРМАЦИЈА

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Сетова теорија транзиције је примењена на проблем варијације дебљине танког ротирајућег диска помоћу коначних деформација. Није претпостављен ни критеријум попуштања као ни правило течења. Добијени резултати су применљиви на компресибилне материјале. Уколико су примењени услови некомпресибилности, изрази за напоне одговарају оним добијеним према критеријуму Треска. Примећено је да утицај дебљине код некопресибилних материјала ротирајућег диска захтева већу угаону брзину да би се могли сматрати пластичним у поређењу са компресибилним материјалима. Са ефектом дебљине ободни напони имају максимум на спољним површинама за компресибилне материјале, док је код дискова направљених од некомпресибилних материјала ободни напон на свом максимуму на унутрашњим површинама диска.

Meaning: sigma theta σ_θ (*circumferential stress*); sigma r- σ_r (*Radial stress*) and displacement-U

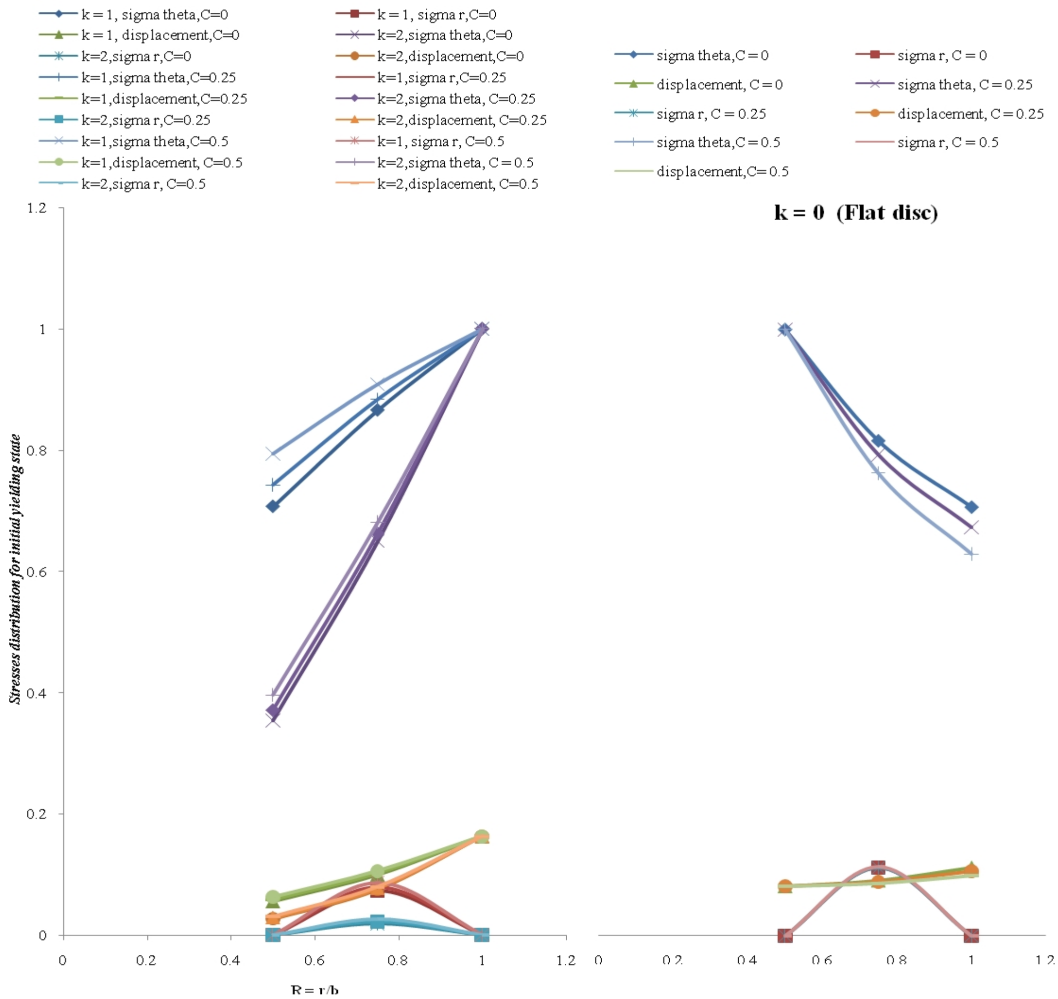

Figure 2 Stresses and displacement at the elastic-plastic transition state in a thin rotating disc having variable thickness (k = 1, 2) and flat disc (k =0) with respect to radii ratio R = r/b.

Meaning: sigma theta σ_θ (circumferential stress); sigma r- σ_r (Radial stress) and displacement-U

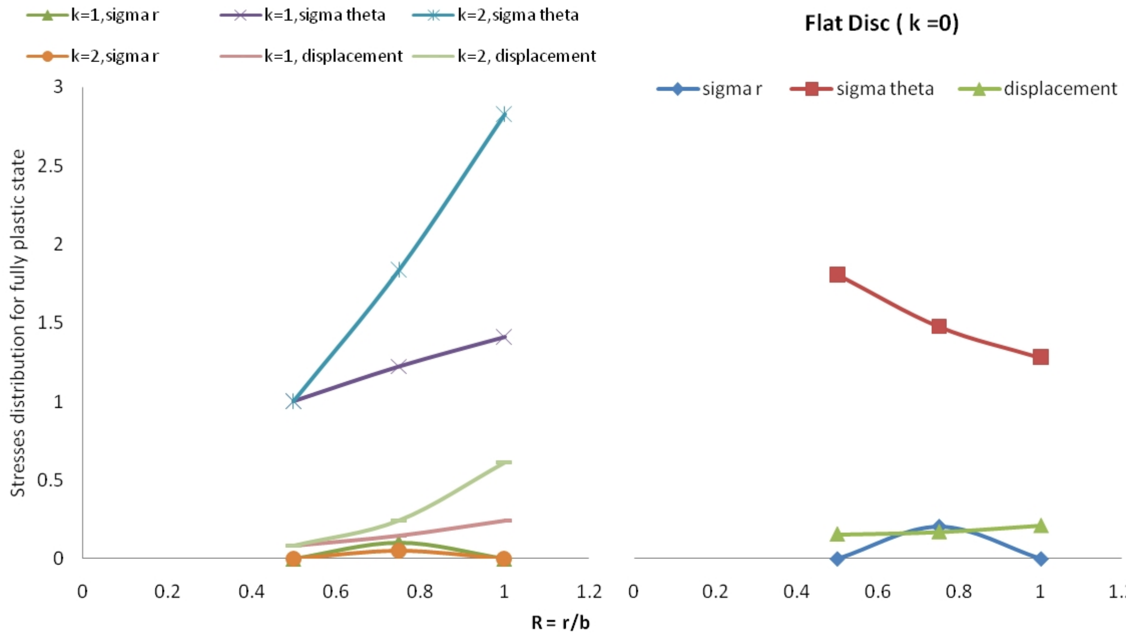

Figure 3 Stresses and displacement for fully plastic state of rotating disc having variable thickness (k = 1, 2) and flat disc (k =0) with respect to radii ratio *R = r/b*