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Vibration Analysis of Cantilever Beam for Damage Detection

Mechanical structure during their functional operations may be vulnerable to damages and therefore cannot to be guaranteed definite fault free operational mode and successful exploitation. In this paper, vibration analysis and frequency response analysis of cantilever aluminium beam with bonded piezoelectric transducer are presented by using finite element method in finite element analysis software ANSYS. Cantilever beam vibration response are analysed and numerical results of undamaged beam model are compared to different scenarios of damage presence in structure, by location and depth of single transversal crack. Technique is based on the idea, if a crack appears in mechanical structure, this can be recognized as changes in the physical properties, which leads to cause changes in the modal properties of the structure.

Keywords: *damage detection, vibration analysis, frequency response.*

1. INTRODUCTION

Damage detection techniques in mechanical structures and their application are becoming more important in recent years in almost all industries of mechanical, aerospace and civil engineering fields. Mechanical systems with ability for detection of interpret adverse changes in a structure can improve their future reliability and reduce life-cycle costs. The main objective for structural health monitoring is the detection and characterization of damages that may affect the integrity and the functional operability of the mechanical structure.

Conventional inspection techniques and methods can be expensive and time consuming. These issues can be considerable overcome by development and implementation of methods and techniques based on equipment that can effectively detect the existence of damage and can provide information regarding the location and the severity of damage in the structure. Therefore piezoelectric transducers, as both sensors and actuators, are commonly used for damage detection in systems for structural health monitoring [1]. These devices have capability for utilization of the converse piezoelectric effect to actuate the structure in addition to the direct effect to sense structural deformation. Piezoelectric transducers are small, lightweight, reasonably priced devices and can be produced in different geometric forms. Piezoelectric sensors and actuator can be bonded onto the surface or can be embedded in to the structures, hence they have great potential to improve significantly monitoring and damage detection by nondestructive evaluation.

Many methods have been developed to detect and locate the crack by measuring the change in the natural frequencies of the structure, due to modal frequencies are properties of the whole structure and decreases as a result of crack. Comprehensive survey for detection, location, and characterization of structural damage via techniques that examine changes in measured structural vibration response, are presented by S. W. Doebling et al. [2]. The survey categorizes the methods according to required measured data and analysis technique, changes in modal frequencies, mode shapes, and changes in measured flexibility coefficients. Methods that use property (stiffness, mass, damping) matrix updating, detection of nonlinear response, and damage detection via neural networks are also presented. The types of structures include in this survey are: beams, trusses, plates, shells, bridges, offshore platforms, other large civil structures, aerospace structures, and composite structures.

Structural damage, such as cracks, usually cause a local reduction in stiffness and visual inspection may not locate the damage. J. Penny et al. [3] used damage location techniques rely on the measurement of small changes in natural frequencies and upon adequate theoretical prediction of these frequency changes. Determination of the natural frequencies at higher modes is often difficult and by no means clear in advance which method should be used to predict the theoretical frequency changes, any successful method based on these quantities must be fairly robust to observational errors and model inadequacies.

The quality of the predictions from any method of damage location is critically dependent on the accuracy of the damage model. A. S. Bouboulas et al. [4] used finite element procedures to approach the vibrations analysis of a cantilever beam subjected to an impulse loading with a breathing crack on, and breathing is treated as a full frictional contact problem between the crack surfaces. The solutions they obtained using incremental iterative procedures, and by quasi-static and non-linear dynamic analyses are aiming to predict of vibration characteristics of cracked cantilever beam.

D. M. Reddy and S. Swarnamani [5], show the effectiveness of using frequency response function (FRF) curvature energy damage index and establish its

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capability to detect and localize damage. By testing the frequency intervals, they presented that the FRF curvature energy damage index method defined in the range of frequencies include the eigen frequencies. The damage index is found to be a function of the frequency bandwidth and variation of FRF curvature energy damage index versus frequency range (band width) provide further information in choice of optimum frequency range response analysis.

In this paper, natural frequencies have been calculated using finite element method in commercially available software package ANSYS. Modal analysis and frequency response analysis in different cases of damage presence on cantilever aluminium beam with bonded piezoelectric transducer, are presented.

2. ANALYSIS OF DYNAMIC CHARACTERISTICS FOR DAMAGE DETECTION

Changes in vibration characteristics of the mechanical structure is the basis of many methods and techniques which are developed and implemented for monitoring systems, intended for effectively, quickly and economically damage detection. Model properties of the structure as modal frequencies, mode shapes and modal damping can be determine as function of physical properties (material, geometry characteristics) [6]. In addition, if damage appears in mechanical structure, this can be recognized as changes in the physical properties, which leads to cause changes in the modal properties of the structure. For example, reduction in stiffness induced by crack or disconnection will cause evident changes in these modal properties [7]. In case where this relation can be model by linear equation, effects of faults on a structure can be classified as linear otherwise as non-linear. For example, opening and closing on crack situated on a beam and depending on the manifestation and position of the applied force, bending moment will induce non-linearity case of damage detection [8]. This paper will not cover methods and techniques in non-linear conditions, but will refer to damage detection due to open transversal crack by changes of natural frequencies and frequency response.

Change in natural frequencies, as approach for damage detection is a classic method, because of easy determinations, high accuracy and sensibility to all kind of damages in structures [9]. When a fault happens in a mechanical structure, natural frequencies of the system are consequently decreasing because of stiffness reduction. A quite huge number of researchers used this method via practicing classical vibrational measurement techniques through experimental methods to determinate the resonant frequencies.

2.1 Forced Vibration

Dynamic behaviour of linear system is described by the following differential equation:

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{f(t)\} \quad (1)$$

where, [M] is the mass matrix, [C] the damping matrix and [K] the stiffness matrix, describing the spatial properties of the system. A vector of displacement is ***x***, a vector of applied forces is ***f***, and the over dots represent differentiation with respect to time *t*. In case of harmonic excitation, the vector of applied forces can be defined as:

$$f(t)=Fe^{i\omega t} \quad (2)$$

following this, the vector of displacement response can be assumed as:

$$x(t)=Xe^{i\omega t} \quad (3)$$

From here, by taking the Fourier transform, the differential equation of motion (1) can be expressed as follows, respectively the system has the following frequency domain representation:

$$[-\omega^2 M+i\omega C+K]\{X(\omega)\}=\{F(\omega)\} \quad (4)$$

where, $X(\omega)$ and $F(\omega)$ are, respectively the Fourier transform of the displacement $x(t)$ and applied force $f(t)$. As a consequence of this and by following equation:

$$[H(\omega)]=[-\omega^2 M+i\omega C+K]^{-1} \quad (5)$$

can be obtain the relation between the response - the output $X(\omega)$ and excitation - the input $F(\omega)$ for each frequency of the system ω:

$$\{X(\omega)\}=[H(\omega)]\{F(\omega)\} \quad (6)$$

$H(\omega)$ is frequency response function and is related to mass, damping and stiffness as dynamics characteristics [10, 11, 12]. Any changes to these spatial properties of the system leads to changes of the frequency response function and creates conditions for appropriate modification of the system as a reflection of the defined changes. The presence and damage rate can be verify by obtaining this function based on the dynamic behaviour of mechanical systems before and after crack occurrence. For each size and location of the crack can be defined frequency response function for further analysis and determination of system integrity.

3. MODEL SET-UP AND PROCEDURE

3.1 Model description

Cantilever beam model was created in software for finite element analysis ANSYS 14.5. The beam model is based from laboratory set-up experiment for cantilever aluminium beam with following dimensional properties: thickness b=0.002 m, height h_b= 0.035 m, length from fixed end l_b = 0.88 m, and material properties: Young's modulus E=69x10^9 N/m^2, Density ρ =2700 kg/m^3, Poisson ratio μ=0.35.

Model of damage beam is created and damage is presented by single transversal crack, and it's assumed always to be open during dynamic analysis. To find out how the crack affects the dynamic behaviour of the beam, different crack scenarios are obtained by two crack parameters, different depth h_c and at different locations l_c (different distance measured from the fixed end), shown in Fig. 1.

The following cases of beam model are considered:

- Cantilever beam with a crack at location l_{c1}=0.079m, and three different depth case: h_{c1}=0.005m, h_{c2}=0.01m, h_{c3}=0.015m, separately.
- Cantilever beam with a crack at location l_{c2}=0.52m, and three different depth case: h_{c1}=0.005m, h_{c2}=0.01m, h_{c3}=0.015m, separately.
- Cantilever beam with a crack at location l_{c3}=0.86m, and three different depth case: h_{c1}=0.005m, h_{c2}=0.01m, h_{c3}=0.015m, separately.

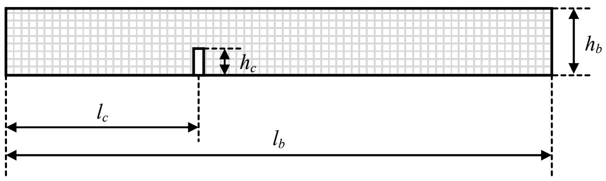

Figure 1. Beam model with single transversal crack

3.2 Simulation procedure

Measured natural frequencies are used for detection process only and through further analysis the same natural frequencies can be used for identification of crack location and size. Determination of the natural frequencies at higher modes are often difficult and only the first three natural frequencies were obtained by simulation of the uncrack and for all cases of crack cantilever beam models. In each case of examples for created cantilever beam model, only one crack exist at only one location. The crack depth varied from 5 *mm* to 15 *mm* at each crack position l_{c1}=0.079 *m*, l_{c2}=0.52 *m*, l_{c3}=0.86 *m*.

The crack geometrical parameters are specified by using the following dimensionless crack parameters as: $l' = l_c / l_b$ non dimensional crack position, $h' = h_c / h_b$ non-dimensional crack depth. The non-dimensional natural frequency for the n^{th} mode is introduced and it is defined as a frequency ratio $\omega' = \omega_c / \omega$, where ω_c is natural frequency of the cracked beam model and ω natural frequency of the beam model without crack.

4. NUMERICAL RESULTS AND DISCUSSIONS

4.1 Changes in Natural Frequencies

The variation of the frequency ratio as a function of the crack depth and crack location for cantilever beam models are shown in Tables 1 to 3. The plots of the variation of the first three natural frequency ratio, as a function of crack depths for different crack positions are show in Figure 2 to 4.

Table 1. First natural frequency ratio ω_1' as a function of the crack depth h' and crack location l'

	l_1'	l_2'	l_3'
h_1'	0.99676	0.999703	1.000559
h_2'	0.989696	0.999189	1.001561
h_3'	0.972275	0.997895	1.002189

Table 2. Second natural frequency ratio ω_2' as a function of the crack depth h' and crack location l'

	l_1'	l_2'	l_3'
h_1'	0.998457	0.998127	1.000506
h_2'	0.995293	0.993018	1.001363
h_3'	0.987748	0.982767	1.001886

Table 3. Third natural frequency ratio ω_3' as a function of the crack depth h' and crack location l'

	l_1'	l_2'	l_3'
h_1'	0.999391	0.999006	1.000443
h_2'	0.998374	0.99631	1.001177
h_3'	0.995829	0.991082	1.001604

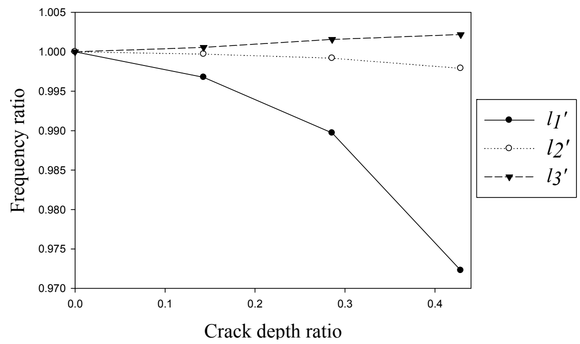

Figure 2. First natural frequency ratio ω_1' in terms of crack depth ratio for different crack positions

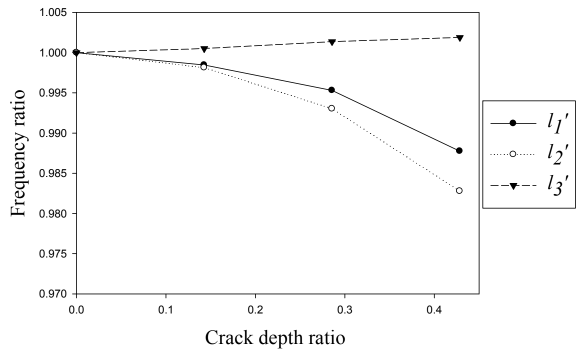

Figure 3. Second natural frequency ratio ω_2' in terms of crack depth ratio for different crack positions

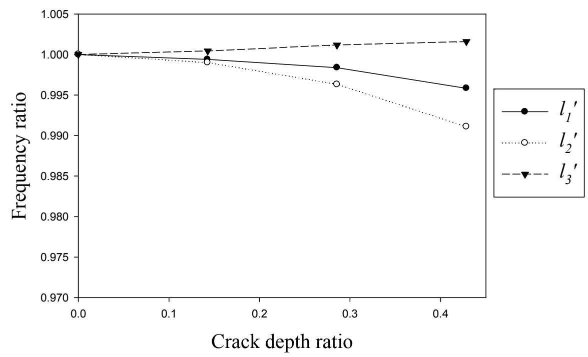

Figure 4. Third natural frequency ratio ω_3' in terms of crack depth ratio for different crack positions

The variation of curves for the natural frequency ratio ω' in terms of crack depth ratio for different crack positions, can be seen as an indication for possible cracked beam, because numerical results shows that there is an noticeable variation between natural frequency of cracked and uncracked cantilever beam model. Crack have largest effect at the fixed end of the beam model for variation of the first natural frequencies, but variation of second natural frequencies is more effected by cracked depth ratio at location l_2' near the midpoint of the beam. The variation of the third natural frequency is less effected for a crack location and cracked depth ratio compared with variation of the first two natural frequencies, but cracked depth ratio at location l_2' contributes to greater variation in terms of other two positions. Numerical results shows that cracked depth ratio located near the free end of the cantilever beam model is with almost negligible effect on the frequencies changes, although changes in frequencies is not only a function of crack depth, and crack location, but also of the mode number [13]. Prior to further assessment of crack size, from the above observations it could be specified that knowing the crack position could lead to estimation of possible crack extension if uses one mode of vibration [14].

4.2 Harmonic analysis of cantilever beam model

Shifts in natural frequencies, as a global properties of the beam, can be obtained from harmonic analysis and by observation of frequencies response function (FRF) for different cases of cracked beam model. Figure 5 and 6, shows the FRFs for second mode of vibration for different crack depth h_{c1}, h_{c2} and h_{c3} at different location l_{c2} and l_{c3}, respectively. FRF amplitude for uncracked beam model in comparison with cracked beam model has noticeable reduction for crack position and l_{c2} which is near the midpoint of the beam. For location l_{c3} near the free end, FRF amplitude has slightly reduction form uncracked beam model (Fig. 6).

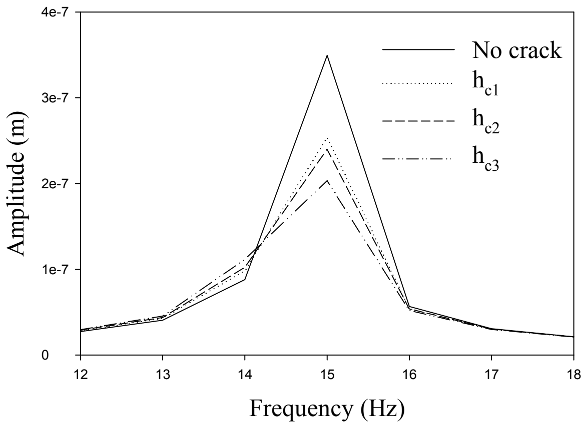

Figure 5. FRF for crack position l_{c2} for different crack depth at second natural frequency

Figure 7 and 8, shows the FRFs for third mode of vibration for different crack depth h_{c1}, h_{c2} and h_{c3} at different location l_{c2} and l_{c3}, respectively. Figure 7, shows natural frequencies shifts and FRF amplitude reduction for crack position at l_{c2} in comparison with uncracked beam model. At location l_{c3} near the free end, there is no difference in natural frequencies shifts and FRF amplitude reduction in consideration of different crack depth h_{c1}, h_{c2} and h_{c3} (Fig. 8). Despite this, there is a changes by frequency and amplitude on FRF curve in terms of cracked in comparison with uncracked beam model.

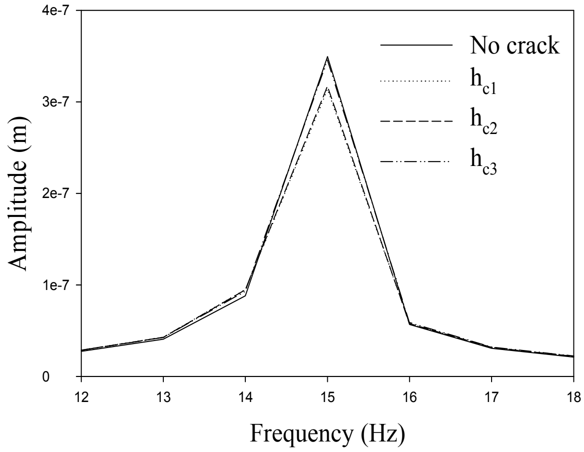

Figure 6. FRF for crack position l_{c3} for different crack depth at second natural frequency

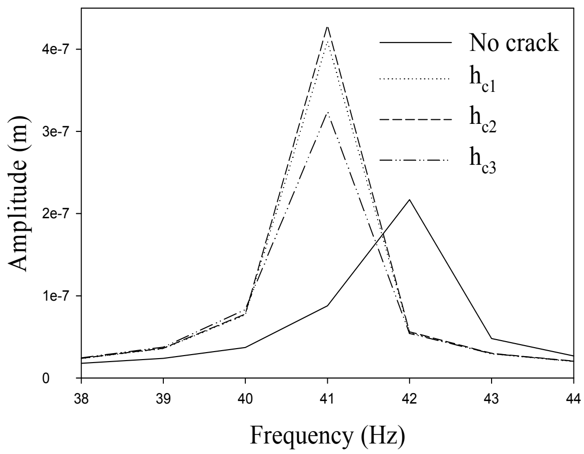

Figure 7. FRF for crack position l_{c2} for different crack depth at third natural frequency

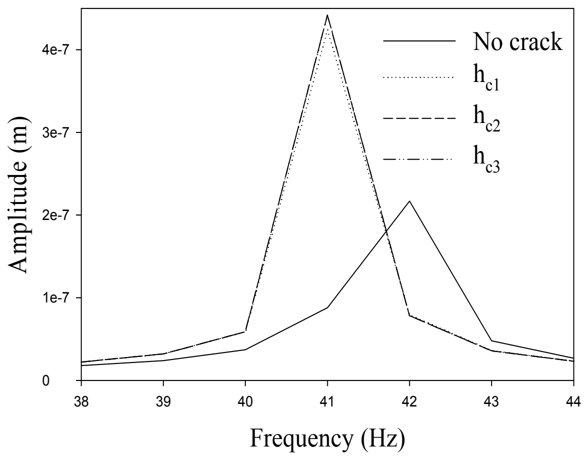

Figure 8. FRF for crack position l_{c3} for different crack depth at third natural frequency

5. CONCLUSION

Vibration analysis of cantilever beam model have been presented in this paper for damage detection. Damage was introduced as single transversal crack at the surface, and it's assumed always to be open during analysis. Effect of different crack depth at different crack position on beam model, changes in first three natural frequency and FRF amplitude are also presented. Vibration behaviour of the beam simulated in FEA software ANSYS and obtained results, shows that cantilever beam model is sensitive to the crack location, crack depth and vibration modes. Numerical results shows the highest variation of natural frequencies occurs for the first mode of beam vibration at crack position near the fixed end, and FRF by changes in amplitude and frequency shifts is most affected when crack is located near the midpoint of the beam where severity of the crack depth is perceptible.

For future robust experimental tests under right measurement conditions, this technique can be applicable wherever similar beams are tested and responses measured. This is due to the fact that the measured parameters of frequencies are unique values, which will remain the same within a tolerance level [14], and if a crack appears in mechanical structure, this can be recognized as changes in the physical properties which leads to cause changes in modal properties of the structure.

Presents of damage in structure may cause serious failure of the mechanical systems, therefore crack must be detected in the early state by uses transducers that are permanently bonded or embedded in the structure. In this paper, analysed cantilever beam model in FEA software has bonded piezoelectric transducers near the fixed end. This patches can perform as sensor for model analysis or by harmonic excitation as actuator for response analysis, and through appropriate integration in mechanical systems can improve damage detection by non-destructive evaluation.

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АНАЛИЗА ВИБРАЦИЈА НА КОНЗОЛЕ ЗА ДЕТЕКЦИЈУ ОШТЕЋЕЊА

Марјан Ђидров, Виктор Гаврилоски, Јована Јованова

Механичка структура може бити склона оштећењима у току рада и стога се не може гарантовати активност без грешака у радном режиму и успешна експлоатација. У овом раду представљена је анализа вибрација и фреквентни одзив конзоле од алуминијума са пиезоелектричним трансдуктором. Ова анализа урађена је методом коначних елемената у програмском пакету ANSYS.

Одзив конзоле је анализиран и резултати неоштећеног модела су упоређивани са резултатима модела код којих постоји оштећење у виду попречне пукотине различите дубине у структури на различитим локацијама. Техника је базирана на томе да ако се пукотина појави у механичкој структури, то може да се посматра као промена физичке карактеристике која узрокује промене у квалитету структуре.