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Effect of Variable Gravity on Thermal Instability of Rotating Nanofluid in Porous Medium

Effect of variable gravity on the onset of thermal convection in a horizontal layer of rotating nanofluid in a porous medium is investigated. For the porous medium Darcy model is used. A linear stability analysis based upon normal mode technique is used to find solution of the fluid layer confined between two free-free boundaries. Rayleigh numbers on the onset of stationary convection and oscillatory convection have been derived by using Galerkin method. Graphs have been plotted to study the effect of Taylor number (rotation), Lewis number and variable gravity parameter on the stationary convection.

Keywords: *Nanofluid, Variable Gravity, Rayleigh-Darcy Number, Galerkin method, Rotation, Porous medium.*

1. INTRODUCTION

Recently a new class of fluid known as nanofluid has been successfully applied in heat transfer devices. Nanofluid is suspensions of nanoparticles (e.g. carbon, metals and metal oxides) of nano size in a carrier fluid (e.g. water, ethylene glycol and lubricants), which was first coined by Choi [1]. Owing to their enhanced properties as thermal transfer fluid for instance, nanofluid can be used in a plethora of engineering applications ranging from use in the automotive industry to the medical arena to use in power plant cooling systems as well as computers. Tzeng et al. [2], Kim et al. [3], Routbort et al. [4], Donzelli et al. [5] reported the various applications of nanofluid.

Theoretical and experimental results on the stability of cellular convection of a Newtonian fluid layer in nonporous medium, in the presence of rotation and magnetic field, have been given by Chandrasekhar [6]. Thermal instability in a porous medium is a phenomenon related to various fields. It has many applications in geophysics, food processing, oil reservoir modeling, building of thermal insulations and nuclear reactors. Many researchers have investigated thermal instability problems by taking different types of fluids. Lapwood [7] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [8]. A good account of convection problems in a porous medium is given by Vafai and Hadim [9], Ingham and Pop [10] and Nield and Bejan [11].

Bénard convection (onset of convection in a horizontal layer of nanofluid uniformly heated from below) of nanofluids based upon the Buongiorno's model has attracted great interest. Buongiorno [12] was the first researcher who dealt with convective transport in nanofluids. He noted that the nanoparticles absolute velocity can be viewed as the sum of the base fluid velocity and a relative (slip) velocity. Vadasz [13] studied heat conduction in nanofluid suspensions whereas Alloui et al. [14] studied the natural convection of nanofluids in a shallow cavity heated from below. The onset of convection in a horizontal layer uniformly heated from below for a horizontal layer of nanofluid was studied by Tzou [15, 16]. The thermal instability of nanofluid on the basis of the transport equations of Buongiorno have been studied by Nield and Kuznetsov [17-19], Kuznetsov and Nield [20-22], Chand and Rana [23-25], Chand et al. [26], Chand [27] and Rana et al. [28-29]. Rotation also play important role in the thermal instability of fluid layer and has applications in rotating machineries such as nuclear reactors, petroleum industry biomechanics etc. Chand [30], Yadav et al. [31], Chand and Rana [32], studied the effect of rotation in a horizontal layer of nanofluid in porous medium and observed that rotation play important role in the stability of fluid layer.

The idealization of uniform gravity assumed in theoretical investigations, although valid for laboratory purposes can scarcely be justified for large-scale convection phenomena occurring in atmosphere, the ocean or mantle of the Earth. It then becomes imperative to consider gravity as variable quantity varying with distance from surface or reference point. Pradhan et al. [33] studied the thermal instability of a fluid layer in a variable gravitational field while Alex et al. [34] studied the variable gravity effects on the thermal instability in a porous medium with internal heat source and inclined temperature gradient. Strughan [35] discuss the various type of variable gravity parameter on the stability convection and recently Chand [36-37] studied the variable gravity effects on the thermal instability in fluid layer. Effect of variable gravity in layer of nanofluid in a porous medium is studied by Chand et al. [38] and found that gravity parameter play significant role on the stability of fluid.

Owing to the various applications variable gravity and rotation in the stability convection an attempt have made to study the effects of variable gravity and

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rotation in a horizontal layer of nanofluid in a porous medium heated from below.

2. MATHEMATICAL FORMULATIONS OF THE PROBLEM

Consider an infinite horizontal layer of nanofluid of thickness 'd' bounded by plane z = 0 and z = d, heated from below in a porous medium of medium permeability k_1 and porosity ε. Fluid layer is rotating uniform about z-axis with angular velocity $\Omega(0, 0, \Omega)$ and is acted upon by gravity force **g**(0, 0,-g) and it is assumed that gravity force vector is varies linearly with z i.e. $\mathbf{g} = (1+\delta\, h(z))g$, where $\delta h(z)$ is the variable gravity parameter. The temperature T and volumetric fraction φ of nano particles are taken to be T_0 and φ_0 at z = 0 and T_1 and φ_1 at z = d $(T_0 > T_1)$. The reference temperature is taken to be T_1. Thermophysical properties of the nanofluid are constant for the analytical formulation but these properties are not constant and strongly depend upon volume fraction of the nanoparticles. Nanoparticles are being suspended in the nanofluid using either surfactant or surface charge technology, preventing the agglomeration and deposition of these on the porous matrix. For simplicity Darcy's law is assumed to be hold and Oberbeck-Boussinesq approximation is employed. Thus relevant governing equations for the study of rotating nanofluid in porous medium (Chandrasekhar [6], Kuznetsov and Nield [17], Chand and Rana [32]) are

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$0 = \nabla p + \left(\varphi\rho_p + (1-\varphi)\{\rho_0(1-\alpha(T-T_0))\}\right)\mathbf{g} - \frac{\mu}{k_1} + \frac{2\rho_0}{\varepsilon}(\mathbf{q}\times\mathbf{\Omega}), \tag{4}$$

$$\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}\cdot\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T, \tag{3}$$

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f\mathbf{q}\cdot\nabla T = k_m\nabla^2 T + \varepsilon(\rho c)_p\left(D_B\nabla\varphi\cdot\nabla T + \frac{D_T}{T_1}\nabla T\cdot\nabla T\right), \tag{4}$$

where **q** is the velocity of fluid, p is the pressure, ρ_0 is the density of nanofluid at z = 0, ρ_p is the density of nanoparticles, φ is the volume fraction of the nanoparticles, T is the temperature, T_1 is the reference temperature, α is coefficient of the thermal expansion, **g** is acceleration due to gravity, k_1 is medium permeability of fluid, ε is the porosity of porous medium, μ is the viscosity, $(\rho c)_m$ is the heat capacity of fluid in porous medium, $(\rho c)_p$ is the heat capacity of nanoparticles, $(\rho c)_f$ is the heat capacity of fluid, k_m is the thermal conductivity of the fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient of the nanoparticles.

We assume the temperature and volumetric fraction of the nanoparticles are constant. Thus boundary conditions (Chandrasekhar [6] and Nield and Kuznetsov [17]) are

$$w = 0, T = T_0, \quad \varphi = \varphi_0 \;\; at \quad z = 0$$
$$w = 0, \;\; T = T_1, \quad \varphi = \varphi_1 \;\; at \quad z = \mathrm{d}. \tag{5}$$

Introduce non-dimensional variables as

$$(x^*, y^*, z^*,) = \left(\frac{x, y, z}{d}\right),\; (u^*, v^*, w^*,) = \left(\frac{u, v, w}{\kappa}\right)d,\; t^* = \frac{t\kappa}{\sigma d^2},$$
$$p^* = \frac{pk_1}{\mu\kappa},\; \varphi^* = \frac{(\varphi-\varphi_0)}{(\varphi_1-\varphi_0)},\; T^* = \frac{(T-T_1)}{(T_0-T_1)},$$

where $\sigma = \frac{(\rho c)_m}{(\rho c)_f}$ and $\kappa = \frac{k_m}{(\rho c)_p}$ is thermal diffusivity of the fluid.

Equations (1) - (4) in non- dimensional form can be written as

$$\nabla\cdot\mathbf{q} = \mathbf{0}, \tag{6}$$

$$0 = -\nabla p - \mathbf{q} - Rm(1+\delta\, h(z))\hat{e}_z + Ra(1+\delta\, h(z))T\hat{e}_z - Rn(1+\delta\, h(z))\varphi\hat{e}_z + \sqrt{Ta}(v\hat{e}_x - u\hat{e}_y), \tag{7}$$

$$\frac{1}{\sigma}\frac{\partial\varphi}{\partial t} + \frac{1}{\varepsilon}\mathbf{q}\cdot\nabla\varphi = \frac{1}{Le}\nabla^2\varphi + \frac{N_A}{Le}\nabla^2 T, \tag{8}$$

$$\frac{\partial T}{\partial t} + \mathbf{q}\cdot\nabla T = \nabla^2 T + \frac{N_B}{Le}\nabla\varphi\cdot\nabla T + \frac{N_A N_B}{Le}\nabla T\cdot\nabla T. \tag{9}$$

[The asterisks have been dropped for simplicity].

Here the non-dimensional parameters are given as

$Le = \frac{\kappa}{D_B}$ is the Lewis number,

$Ra = \frac{\rho_0\alpha g k_1 d(T_0-T_1)}{\mu\kappa}$ is Rayleigh-Darcy number,

$Rm = \frac{(\rho_p\varphi_0 + \rho_0(1-\varphi_0))g k_1 d}{\mu\kappa}$ is density Rayleigh number,

$Rn = \frac{(\rho_p - \rho)\varphi_0 g k_1 d}{\mu\kappa}$ is nanoparticles Rayleigh number,

$N_A = \frac{D_T(T_0-T_1)}{D_B T_1(\varphi_1-\varphi_0)}$ is modified diffusivity ratio,

$N_B = \frac{\varepsilon(\rho c)_p\varphi_0}{(\rho c)_f}$ is modified particle-density increment,

$Ta = \left(\frac{2\Omega d^2}{\varepsilon\nu}\right)^2$ is the Taylor number and

$\hat{\mathbf{e}}_\mathbf{x}, \hat{\mathbf{e}}_\mathbf{y}, \hat{\mathbf{e}}_\mathbf{z}$ are the unit vectors along x- axis, y-axis and z-axis respectively.

The dimensionless boundary conditions are

$$w = 0, \quad T = T_{0,} \quad \varphi = \varphi_0 \quad at \quad z = 0$$

$$w = 0, \quad T = T_{1,} \quad \varphi = \varphi_1 \quad at \quad z = 1. \qquad (10)$$

The basic state was assumed to be quiescent and is given by

$$u = v = w = 0, \quad p = p(z), \quad T = T_b(z) \quad \varphi = \varphi_b(z).$$

The approximate solution is given by

$T_b = 1 - z$ and

$\varphi_b = z.$

These results are obtained by Nield and Kuznetsov [17] and Kuznetsov and Nield [20].

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are written in following forms

$$q\,(u,v,w) = 0 + q'\,(u,v,w), T = T_{b,} + T', \\ \varphi = \varphi_b \quad + \varphi', p = p_b + p', with\, T_b = 1 \text{ - } z, \varphi_b = z. \qquad (11)$$

Using equation (11) in equations (6) – (9) and linearise by neglecting the product of the prime quantities we obtain the following equations

$$\nabla \cdot \mathbf{q} = 0, \qquad (12)$$

$$0 = -\nabla p \text{ - } \mathbf{q} + Ra(1 + \delta\, h(z))T\hat{e}_z - Rn(1 + \delta\, h(z))\varphi\hat{e}_z \\ + \sqrt{Ta}\left(v\hat{e}_x \text{ - } u\hat{e}_y\right), \qquad (13)$$

$$\frac{1}{\sigma}\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon}w = \frac{1}{Le}\nabla^2\varphi + \frac{N_A}{Le}\nabla^2 T, \qquad (14)$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{Le}\left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) - \frac{2N_A N_B}{Le}\frac{\partial T}{\partial z}. \qquad (15)$$

[dashes (') are suppressed for convenience].

Boundary conditions are

$$w = 0, \quad T = T_{0,} \quad \varphi = \varphi_0 \quad at \quad z = 0 \\ w = 0, \quad T = T_{1,} \quad \varphi = \varphi_1 \quad at \quad z = 1. \qquad (16)$$

It will be noted that the parameter Rm is not involved in these and subsequent equations. It is just a measure of the basic static pressure gradient.

The six unknown's u, v, w, p, T and φ can be reduced to three unknown's by operating equation (13) with $\hat{\mathbf{e}}_\mathbf{z}.curl\,curl$, we get

$$\nabla^2 w = Ra(1 + \delta\, h(z))\nabla_H^2 T - Rn(1 + \delta\, h(z))\nabla_H^2 \varphi \\ \text{ - }\sqrt{Ta}\frac{\partial}{\partial z}\xi, \qquad (17)$$

where $\nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplacian operator on the horizontal plane and $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is vorticity.

Eliminating p and introducing vorticity ξ from equation (13), we get

$$\xi = \sqrt{Ta}\frac{\partial w}{\partial z}. \qquad (18)$$

Now eliminating ξ from equations (17) and (18), we have

$$\left(\nabla^2 + Ta\frac{\partial^2}{\partial z^2}\right)w - Ra(1 + \delta\, h(z))\nabla_H^2 T \\ + Rn(1 + \delta\, h(z))\nabla_H^2\varphi = 0. \qquad (19)$$

3. NORMAL MODES AND STABILITY ANALYSIS

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w, T, \varphi] = [W(z), \Theta(z), \Phi(z)]exp\left(ik_x x + ik_y y + nt\right), \quad (20)$$

where k_x and k_y are wave numbers in x and y directions respectively, while n (complex constant) is the growth rate of disturbances.

By using equation (20), equations (14), (15) and (19) become

$$\left(\left(D^2 - a^2\right) + TaD^2\right)W - a^2 Ra(1 + \delta\, h(z))\Theta \\ + a^2 Rn(1 + \delta\, h(z))\Phi = 0, \qquad (21)$$

$$\frac{W}{\varepsilon} \text{ - } \frac{N_A}{Le}\left(D^2 - a^2\right)\Theta - \left(\frac{1}{Le}\left(D^2 - a^2\right) \text{ - } \frac{n}{\sigma}\right)\Phi = 0, \qquad (22)$$

$$W + \left(D^2 - a^2 - n + \frac{N_A}{Le}D - \frac{2N_A N_B}{Le}D\right)\Theta \text{ - } \frac{N_B}{Le}D\Phi = 0. \qquad (23)$$

Where $D^2 = \frac{\partial^2}{\partial z^2}$ and $a = \sqrt{k_x^2 + k_y^2}$ is the dimensionless the resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are written as

$$W = 0, \quad D^2 W = 0, \Theta = 0, \Phi = 0\, at \quad z = 0$$

$$W = 0, \quad D^2 W = 0, \Theta = 0, \Phi = 0\, at \quad z = 1. \qquad (24)$$

We assume the solution to W, Θ and Φ is of the form

$$W = W_0 \sin\pi \; z \;\; \Theta = \Theta_0 \sin\pi\, z, \Phi = \Phi_0 \sin\pi\, z \quad (25)$$

which satisfies boundary conditions (24).

Substituting solution (25) in the equations (21) - (23), we obtain following matrix equation

$$\begin{bmatrix} J+Ta\pi^2 & -a^2Ra(1+\delta h(z)) & a^2Rn(1+\delta h(z)) \\ 1 & -(J+n) & 0 \\ \frac{1}{\varepsilon} & \frac{N_A}{Le}J & \frac{J}{Le}+\frac{n}{\sigma} \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where $J = \pi^2 + a^2$.

For non-trivial solution the determinant of above matrix is zero, thus we obtain the eigen-value equation as follows

$$Ra = \frac{J(J+n)}{a^2(1+\delta h(z))} + \frac{(J+n)Ta\pi^2}{a^2(1+\delta h(z))} - \frac{\frac{N_A}{Le}J + \frac{(J+n)}{\varepsilon}}{\frac{J}{Le}+\frac{n}{\sigma}} Rn. \quad (26)$$

For neutral stability the real part of n is zero. Hence on putting $n = i\omega$, (where ω is real and dimensionless frequency) in equation (26), we get

$$Ra = \frac{J(J+i\omega)}{a^2(1+\delta h(z))} + \frac{(J+i\omega)Ta\pi^2}{a^2(1+\delta h(z))} - \frac{\frac{N_A}{Le}J + \frac{(J+i\omega)}{\varepsilon}}{\frac{J}{Le}+\frac{i\omega}{\sigma}} Rn. \quad (27)$$

4. STATIONARY CONVECTION

When the stability sets as stationary convection, the marginal state will be characterized by $\omega = 0$, then equation (27) gives the stationary Rayleigh number at the margin of stability, in the following form

$$(Ra)_s = \frac{(\pi^2+a^2)^2}{a^2(1+\delta h(z))} + \frac{(\pi^2+a^2)Ta\pi^2}{a^2(1+\delta h(z))} - \left(N_A + \frac{Le}{\varepsilon}\right)Rn. \quad (28)$$

In the absence of rotation ($Ta = 0$) and of constant gravity ($\delta h(z) = 0$), then equation (28) reduces to

$$(Ra)_s = \frac{(\pi^2+a^2)^2}{a^2} - \left(N_A + \frac{Le}{\varepsilon}\right)Rn. \quad (29)$$

It is the good agreement of result derived by Nield and Kuznetsov [17] and Chand and Rana [32].

The minimum of the equation (29) is attained at $a = \pi$, so that the critical Rayleigh number for stationary number is given by

$$(Ra)_c = 4\pi^2 - \left(N_A + \frac{Le}{\varepsilon}\right)Rn. \quad (30)$$

It is the good agreement of result obtained by Nield and Kuznetsov [17], Kuznetsov and Nield [20] for particular case.

In the absence of nanoparticles we obtain the critical Rayleigh-Darcy number given by $(Ra)_c = 4\pi^2$. This is the good agreement of the classical result obtained by Lapwood [7] for regular fluid.

5. OSCILLATORY CONVECTION

Here we considered the possibility of oscillatory convection. For oscillatory convection $\omega \neq 0$. Equating real and imaginary parts of equation (27), we have

$$\frac{a^2RaJ(1+\delta h(z))}{Le} + \left(\frac{N_AJ}{Le} + \frac{J}{\varepsilon}\right)a^2Rn(1+\delta h(z)) = \frac{J^2}{Le}(J+\pi^2Ta) - \frac{\omega^2}{\sigma}(J+\pi^2Ta), \quad (31)$$

and

$$\omega\left(\frac{a^2Ra(1+\delta h(z))}{\sigma} + \frac{a^2Rn(1+\delta h(z))}{\varepsilon} - J(J+\pi^2Ta)\left(\frac{1}{Le}+\frac{1}{\sigma}\right)\right) = 0 \quad (32)$$

Since $\omega \neq 0$, thus from equation (32), we have

$$\frac{Ra}{\sigma} + \frac{Rn}{\varepsilon} - \frac{(\pi^2+a^2)}{a^2}\frac{(\pi^2+a^2+\pi^2Ta)}{(1+\delta h(z))}\left(\frac{1}{Le}+\frac{1}{\sigma}\right) = 0 . \quad (33)$$

The frequency of the oscillatory mode is obtained in the following form

$$\frac{\omega^2 L_e}{a^2\sigma} = \frac{(\pi^2+a^2)^2}{a^2} - \frac{(\pi^2+a^2)(1+\delta h(z))}{(\pi^2+a^2+\pi^2Ta)}Ra - \frac{(\pi^2+a^2)(1+\delta h(z))}{(\pi^2+a^2+\pi^2Ta)}\left(N_A + \frac{Le}{\varepsilon}\right)Rn. \quad (34)$$

In order for ω to be real it is necessary that

$$\frac{(\pi^2+a^2)(1+\delta h(z))}{(\pi^2+a^2+\pi^2Ta)}\left(Ra + \left(N_A + \frac{Le}{\varepsilon}\right)Rn\right) \le \frac{(\pi^2+a^2)^2}{a^2} . \quad (35)$$

In the absence of rotation (Ta = 0) and for constant gravity ($\delta h(z) = 0$), we have

$$\frac{Ra}{\sigma} + \frac{Rn}{\varepsilon} - \frac{(\pi^2+a^2)^2}{a^2}\left(\frac{1}{Le}+\frac{1}{\sigma}\right) = 0 , \quad (36)$$

$$\frac{\omega^2 L_e}{a^2\sigma} = \frac{(\pi^2+a^2)^2}{a^2} - Ra - \left(N_A + \frac{Le}{\varepsilon}\right)Rn. \quad (37)$$

and

$$\left(Ra + \left(N_A + \frac{Le}{\varepsilon}\right)Rn\right) \le \frac{(\pi^2+a^2)^2}{a^2}. \quad (38)$$

These are good agreements of results obtained by Nield and Kuznetsov [17], Chand and Rana [23].

The critical value of the wave number is attained at $a = \pi$, hence one can obtain the results for the case of stability boundary as

$$\frac{Rn}{\varepsilon} + \frac{Ra}{\sigma} = 4\pi^2\left(\frac{1}{Le}+\frac{1}{\sigma}\right), \quad (39)$$

$$\frac{\omega^2 L_e}{a^2\sigma} = 4\pi^2 - \left(Ra + \left(N_A + \frac{Le}{\varepsilon}\right)Rn\right). \quad (40)$$

$$\left(Ra + \left(N_A + \frac{Le}{\varepsilon}\right)Rn\right) \le 4\pi^2. \quad (41)$$

These results are same as obtained by Nield and Kuznetsov [17] for particular case.

6. RESULTS AND DISCUSSION

The effects of variable gravity parameter, Lewis number and rotation on stationary convection have been presented graphically. Stability curves for variable gravity parameter, Lewis number and rotation are shown in figures 1-3.

Fig.1 indicate the effect of variable gravity parameter on the stationary convection and it is found that fluid layer has stabilizing effect when the gravity parameter varies as $h(z) = z^2 - 2z$, $h(z) = -z$, $h(z) = -z^2$ and has destabilizing effect when gravity parameter is $h(z) = z$. These results are good agreements of the results obtained by Straughan [35].

Fig.2 indicates the effect of Lewis number on stationary convection and it is found that the critical Rayleigh number increases with an increase in the value of Lewis number, indicating that the effect of Lewis number is to inhibit the onset of convection. Also it is found that the value of stationary Rayleigh number increases when we taken decreasing gravity profile i.e. when variable gravity parameter is $h(z) = -z$, while the value of stationary Rayleigh number decreases when we take increasing gravity profile i.e. when variable gravity parameter is $h(z) = z$. Thus decreasing gravity parameter has stabilizing effect while increasing gravity parameter has destabilizing effect on the stationary convection.

Fig.3 indicates the effect of Taylor number on stationary convection and it is found that the critical Rayleigh number increases with an increase in the value of Taylor number, indicating that the effect of Taylor number is to inhibit the onset of convection. Also it is found that the value of stationary Rayleigh number increases when we taken decreasing gravity profile i.e. when variable gravity parameter is $h(z) = -z$, while the value of stationary Rayleigh number decreases when we take increasing gravity profile i.e. when variable gravity parameter is $h(z) = z$. Thus decreasing gravity parameter has stabilizing effect while increasing gravity parameter has destabilizing effect on the stationary convection.

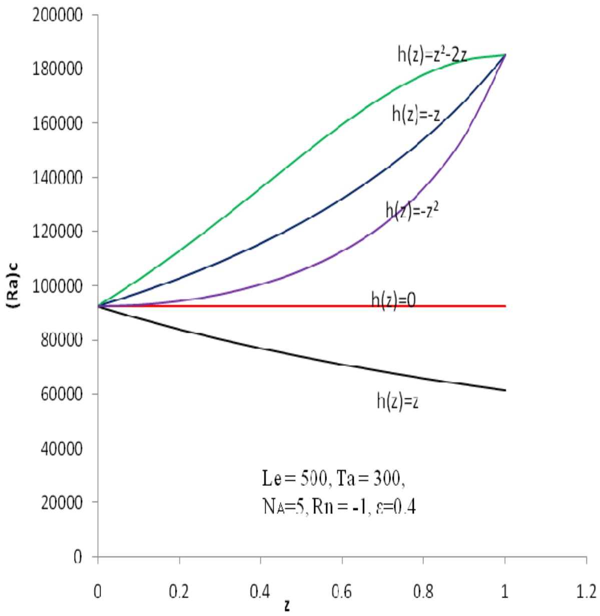

Fig.1 Stability curve for different values of variable gravity parameter.

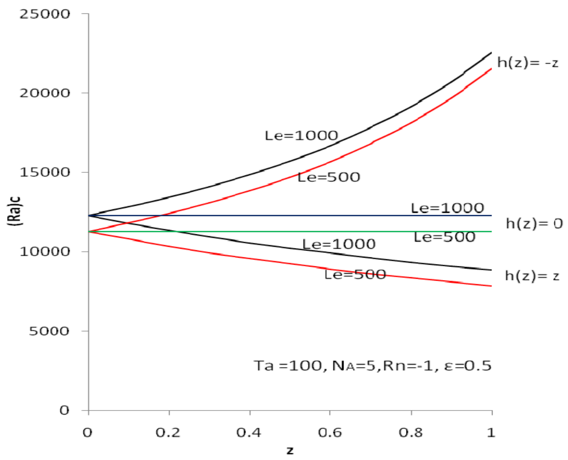

Fig.2 Stability curve for different values of Lewis number

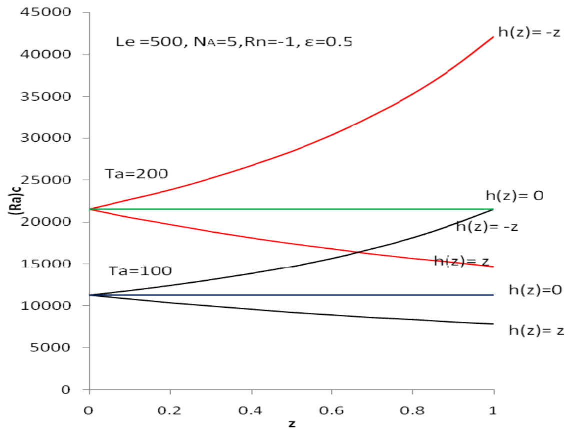

Fig.3 Stability curve for different values of Taylor number.

7. CONCLUSIONS

A linear analysis of thermal instability for rotating nanofluid with variable gravity in a porous medium is investigated. The problem is analyzed for free-free boundary conditions by employing the normal mode technique.

The main conclusions are:

(i) The critical cell size is not a function of any thermophysical properties of nanofluid.

(ii) Instability is purely phenomenon due to buoyancy coupled with the conservation of nanoparticles. It is independent of the contributions of Brownian motion and thermophoresis.

(iii) Critical value of Rayleigh number is independent of modified particle-density increment N_B.

(iv) Stationary convection has stabilizing effect when the gravity parameter varies as $h(z) = z^2 - 2z$, $h(z) = -z$, $h(z) = -z^2$ and has destabilizing effect when gravity parameter varies as $h(z) = z$.

(v) Lewis number and Taylor number is to inhibit the onset of stationary convection.

(vi) Decreasing gravity parameter has stabilizing effect while increasing gravity parameter has destabilizing effect on the stationary convection.

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NOMENCLATURE

a wave number

$\hat{\mathbf{e}}_x$ unit vector along x- axis

$\hat{\mathbf{e}}_y$ unit vector along y- axis

$\hat{\mathbf{e}}_z$ unit vector along z- axis

d thickness of fluid layer

D_B Brownian diffusion coefficient

D_T thermophoretic diffusion coefficient

$\mathbf{g}$ acceleration due to gravity

$h(z)$ variable gravity parameter

k_1 medium permeability

k_m thermal conductivity

Le Lewis number

n growth rate of disturbances

N_A modified diffusivity ratio

N_B modified particle -density increment

p pressure

$\mathbf{q}$ Darcy velocity vector

Ra thermal Rayleigh Darcy number

Ra_c critical Rayleigh Darcy number

Rm density Rayleigh number

Rn concentration Rayleigh number

Ta Taylor number

t time

T temperature

T_1 reference temperature

(u,v,w) velocity components

(x,y,z) space co-ordinates

Greek Symbols

α thermal expansion coefficient

μ viscosity of fluid

ε porosity parameter

Ω angular velocity

ρ density of the nanofluid

$(\rho c)_m$ heat capacity in porous medium

$(\rho c)_p$ heat capacity of nanoparticles

φ volume fraction of the nanoparticles

ρ_p density of nanoparticles

ρ_f density of base fluid

ξ vorticity

κ thermal diffusivity

ω frequency of oscillation

Superscripts

* non-dimensional variables

' perturbed quantities

Subscripts

p particle

f fluid

b basic state

s stationary convection

0 lower boundary

1 upper boundary

H horizontal plane

УТИЦАЈ ПРОМЕНЉИВЕ ГРАВИТАЦИЈЕ НА ТОПЛОТНУ НЕСТАБИЛНОСТ РОТИРАЈУЋЕГ НАНОФЛУИДА У ПОРОЗНОМ МЕДИЈУМУ

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У раду се истражује утицај промењливе гравитације на почетак термалне конвекције у хоризонталном слоју ротирајућег нанофлуида у порозном медијуму. Дарсијев модел се користи као порозни медијум. Анализа линеарне стабилности заснована на техници нормалног режима се користи да би се нашло решење за слој флуида ограниченог двема слободним границама. Галеркинова метода је примењена за извођење Рејлијевих бројева за почетак стационарне и осцилаторне конвекције. У циљу проучавања утицаја Тејлоровог коефицијента (ротације), Луисовог коефицијента и параметра промењливе гравитације на стационарну конвекцију израђени су графици.