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On the Onset of Thermosolutal Instability in a Layer of an Elastico-Viscous Nanofluid in Porous Medium

The thermosolutal instability problem in a horizontal layer of an elastico-viscous nanofluid in porous medium is considered. Walters' (model B') fluid model is employed to describe the rheological behavior of the nanofluid and for the porous medium, the Darcy model is employed. From the linear stability analysis based upon normal modes analysis method, the dispersion relation accounting for the effect of various parameters is derived. The onset criterion for stationary convection is analytically derived and it is observed that the Walters' (model B') elastico-viscous nanofluid fluid behaves like an ordinary Newtonian nanofluid. The effects of solutal Rayleigh Number, thermo-nanofluid Lewis number, thermo-solutal Lewis number, Soret and Dufour parameter on the stability of stationary convection are investigated.

Keywords: Nanofluid; Porous medium; Thermosolutal instability; Walters' (model B') fluid..

1. INTRODUCTION

The thermosolutal instability is an important phenomenon that has applications to many different areas such as geophysics, soil sciences, food processing, oil reservoir modeling, oceanography, limnology and engineering, among others. Thermosolutal instability problems related to different types of fluids and geometric configurations have been extensively studied. The thermal instability of a Newtonian fluid under a wide range of hydrodynamics and hydromagnetic assumptions was discussed in detail by Chandrasekhar [1]. The thermal instability of a Maxwellian visco-elastic fluid in the presence of a magnetic field was analyzed by Bhatia and Steiner [2]. Veronis [23] has investigated the problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient. Brakke [4] explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is layered over a denser solution of more rapidly diffusing sucrose.

Much research in recent years has focused on the study of nanofluids with a view to applications in several industries such as the automotive, pharmaceutical or energy supply industries. A nanofluid is a colloidal suspension of nano sized particles, that is, particles the size of which is below 100 nm, in a base fluid. Common fluids such as water, ethanol or engine oils are typically used as base fluids in nanofluids. Among the variety of nanoparticles that have been used in nanofluids it can be found oxide ceramics such as Al_2O_3 or CuO, nitride ceramics such as AlN or SiN and several metals such as Al or Cu. Since the term nanofluid was coined by Choi [5] the understanding of the so-called anomalous increase in thermal conductivity of nanofluids has generated considerable research interest. Buongiorno [6] proposed that the absolute nanoparticle velocity can be viewed as the sum of the base fluid velocity and a relative slip velocity. After analyzing the effect of the following seven slips mechanisms: inertia, Brownian diffusion, thermophoresism, diffusiophoresis, Magnus effect, Fluid drainage and gravity, he concluded that in the absence of turbulent eddies Brownian diffusion and thermophoresis are the dominant slip mechanisms.

The onset of convection in a horizontal layer heated from below (Bénard problem) for a nanofluid was studied by Tzou [7]. Alloui et al. [8] performed an analytical and numerical study of a natural convection problem in a shallow cavity filled with a nanofluid and heated from below. These authors reported that the presence of nanoparticles in a fluid reduced the strength of flow field, being these reductions especially relevant at low values of the Rayleigh number. Furthermore, they found that there is an optimum nanoparticle volume fraction, which depends on both the type of nanoparticle and the Rayleigh number, at which the heat transfer through the system is maximum. A considerable number of thermal instability problems in a horizontal layer of porous medium saturated by a nanofluid have also been numerically and analytically investigated by Kuznetsov and Nield [9,10] and Nield and Kuznetsov [11,12,13]. Furthermore, the effect of rotation on thermal convection in the nanofluid layer saturating a Darcy-Brinkman porous medium has been reported by Chand and Rana [14,15].

All the studies referred above deal with Newtonian nanofluids. However, with the growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology and petroleum industry attracted widespread interest in the study on non-Newtonian nanofluids. Although experiments performed by Tom et al. [16] revealed that the behavior of a dilute solution of methyl methacrylate in n-butyl acetate agrees well with

the theoretical model of Oldroyd [17], it is widely known that there are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or by Oldroyd's constitutive relations. . One such type of fluids is Walters' (model B') elastico-viscous fluid having relevance in chemical technology and industry. Walters' [18] reported that the mixture of polymethyl methacrylate and pyridine at 25^0C containg 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters' (model B') elastico-viscous fluid. Walters' (model B') elastico-viscous fluid form the basis for the manufacture of many important polymers and useful products. A good account of thermal and thermosolutal instabilities problems in a Walters' (Model B') elastico-viscous fluid in a porous medium is given by Gupta and Aggarwal [19], Rana and Sharma [20], Rana et al. [21], Rana [22] and Shivakumara et al. [23]. Sheu [24] used the Oldroyd-B fluid model to describe the rheological behavior of the nanofluid in his investigation about thermal instability in a porous medium layer saturated with a viscoelastic fluid.

The growing number of applications of nanofluids, which include several medical fields, such as cancer therapy, motivated the current study. Our main aim is to study the thermosolutal instability problem in a horizontal layer of an elastico-viscous nanofluid and Walters' (Model B') fluid model is used to describe the rheological behavior of nanofluid.

2. MATHEMATICAL MODEL

Let $T_{ij}, \tau_{ij}, e_{ij}, \mu, \mu', p, \delta_{ij}, q_i, x_i$ and d/dt denote, respectively, the total stress tensor, the shear stress tensor, the rate-of-strain tensor, the viscosity, the viscoelasticity, the isotropic pressure, the Kronecker delta, the velocity vector, the position vector and the convective derivative. Then the Walters' (model B') elastico-viscous fluid is described by the constitutive relations

$$T_{ij} = -p\delta_{ij} + \tau_{ij},$$

$$\tau_{ij} = 2\left(\mu + \mu'\frac{d}{dt}\right)e_{ij},$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_j}{\partial x_i}\right).$$

The above relations were proposed and studied by Walters' [18].

Here we consider an infinite horizontal layer of a Walters' (model B') elastico-viscous nanofluid of thickness d, bounded by the planes z = 0 and z = d as shown in fig.1. The layer is heated and soluted from below, which is acted upon by a gravity force *g = (0, 0, -g)* aligned in the *z* direction. The temperature, *T*, concentration, *C* and the volumetric fraction of nanoparticles, *φ*, at the lower (upper) boundary is assumed to take constant values T_0, C_0 and **φ₀** (T_1, C_1 and φ_1), respectively. We know that keeping a constant volume fraction of nanoparticles at the horizontal boundaries will be almost impossible in a realistic

situation. However, we assumed these conditions, which have also been previously adopted by several authors (Tzou [7], Kuznetsov and Nield [10], Nield and Kuznetsov [13], Sheu [24], Chand and Rana [14,15]) because they allow the linear stability analysis to be analytically performed.
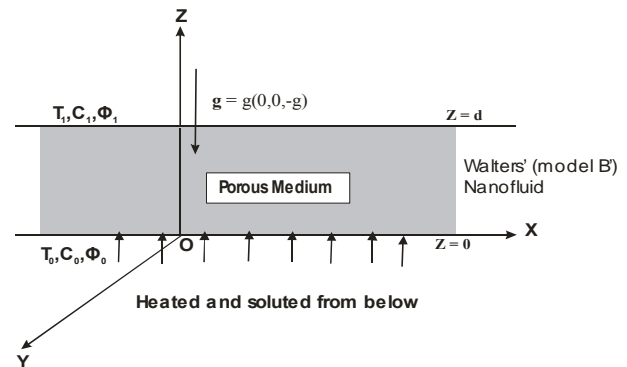

Figure.1. Physical Configuration

2.1 Assumptions

The mathematical equations describing the physical model are based upon the following assumptions:

- All thermo physical properties, except for the density in the buoyancy term, are constant (Boussinesq hypothesis);
- Base fluid and nano particles are in thermal equilibrium state;
- Nanofluid is incompressible and laminar;
- Negligible radiative heat transfer;
- Size of nanoparticles is small as compared to pore size of the matrix;
- Nanoparticles are being suspended in the nanofluid using either surfactant or surface charge technology, preventing the agglomeration and deposition of these on the porous matrix;
- The temperature, the solute concentration and the volumetric fraction of the nanoparticles are constant on the boundaries;
- The base fluid of the nanofluid is a Walters' (model B') elastico-viscous fluid;
- Nanoparticles do not affect the solute concentration.

2.2 Governing Equations

Let $\rho, \mu, \mu', p, \varepsilon$, k_1 and **q**(u, v, w), denote respectively, the density, viscosity, viscoelasticity, pressure, medium porosity, medium permeability and Darcy velocity vector. Then the equations of continuity and motion for Walters' (model B') elastico-viscous fluid (Chandrasekhar [1], Gupta and Aggarwal [19], Rana and Sharma [20] and Rana et al. [21]) in porous medium are:

$$\nabla.\mathbf{q} = 0, \qquad (1)$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\mathbf{q}\right] = -\nabla p + \rho\mathbf{g} - \frac{1}{k_1}\left(\mu - \mu'\frac{\partial}{\partial t}\right)\mathbf{q}, \quad (2)$$

The ρ density of the nanofluid can be written as Buongiorno (2006)

$$\rho = \varphi \rho_p + (1-\varphi)\rho_f \qquad (3)$$

where φ is the volume fraction of nano particles, ρ_p is the density of nano particles and ρ_f is the density of base fluid. Following Tzou [7] and Nield and Kuznetsov [13], we approximate the density of the nanofluid by that of the base fluid, that is, we consider $\rho = \rho_f$. Now, introducing the Boussinesq approximation for the base fluid, the specific weight, ρg in equation (2) becomes

$$\rho \mathbf{g} \cong \left(\varphi \rho_p + (1-\varphi) \cdot \right.$$
$$\left. \cdot \left\{ \rho \left(1 - \alpha_T (T-T_0) - \alpha_C (C-C_0) \right) \right\} \right) \mathbf{g} \qquad (4)$$

where α_T is the coefficient of thermal expansion and α_C is analogous to solute concentration.

If one introduces a buoyancy force, the equation of motion for Rivlin-Ericksen nanofluid by using Boussinesq approximation and Darcy model for porous medium (Kuznetsov and Nield [13]) is given by

$$0 = -\nabla p +$$
$$\left(\varphi \rho_p + (1-\varphi)\left\{ \rho \left(1 - \alpha_T (T-T_0) - \alpha_C (C-C_0) \right) \right\} \right) \mathbf{g}$$
$$-\frac{1}{k_1}\left(\mu - \mu' \frac{\partial}{\partial t} \right) \mathbf{q} \qquad (5)$$

The continuity equation for the nanoparticles (Buongiorno [4]) is

$$\frac{\partial \varphi}{\partial t} + \mathbf{q}.\nabla \varphi = D_B \nabla^2 \varphi + \frac{D_T}{T_1}\nabla^2 T. \qquad (6)$$

The thermal energy equation for a nanofluid is:

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{q}.\nabla T = k_m \nabla^2 T +$$
$$+\varepsilon(\rho c)_p \left(D_B \nabla \varphi.\nabla T + \frac{D_T}{T}\nabla T.\nabla T \right) +$$
$$+\rho c D_{TC} \nabla^2 C \qquad (7)$$

where $(\rho c)_m$ is heat capacity of fluid in porous medium, $(\rho c)_p$ is heat capacity of nanoparticles and k_m is thermal conductivity and is a diffusivity of Dufour type.

The conservation equation for solute concentration (Nield and Kuznetsov [14]) is

$$\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}.\nabla C = D_{Sm}\nabla^2 C + D_{CT}\nabla^2 T. \qquad (8)$$

where D_{Sm} and D_{CT} are respectively, the solute diffusivity of the porous medium and diffusivity of the Soret type.

The boundary conditions:

$$w = 0,\ T = T_0,\ \varphi = \varphi_0,\ C = C_0\ at\ z = 0, \qquad (9)$$

$$w = 0,\ T = T_1,\ C = C_1\ \varphi = \varphi_1\ at\ z = 1. \qquad (10)$$

We introduce non-dimensional variables as

$$(x',y',z',) = \left(\frac{x,y,z}{d} \right), (u',v',\ w',) = \left(\frac{u,v,w}{\kappa_m} \right)d,$$

$$t' = \frac{t\kappa}{\sigma d^2}, p' = \frac{pk}{\mu \kappa_m}, \varphi' = \frac{(\varphi - \varphi_0)}{(\varphi_1 - \varphi_0)},$$

$$T' = \frac{(T-T_1)}{(T_0-T_1)}, C' = \frac{(C-C_1)}{(C_0-C_1)},$$

where $\kappa_m = \dfrac{k_m}{(\rho c_P)_f}$ is thermal diffusivity of the fluid

and $\sigma = \dfrac{(\rho c_P)_m}{(\rho c_P)_f}$ is the thermal capacity ratio.

Thereafter dropping the dashes (') for convenience.

Equations (1),(5),(6),(7) and (8) in non-dimensional form can be written as

$$\nabla.\mathbf{q} = 0, \qquad (11)$$

$$0 = -\nabla p - \left(1 - F\frac{\partial}{\partial t} \right)\mathbf{q} - Rm\ \hat{e}_z + R_D T\ \hat{e}_z - Rn\varphi\ \hat{e}_z, \qquad (12)$$

$$\frac{1}{\sigma}\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}.\nabla \varphi = \frac{1}{Ln}\nabla^2 \varphi + \frac{N_A}{Ln}\nabla^2 T, \qquad (13)$$

$$\frac{\partial T}{\partial t} + \mathbf{q}.\nabla T = \nabla^2 T + \frac{N_B}{Ln}\nabla \varphi.\nabla T +$$
$$+\frac{N_A N_B}{Ln}\nabla T.\nabla T + N_{TC}\nabla^2 C, \qquad (14)$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}.\nabla C = \frac{1}{Le}\nabla^2 \varphi + N_{CT}\nabla^2 T, \qquad (15)$$

where we have dimensionless parameters as:

Thermo-solutal Lewis number: $Le = \dfrac{\kappa_f}{D_S}$;

Thermo-nanofluid Lewis number: $Ln = \dfrac{\kappa_m}{D_B}$;

Kinematic visco-elasticity parameter: $F = \dfrac{\mu'\kappa_m}{\mu \sigma d^2}$;

Thermal Rayleigh-Darcy Number:

$$R_D = \frac{\rho g \alpha d k_1 (T_0 - T_1)}{\mu \kappa_m}\ ;$$

Solutal Rayleigh Number:

$$Rs = \frac{\rho g \alpha d k_1 (T_0 - T_1)}{\mu \kappa_m}\ ;$$

Density Rayleigh number:

$$R_m = \frac{\rho_p \varphi_0 + \rho(1-\varphi_0) g k_1 d}{\mu \kappa_m}\ ;$$

Nanoparticle Rayleigh number:

$$Rn = \frac{(\rho_p - \rho)(\varphi_1 - \varphi_0) g k_1 d}{\mu \kappa_m}\ ;$$

Modified diffusivity ratio:

$$N_A = \frac{D_T(T_0 - T_1)}{D_B T_1(\varphi_1 - \varphi_0)} \; ;$$

Modified particle- density ratio:

$$N_B = \frac{(\rho c)_p (\varphi_1 - \varphi_0)}{(\rho c)_f} \; ;$$

Soret parameter:

$$N_{CT} = \frac{D_{TC}(C_0 - C_1)}{\kappa_m(T_0 - T_1)} \; ;$$

Dufour parameter:

$$N_{TC} = \frac{D_{CT}(T_0 - T_1)}{\kappa_m(C_0 - C_1)} \; ;$$

The dimensionless boundary conditions are:

$$w = 0, \; T = t, \; C = 1, \; \varphi = 0 \; at \; z = 0, \quad (16)$$

$$w = 0, \; T = 0, \; C = 0, \; \varphi = 1 \; at \; z = 1. \quad (17)$$

2.3 Basic Solutions

Following Nield and Kuznetsov [13], and Sheu [24] we assume a quiescent basic state that verifies

$$u = v = w = 0, \; p = p(z),$$
$$C = C_b(z), \; T = T_b(z), \; \varphi = \varphi_b(z). \quad (18)$$

Therefore, when the basic state defined in (18) is substituted into equations (11) – (15), these equations reduce to:

$$0 = -\frac{dp_b(z)}{dz} - Rm + R_D T_b(z) +$$
$$+ \frac{Rs}{Le} C_b(z) - Rn\varphi_b(z), \quad (19)$$

$$\frac{d^2\varphi_b(z)}{dz^2} + N_A \frac{d^2 T_b(z)}{dz^2} = 0, \quad (20)$$

$$\frac{d^2 T_b(z)}{dz^2} + \frac{N_B}{Ln} \frac{d\varphi_b(z)}{dz} \frac{dT_b(z)}{dz} +$$
$$+ \frac{N_A N_B}{Le} \left(\frac{dT_b(z)}{dz}\right)^2 + N_{TC} \frac{d^2 C_b(z)}{dz^2} = 0, \quad (21)$$

$$\frac{1}{Le} \frac{d^2 C_b(z)}{dz^2} + N_{CT} \frac{d^2 T_b(z)}{dz^2} = 0, \quad (22)$$

Using boundary conditions in equations (16) and (17), the solution of equation (20) is given by

$$\varphi_b(z) = (1 - T_b) N_A + (1 - N_A) z. \quad (23)$$

Using boundary conditions (16) and (17), the solution of equation (22) is given by

$$C_b(z) = (1 - T_b) Le N_{CT} - (1 + Le N_{CT}) z + 1. \quad (24)$$

Substituting the values of $\varphi_b(z)$ and $C_b(z)$ respectively, from equations (23) and (24) in equation (21), we get:

$$\frac{d^2 T_b(z)}{dz^2} + \frac{(1 - N_A) N_B}{Ln(1 + N_{TC} N_{CT} Le)} \frac{dT_b(z)}{dz} = 0. \quad (25)$$

The solution of differential equation (21) with boundary conditions in equations (16) and (17) is

$$T_b(z) = \frac{1 - e^{-(1-N_A)N_B(1-z)/Ln(1+N_{TC}N_{CT}Le)}}{1 - e^{-(1-N_A)N_B/Ln(1+N_{TC}N_{CT}Le)}}. \quad (26)$$

According to Buongiorno [4], for most nanofluid investigated so far $L_n/(\varphi - \varphi_0)$ is large, of order 10^5-10^6 and since the nanoparticle fraction decrement $(\varphi_1 - \varphi_0)$ in not smaller than 10^{-3} which means L_n is large. Typical value of N_A is no greater than about 10. Then, the exponents in equation (20) are small. By expanding the exponential function into the power series and retaining up to the first order is negligible and so to a good approximation for the solution

$$T_b = 1 - z, C_b = 1 - z \; and \; \varphi_b = z. \quad (27)$$

These results are identical with the results obtained by Kuznetsov and Nield [10], Sheu [24] and Nield and Kuznetsov [13].

2.4 Perturbation Solutions

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, so that

$$q(u,v,w) = 0 + q''(u,v,w),$$
$$T = (1-z) + T'', \; C_b = (1-z) + C'', \quad (28)$$
$$\varphi = z + \varphi'', \; p = p_b + p''.$$

Introducing equation (28) into equations (8) – (11), linearizing the resulting equations by neglecting nonlinear terms that are product of prime quantities and dropping the primes (") for convenience, the following equations are obtained:

$$\nabla \cdot \mathbf{q} = 0, \quad (29)$$

$$0 = -\nabla p - \left(1 - F \frac{\partial}{\partial t}\right) \mathbf{q} +$$
$$+ R_D T \, \hat{e}_z + \frac{Rs}{Le} C \hat{e}_z - Rn\varphi \, \hat{e}_z, \quad (30)$$

$$\frac{1}{\sigma} \frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} w = \frac{1}{Ln} \nabla^2 \varphi + \frac{N_A}{Ln} \nabla^2 T, \quad (31)$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{Ln} \left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) -$$
$$- \frac{2 N_A N_B}{Ln} \frac{\partial T}{\partial z} + N_{TC} \nabla^2 C. \quad (32)$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}-\frac{1}{\varepsilon}w=\frac{1}{Le}\nabla^2 C+N_{CT}\nabla^2 T. \qquad (33)$$

Boundary conditions for equations (29)-(31) are

$$w=0,\ T=0,\ C=0,\ \varphi=0\ at\ z=0,\ 1. \qquad (34)$$

Note that as the parameter *Rm* is not involved in equations (29)-(33) it is just a measure of the basic static pressure gradient.

The seven unknowns *u, v, w, p, T, C* and *φ* can be reduced to four by operating equation (30) with e_z.curl curl, which yields

$$\nabla^2 w=F\frac{\partial}{\partial t}\nabla^2 w+R_D\nabla_H^2 T+\frac{Rs}{Le}C\nabla_H^2-Rn\nabla_H^2\varphi, \quad (35)$$

where ∇_H^2 is the two-dimensional Laplace operator on the horizontal plane, that is:

$$\nabla_H^2=\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}.$$

3. NORMAL MODES

We express the disturbances into normal modes of the form:

$$\begin{aligned}[w,\ T,C,\ \varphi]&=\left[W(z),\ \Theta(z),\ \Gamma(z),\Phi(z)\right]\cdot\\ &\cdot\exp\left(ik_x x+ik_y y+\omega t\right),\end{aligned} \qquad (34)$$

where k_x, k_y are the wave numbers in the *x* and *y* direction, respectively, and *ω* is the growth rate of the disturbances.

Substituting equation (34) into equations (35) and (31)-(33), we obtain the following eigen value problem:

$$\left(1-\omega F\right)\left(D^2-a^2\right)W+a^2R_D\Theta+$$
$$+\frac{Rs}{Le}k^2\Gamma-a^2Rn\Phi=0, \qquad (29)$$

$$\frac{1}{\varepsilon}W+N_{CT}\left(D^2-a^2\right)\Theta+\frac{1}{Le}\left(D^2-a^2-\frac{\omega}{\sigma}\right)\Gamma=0, (30)$$

$$W+\left(D^2+\frac{N_B}{Ln}D-\frac{2N_A N_B}{Ln}D-a^2-\omega\right)\Theta-$$
$$-\frac{N_B}{Ln}D\Phi+N_{TC}\left(D^2-a^2\right)\Gamma=0, \qquad (31)$$

$$\frac{1}{\varepsilon}W-\frac{N_A}{Ln}\left(D^2-a^2\right)\Theta-\left(\frac{1}{Ln}\left(D^2-a^2\right)-\frac{\omega}{\sigma}\right)\Phi=0, (32)$$

$$W=0,\ D^2W=0,\ \Gamma=0,\ \Theta=0, \Phi=0\ at\ \ z=0 \ (33)$$

and

$$W=0,\ D^2W=0,\ \Gamma=0,\ \Theta=0,\ \Phi=0\ at\ z=1. \ (34)$$

where $D=d/dz$ and $a^2=k_x^2+k_y^2$ is the dimensionless horizontal wave number.

4. LINEAR STABILITY ANALYSIS AND DISPERSION RELATION

The eigen functions $f_j(z)$ corresponding to the eigen value problem (29)-(34) are $f_j=sin(jz)$. Considering solutions W, Θ, Γ and Φ of the form:

$$W=W_0\sin\left(\pi z\right),\ \Theta=\Theta_0\sin\left(\pi z\right),$$
$$\Gamma=\Gamma_0\sin\left(\pi z\right),\Phi=\Phi_0\sin\left(\pi z\right). \qquad (35)$$

Substituting (35) into equations (29) – (32) and integrating each equation from *z = 0* to *z = 1,* we obtain the following matrix equations

$$\begin{bmatrix}\left(1-\omega F\right)J^2 & -a^2R_D & -\dfrac{Rs}{Le}a^2 & a^2Rn\\[2mm] -\dfrac{1}{\varepsilon} & J^2N_{CT} & \dfrac{J^2}{Le}+\dfrac{\omega}{\sigma Le} & 0\\[2mm] -1 & J^2+\omega & -J^2N_{TC} & 0\\[2mm] \dfrac{1}{\varepsilon} & \dfrac{N_A}{Ln}J^2 & 0 & \dfrac{J^2}{Ln}+\dfrac{\omega}{\sigma}\end{bmatrix}\begin{bmatrix}W_0\\ \Theta_0\\ \Gamma_0\\ \Phi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \qquad (35)$$

where $J^2=\pi^2+a^2$ is the total wave number.

The linear system (35) has a non-trivial solution if and only if:

$$R_D=\frac{1}{\varepsilon\sigma J^2+\omega\varepsilon+N_{TC}J^2 Le\sigma}\times\left\{\begin{array}{l}\dfrac{\left(1-\omega F\right)J^2\varepsilon}{a^2}\left[\left(J^2+\omega\right)\left(\sigma J^2+\omega\right)+\sigma LeN_{TC}N_{CT}J^4\right]\\[3mm] +Rs\sigma\left[\varepsilon N_{CT}J^2-\left(J^2+\omega\right)\right]-\dfrac{Rn\sigma}{\sigma J^2+\omega Ln}\times\\[3mm] \left(\sigma J^2+\omega\right)\left(\left(J^2+\omega\right)Ln+N_A J^2\varepsilon\right)+N_{TC}J^4 Le\sigma\left(N_A+LnN_{CT}\right)\end{array}\right\}. \qquad (36)$$

Equation (36) is the required dispersion relation accounting for the effect of thermo-solutal Lewis number, thermo-nanofluid Lewis number, kinematic visco-elasticity parameter, solutal Rayleigh Number, nanoparticle Rayleigh number, modified diffusivity ratio, Soret and Dufour parameter on thermosolutal instability in a layer of Walters' (model B') elastico-viscous nanofluid saturating a porous medium

5. THE STATIONARY CONVECTION

For stationary convection, putting ω = 0 in equation (36) reduces it to

$$R_D = \frac{1}{\varepsilon + N_{TC}Le} \times$$
$$\times \left\{ \frac{\left(\pi^2 + a^2\right)^2 \varepsilon}{a^2}\left(1 + LeN_{TC}N_{CT}\right) + Rs\left(\varepsilon N_{CT} - 1\right) \atop -Rn\left[Ln + N_A\varepsilon + N_{TC}Le\left(N_A + LnN_{CT}\right)\right] \right\}. \quad (37)$$

Equation (37) expresses the thermal Darcy-Rayleigh number as a function of the dimensionless resultant wave number a and the parameters N_{TC}, N_{CT}, R_S, L_n, Rn, Le, N_A. Since the elastico-viscous parameter F vanishes with ω so the Walters' (model B') elastico-viscous nanofluid fluid behaves like an ordinary Newtonian nanofluid fluid. Equation (37) is identical to that obtained by Kuznetsov and Nield [13], and Chand and Rana [14]. Also, in equation (37) the particle increment parameter N_B does not appear and the diffusivity ratio parameter N_A appears only in association with the nanoparticle Rayleigh number R_n. This implies that the nanofluid cross-diffusion terms approach to be dominated by the regular cross-diffusion term.

In the absence of the Dufour and Soret parameters N_{TC}, N_{CT}, and equation (37) reduces to

$$R_D = \frac{\left(\pi^2 + a^2\right)^2}{a^2} - \frac{Rs}{\varepsilon} - \left(\frac{Ln}{\varepsilon} + N_A\right)Rn, \quad (38)$$

which is identical with the result derived by Kuznetsov and Nield [10].

In the absence of the stable solute gradient parameter Rs, equation (38) reduces to

$$R_D = \frac{\left(\pi^2 + a^2\right)^2}{a^2} - \left(\frac{Ln}{\varepsilon} + N_A\right)Rn, \quad (39)$$

Equation (39) is identical with the results derived by Sheu [24] and Chand and Rana [14].

The critical cell size at the onset of instability is obtained by minimizing R_D with respect to a. Thus, the critical cell size must satisfy:

$$\left(\frac{\partial R_D}{\partial a}\right)_{a=a_c} = 0,$$

Equation (37) which gives

$$a_c = \pi \cong 3.1416. \quad (40)$$

And the corresponding critical thermal Darcy-Rayleigh number $(R_D)_C$ on the onset of stationary convection is given by:

$$\left(R_D\right)_c = \frac{1}{\varepsilon + N_{TC}Le} \times$$
$$\times \left\{ 4\pi^2\varepsilon\left(1 + LeN_{TC}N_{CT}\right) + Rs\left(\varepsilon N_{CT} - 1\right) \atop -Rn\left[Ln + N_A\varepsilon + N_{TC}Le\left(N_A + LnN_{CT}\right)\right] \right\}. \quad (41)$$

It is noted that if Rn is positive then R_D is minimized by a stationary convection. The result given in equation (41) is a good agreement with the result derived by Sheu [24] and Chand and Rana [14] in the absence of the Dufour, Soret and stable solute gradient parameters, N_{TC}, N_{CT} and Rs respectively.

6. RESULTS AND DISCUSSIONS

The critical thermal Darcy-Rayleigh number on the onset of stationary convection is given by (41) and does not depend on viscoelastic parameter and it takes the same value that the one obtained for an ordinary Newtonian fluid.

Furthermore, the critical wave number, a_c, defined by equation (40) at the onset of steady convection coincides with those reported by Tzou [7], Kuznetsov and Nield [10] and Chand and Rana [14]. Note that this critical value does not depend on any thermo physical property of the nanofluid. Consequently, the interweaving behaviors' of Brownian motion and thermophoresis of nanoparticles does not change the cell size at the onset of steady instability and the critical cell size a_c is identical to the well known result for Bénard instability with a regular fluid [1].

It is noted that the absence of the Dufour and Soret parameters N_{TC} and N_{CT} and nonoparticles, one recovers the well-known results that the critical thermal Darcy-Rayleigh number is equal to $4\pi^2$ as obtained by Sheu [24]. Thus the combined effect of Brownian motion and thermophoresis of nanoparticles on the critical Rayleigh number is reflected in the third term in equation (41). From equation (41), it can be concluded that for the case of top-heavy distribution of nanoparticles ($\varphi_1 > \varphi_0$ and $\rho_p > \rho$), which corresponds to positive values of Rn, the value of the steady critical Rayleigh number for the nanofluid is smaller than that for an ordinary fluid, that is, steady convection sets earlier in these kinds of nanofluids than in an ordinary fluid. This implies that thermal conductivity of ordinary fluids is higher than that of nanofluids with top-heavy distribution of nano particles. On the contrary, for the case of bottom-heavy distribution of nano particles ($\varphi_1 < \varphi_0$ and $\rho_p > \rho$), which corresponds to negative values of Rn, the value of the critical Rayleigh number for the nanofluid is larger than that for an ordinary fluid, that is, convection sets earlier in a ordinary fluid than in a nanofluid with bottom-heavy distribution of

nanoparticles. This implies that thermal conductivity of this kind of nanofluids is higher than that of ordinary fluids.

The dispersion relation (37) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters to depict the stability characteristics.
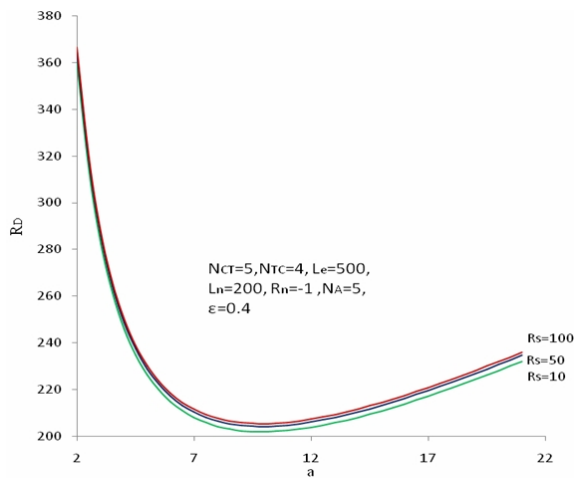

Figure 2. The variations of thermal Rayleigh-Darcy number R_D with the wave number a for different values of the solutal Rayleigh numbers $Rs = 10$, $Rs = 50$ and $Rs = 100$.

The variations of thermal Rayleigh- Darcy number R_D with the wave number a for three different values of the solutal Rayleigh number, namely, $Rs = 10, 50$ and 100 is plotted in Fig. 2 and it is observed that the thermal Rayleigh- Darcy number increases with the increase in solutal Rayleigh number so the solutal Rayleigh number stabilizes the system. In Fig. 3, the variations of thermal Rayleigh- Darcy number R_D with the wave number a for three different values of the thermo-nanofluid Lewis number, namely, $Ln = 500, 1000$ and 1500 which shows that thermal Rayleigh-Darcy number increases with the increase in thermo-nanofluid Lewis number. Thus thermo-nanofluid Lewis number has stabilizing effect on the system.
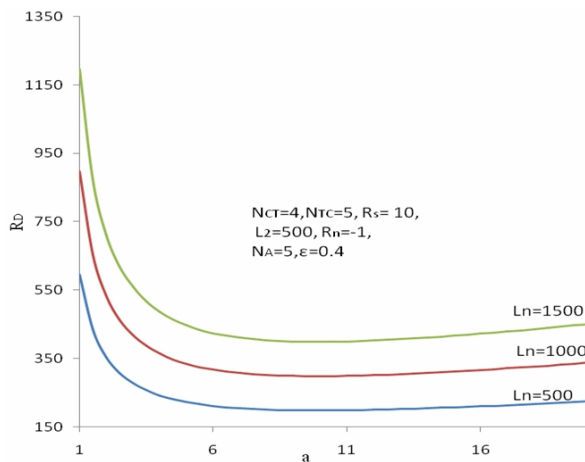

Figure 3. The variations of thermal Rayleigh- Darcy number R_D with the wave number a for different values of the thermo-nanofluid Lewis number $Ln = 500$, $Ln = 1000$ and $Ln = 1500$.

The variations of thermal Rayleigh- Darcy R_D with the wave number a for three different values of the

thermosolutal Lewis number, namely, $Le= 500, 1000$ and 1500 is plotted in Fig. 4 and it is found that thermal Rayleigh- Darcy number decreases with the increase in thermosolutal Lewis number so the thermosolutal Lewis number has destabilizing effect on the system.
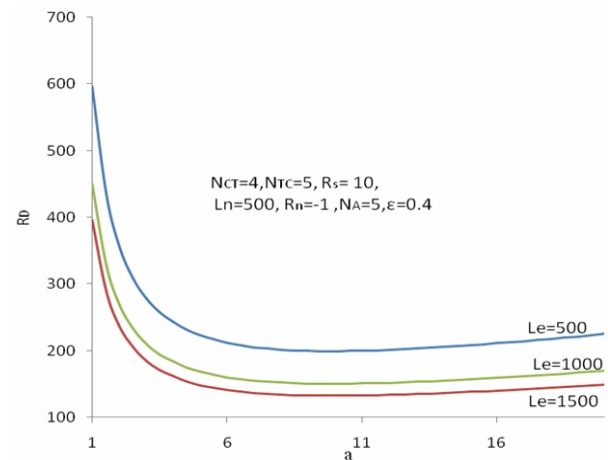

Figure 4. The variations of thermal Rayleigh- Darcy number R_D with the wave number a for different values of the thermosolutal Lewis number $Le = 500$, $Le = 1000$ and $Le = 1500$.

In Fig. 5, the variations of thermal Rayleigh- Darcy number R_D with the wave number a for three different values of the Soret parameter, namely $N_{TC} = 5, 10, 15$ which shows that thermal Darcy-Rayleigh number increases with the increase in Soret parameter. Thus Soret parameter has stabilizing effect on the system. The variations of thermal Rayleigh- Darcy number R_D with the wave number a for three different values of Dufour parameter, namely $N_{TC} = 5, 10$ and 15 is plotted in Fig. 6 and it is found that thermal Darcy-Rayleigh number increases with the increase in Dufour parameter, so the Dufour parameter has stabilizing effect on the onset of stationary convection in a layer of Rivlin-Ericksen elastico-viscous Nanofluid saturating a porous medium. The system becomes more stable when the values of Soret and Dufour parameters are equal. The results obtained in figures 2 to 6 are in good agreement with the result obtained by Chand and Rana [14,15], kuznetsov and Nield [13] and Sheu [24].
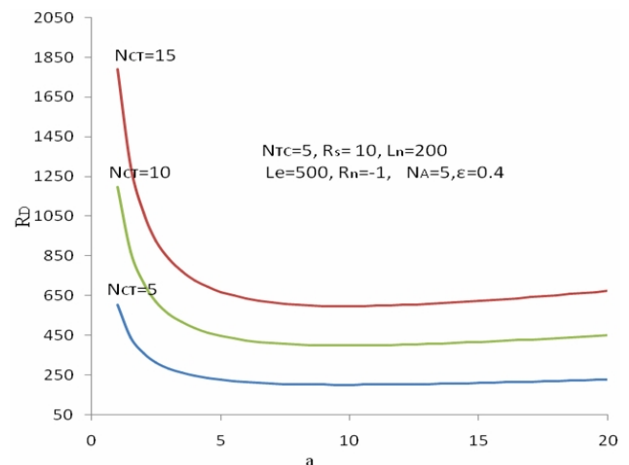

Figure 5. The variations of thermal Rayleigh- Darcy number R_D with the wave number a for different values of the Soret parameter $N_{TC} = 5$, $N_{TC} = 10$, $N_{TC} = 15$.
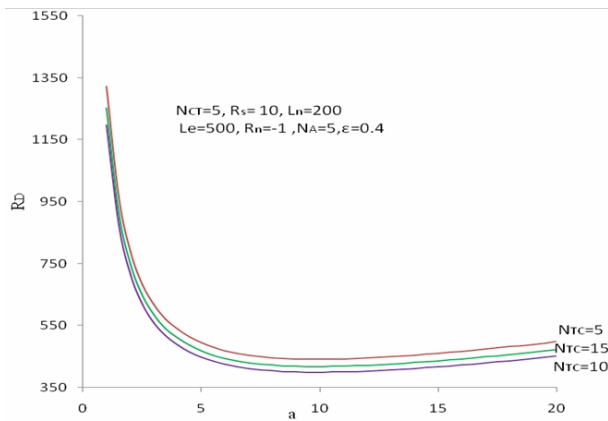
Figure 6. The variations of thermal Rayleigh- Darcy number R_D with the wave number a for different values of the Dufour parameter N_{TC} = 5, N_{TC} = 10, N_{TC} = 15

7. CONCLUSION

Thermosolutal instability on the onset of stationary convection in a layer of Walters' (model B') elastico-viscous Nanofluid in a porous medium is investigated by using a linear stability analysis. The main conclusions are:

- For the case of stationary convection, the Walters' (model B') nanofluid behaves like an ordinary Newtonian nanofluid.
- Kinematic viscoelasticity has no effect on the onset of stationary convection.
- The solutal Rayleigh Number, thermo-nanofluid Lewis number, Soret parameter and Dufour parameter have stabilizing effects on the stationary convection as shown in figures 2, 3, 5 and 6, respectively.
- The thermo-solutal Lewis number has destabilizing effect on the stationary convection of the system as shown in figure 4.

REFERENCES

[1] Chandrasekhar, S.: *Hydrodynamic and Hydromagnetic Stability*, Dover Publication, New York, (1961).

[2] Bhatia, P.K. and Steiner, J.M.: Thermal instability of visco-elastic fluid in hydromagnetics, Journal of Mathematical Analysis and Applications., Vol. 41, , pp. 271-282, 1973.

[3] Veronis G.: On finite amplitude instability in thermohaline convection, Journal of Marine Research, Vol.23, pp. 1-17, 1967.

[4] Brakke M.K.: Zone electrophoresis of dyes, proteins and viruses in density gradient columns of sucrose solutions, Archives of Biochemistry and Biophysics, Vol. 55, pp. 175-190, 1955.

[5] Choi, S.: Enhancing thermal conductivity of fluids with nanoparticles, *In*: *D.A. Siginer and H.P. Wang (Eds), Developments and Applications of Non-Newtonian Flows*, ASME FED, 231/MD, 66, pp. 99-105, 1995.

[6] Buongiorno J.: Convective transport in nanofluids, ASME Journal of Heat Transfer, Vol.128, pp. 240-250, 2006.

[7] Tzou, D.Y.: Thermal instability of nanofluids in natural convection, International Journal of Heat and Mass Transfer, Vol. 51, pp. 2967-2979, 2008.

[8] Alloui, Z., Vasseur P. and Reggio, M.: Natural convection of nanofluids in a shallow cavity heated from below, The International Journal of Thermal Sciences, Vol. 50, pp. 385-393, 2011.

[9] Kuznetsov, A.V. and Nield, D.A.: Thermal instability in a porous medium layer saturated by a nanofluid: Brinkman Model, Transport in Porous Media, Vol. 81, pp. 409-422, 2010.

[10] Kuznetsov, A.V. and Nield, D.A.: The onset of double-diffusive nanofluid convection in a layer of saturated porous medium, Transport in Porous Media, Vol. 85, pp. 941-951, 2010.

[11] Kuznetsov, A.V. and Nield, D.A.: Thermal instability in a porous medium layer saturated by a nanofluid, International Journal of Heat and Mass Transfer, Vol. 52, pp. 5796–5801, 2009.

[12] Kuznetsov, A.V. and Nield, D.A.: The onset of convection in a layer of cellular porous material: effect of temperature-dependent conductivity arising from radiative transfer, Journal of Heat Transfer, Vol. 132, 074503, 2010.

[13] Nield, D.A. and Kuznetsov, A.V.: The onset of double-diffusive convection in a nanofluid layer, International Journal of Heat and Fluid Flow, Vol. 32, pp. 771-776, 2011.

[14] Chand R. and Rana G.C.: On the onset of thermal convection in rotating nanofluid layer saturating a Darcy-Brinkman porous medium, International Journal of Heat and Mass Transfer, Vol. 55, pp. 5417–5424, 2012.

[15] Chand, R. and Rana G.C.: Oscillating convection of nanofluid in porous medium, Transport in Porous Media., Vol. 95, pp. 269-284, 2012.

[16] Tom, B.A. and Strawbridge, D.J.: Elastic and viscous properties of dilute solutions of polymethyl methacrylate in organic liquids, Transactions of the Faraday Society., Vol. 49, pp. 1225-1232, 1953.

[17] Oldroyd, J.G.: Non-Newtonian effects in steady motion of some idealized elastico-viscous liquids, Proceedings of the Royal Society (London), A245, pp. 278-286, 1958.

[18] Walters, K.: The solution of flow problems in the case of materials with memory, J. Mecanique, 2, pp. 469-478, 1962.

[19] Gupta, U. and Aggarwal, P.: Thermal instability of compressible Walters' (model B') fluid in the

presence of hall currents and suspended particles, Thermal Science, Vol. 15, pp. 487-500, 2011.

[20] Rana, G.C and Sharma, V.: Hydromagnetic thermosolutal instability of Walters' (model B') rotating fluid permeated with suspended particles in porous medium, The International Journal of Multiphysics, Vol. ,5, pp. 325-338, 2011.

[21] Rana G.C., Chand, R. and Kumar, S.: Thermosolutal convection in compressible Walters' (model B') fluid permeated with suspended particles in a Brinkman porous medium, Journal of Computational Multiphase Flows, Vol. 4, pp. 211-224, 2012.

[22] Rana, G. C.: Thermosolutal convection in Walters' (Model B') elastico-viscous rotating fluid permeated with suspended particles and variable gravity field in porous medium in hydromagnetics, Journal of Applied Fluid Mechanics, Vol.6 , pp. 87-94, 2013.

[23] Shivakumara, I.S., Lee J., Malashetty M.S. and Sureshkumara, S.: Effect of thermal modulation on the onset of thermal convection in Walters' B viscoelastic fluid in a porous medium, Transport in Porous Media, Vol 87., pp. 291-307, 2011.

[24] Sheu, L.J.: Thermal instability in a porous medium layer saturated with a viscoelastic nanofluid, Transport in Porous Media., Vol. 88 , pp. 461-477, 2011.

О ПОЧЕТКУ ТЕРМО-РАСТВОРЉИВЕ НЕСТАБИЛНОСТИ У СЛОЈУ ЕЛАСТИЧНО-ВИСКОЗНОГ НАНОФЛУИДА У ПОРОЗНОМ МЕДИЈУМУ

G.C.Rana, R.C.Thakur, S.K.Kango

У раду се разматра проблем термо-растворљиве нестабилности у хоризонталном слоју нанофлуида у порозном медијуму. Валтерсов (модел Б') модел флуида се користи за описивање реолошког понашања нанофлуида а Дарсијев модел за описивање порозног медијума. Релација дисперзије којом се објашњавају утицаји различитих параметара изведена је из анализе линеарне стабилности на основу методе анализе нормалних стања. Почетни ктитеријум за стационарно струјање је изведен аналитички и утврђено је да се Валтерсов (модел Б') еластично-растворљиви нанофлуид понаша као обичан њутоновски нанофлуид. У раду се истражује како на стабилност стационарног струјања утичу Рејлијев коефицијент растворљивости, Луисов коефицијент термо-нанофлуида, Луисов коефицијент термо-растворљивости и Сореов и Дифоров параметар

NOMENCLATURE

a	Wave number
c	Specific heat
d	Thickness of the horizontal layer
D_B	Diffusion coefficient (m^2/s)
D_T	Thermophoretic diffusion coefficient
F	Kinematic visco-elasticity parameter
g	Acceleration due to gravity (m/s^2)
$\mathbf{g}$	Gravitational acceleration vector
k	Thermal conductivity (w/ mK)
Le	Thermosolutal Lewis number
Ln	Thermo-nanofluid Lewis number
N_A	Modified diffusivity ratio
N_B	Modified particle-density ratio
N_{CT}	Soret parameter
N_{TC}	Dufour parameter
p	pressure (Pa)
Pr	Prandtl number
$\mathbf{q}$	Darcy velocity vector (m/s)
R_D	Thermal Darcy-Rayleigh number
$(R_D)_C$	Critical Thermal Darcy-Rayleigh number
Rm	Basic-density Rayleigh number
Rn	Concentration Rayleigh number
Rs	Solutal Rayleigh number
t	time (s)
T	temperature (K)
(u,v,w)	Darcy velocity components
(x,y,z)	space co-ordinates (m)

Greek symbols

α_T	Solute volumetric coefficient
α_C	Thermal volumetric coefficient (1/K)
φ	Nanoparticles volume fraction
κ_m	Effective thermal diffusivity of porous medium (m/s^2)
μ	Viscosity of the fluid(Ns/m^2)
μ'	Viscoelasticity
v	Kinematic viscosity
v'	Kinematic viscoelasticity
ρ	Density of fluid (kg/m^3)
ρ_p	Nanoparticle mass density (kg/m^3)
ω	Growth rate of disturbances

Superscripts

'	Non-dimensional variables
f	''

Subscripts

p	Particle
f	Fluid
b	Basic state
0	Lower boundary
1	Upper boundary