Model Based Vibration Control of Smart Flexible Structure Using Piezoelectric Transducers

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This paper focuses on development and implementation of optimal control algorithms for vibration control of flexible beam structures with embedded piezoelectric actuators. Piezoelectric transducers have become the leading active elements in smart structures based on their characteristics and reliability. Piezo laminated beam with collocated pairs of piezoelectric sensors and actuators is modelled using the methodology of system identification. The obtained model has been implemented in the model based optimal control algorithms. Linear quadratic regulator and model predictive control are developed and tested using LabVIEW and NI cRIO platform. The MPC algorithm shows better performance due to the constraint handling and requires more powerfull real-time and FPGA controller target.

Keywords: *smart flexible structures, piezoelectric transducers, vibration control.*

1. INTRODUCTION

The advanced interdisciplinary research area of vibration control has brought a lot of intention to itself in the last decade developing and implementing different systems for vibration suppression in many different technical fields [1]. The trend in design of mechanical systems has the tendency to lean towards more light structures in favor of flexibility, but also vibration. Light structures which have the distinctive feature of having sensors and actuators that are often distributed and have a high degree of integration inside the structure are called smart structures [2].

When smart structures are analyzed in the direction of active vibration control systems, their basic components are:

- flexible structure,
- integrated sensors,
- integrated actuators, and
- controller

The mechanical structure is influenced by some disturbance and the sensors measure the disturbance influence on the structure itself. Controllers acquire these sensor signals in order to intelligently make use of them and to generate the appropriate control signals. The actuators act according to the generated control signal aiming to counteract the influence of the disturbance on the structure. The rapid developing technologies in sensors, actuators and real-time controllers has pushed the limits of vibration control systems to a complete new level introducing the mechatronic approach with high level of integration [3].

The application of piezoelectric transducers for active vibration control of smart flexible structures has been extensively studied over the last few years as they are becoming more commercially available. The best known piezoceramic is the Lead Zirconate Titanate (PZT). PZT patches can be glued on the supporting structure and become part of the structure itself, without significant change to the structure dynamics or functionality. PZT is ideal because of its respectable maximum actuation strain, reasonable cost, and high accessibility [6].

Piezoelectric sensors operate using the direct effect, i.e., electric charge is generated when a piezoelectric material is stressed causing deformation. These sensors are extremely sensitive, have superior signal-to-noise ratio, and high frequency noise rejection. Piezo film is approximately ten times as sensitive as semi-conducting gauges and over 300 times as sensitive as resistance gauges. Sensors can be bonded to another material, which causes the sensor to deform with the base structure. The deformation of the sensor can be measured by measuring the voltage across its electrodes. Typical piezoelectric actuators operate using the inverse effect of piezoelectric materials. This effect states that when a piezoelectric material is placed into an electric field i.e., a voltage is applied across its electrodes; a strain is induced in the material.

The piezo actuators require more power for active vibration control [6], but compared to traditional actuators (motors, hydraulics) offer faster response time and higher reliability [7].

The research presented in this paper includes modeling, simulation and experimental results for an aluminum beam with distributed piezoelectric patches, disturbed by external shaker excitation.

In section 2 the theoretical background for optimal feedback control algorithms is presented. In section 3 and 4 are given the experimental setup and the results obtained during the research followed by conclusions.

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2. OPTIMAL FEEDBACK ALGORITHMS FOR VIBRATION CONTROL

Choosing and developing control algorithm for vibration control application is complex engineering task. This process is done through iterations until the main goal is met taking in consideration many limitation factors in each iteration. Vibration control system design steps are compactly summarised and presented in figure 1.

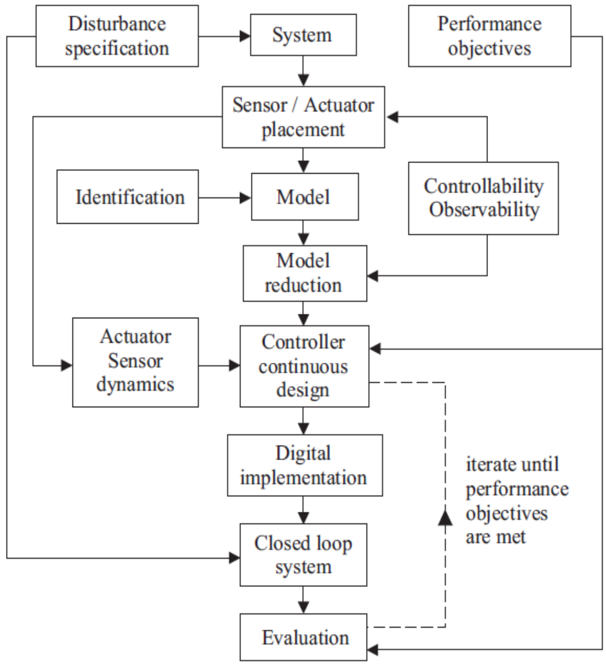

Figure 1. Vibration control system development

The first analyses are about the mechanical system dynaics and the possible disturbance specification. Based on the eigen frequencies and disturbances, active elements are chosen and a methodology for sensor and actuator optimal placement is performed. After sensors and actuators are positioned a system identification can be used for modeling the system. If necessairy active elements dynamics can be incorporated in the mathematical model. After the controller is designed and tested in simulation, it can be discretised for real-time controller implmentation. All the steps in between bring additional analyses that can improve the itterations.

The working principle of a feedback controller algorithm is based on the block diagram showed in figure 2. The system output signal y is compared to the reference signal r and the error signal $e = r - y$ is then fed to the compensator $H(s)$ after which goes into the system $G(s)$.

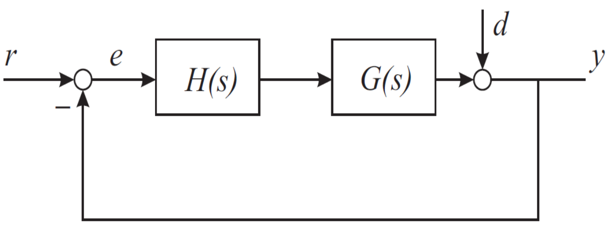

Figure 2. Feedback controller algorithm

The goal is to design the compensator $H(s)$ in order to achieve the desired system dynamics. The idea of the feedback vibration control algorithms is to lower the dynamic influence of the resonant frequencies in the total dynamic response of the system. The system transfer function, disturbance to output is calculated as following:

$$\frac{y(s)}{d(s)} = \frac{1}{1+GH} \quad (1)$$

In the range around the resonant frequencies $GH >> 1$. The goal is to hold the system output y in a certain defined range no matter what the disturbance is, which makes the following transfer function of interest:

$$F(s) = \frac{y(s)}{r(s)} = \frac{GH}{1+GH} \quad (2)$$

For $GH >> 1$ the transfer function $F(s)$ has value close to 1, which means that the output variable could achieve the desired value if the system is modeled accurately.

For smart flexible structures the lower order model only takes in consideration only the few lower most dominant eigen frequencies. The real-time controller sampling frequency needs to be at least twice the highest eigen frequency that is to be controlled, which is one of the main issues for discrete controller design and implementation.

The feedback class of algorithms takes the sensor signals and calculates optimal gains that are fed to the actuators. The general state space model is:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad (3)$$

where A is the state matrix, B is the input matrix, C is the output matrix, and the $Du(t)$ is usually 0 since it is the direct feedthrough between the input and the output.

The position feedback is defined as:

$$u(t) = -Ky(t) \quad (4)$$

where K is the gain feedback matrix.

If the sensor signal is the velocity, then the control signal is defined as:

$$u(t) = -K\dot{y}(t) \quad (5)$$

Accordingly the acceleration gain matrix is:

$$u(t) = -K\ddot{y}(t) \quad (6)$$

Combining the general state space model and the position feedback gain, the control input is:

$$u(t) = -KCx(t) \quad (7)$$

which transforms the state equation in:

$$\dot{x}(t) = (A - BKC) \quad (8)$$

When the sensors and actuators are positioned in collocated way $C = B^T$, which leads to:

$$\dot{x}(t) = (A - BKC)$$
$$y(t) = B^T x(t) \tag{9}$$

The gain matrix is diagonal matrix containing individual amplitudes for each actuator:

$$K = [0 \ diag(A_1 A_2 ... A_i ... A_N)] \tag{10}$$

where $i = 1...N$ is the number of locations where there is actuation.

2.1 LQR algorithm

The linear quadratic regulator (LQR) is based on the presented theory for optimal feedback control where the gain matrix contain the individual gains for each actuator:

$$K = [0 \ diag(K_1 K_2 ... K_i ... K_N)] \tag{11}$$

The optimal control is based on solving the cost function for performance indexes to obtain the optimal control output. The cost function is defined as:

$$J = \frac{1}{2}\int_{t_0}^{t_f} (x^T(t)Qx(t) + u^T(t)Ru(t))dt + \frac{1}{2}x^T(t_f)P_f x(t_f) \tag{12}$$

where Q is the state weighting matrix, R is the input weighting matrix and P_f is the final weighting matrix. All these weighting factors are adjusted in order to obtain the desired system behavior. The optimal control problem is in minimization of the cost function given with:

$$J = \frac{1}{2}\int_{t_0}^{\infty} (x^T(t)Qx(t) + u^T(t)Ru(t)dt \tag{13}$$

The control law form constraint feedback gain matrix, $u(t) = -KCx(t)$ that makes the general state space model:

$$\dot{x}(t) = (A - BK)x(t) \tag{14}$$

The gain matrix is calculated as follows:

$$K = R^{-1}B^T P \tag{15}$$

where P is the solution of the differential Riccati equation:

$$\dot{P} = -PA - A^T P + PBR^{-1}B^T P - Q \tag{16}$$

The control outputs generated with this algorithm cannot take in consideration the physical limits of the actuators meaning maximal voltage levels they could accept. This problem is usually solved by direct saturation of the output signals that can lead to lowering the controller performance and instability problems.

2.2 MPC algorithm

One of the directions to improve optimal feedback control is to introduce advanced model predictive control algorithm that handles constraints on control law level. Model predictive control has proven its advantages in slow processes, but its implementation in fast dynamics became interesting only with the development of powerful real-time controllers that support field programmable gate arrays (FPGA). MPC algorithm is suitable for piezoelectric smart beam because it takes in consideration the constraints of the system [5].

The model predictive control is based on obtaining the optimal model iteratively on a limited horizon. The controller performance is estimated with the cost function defined with the system model. Optimal parameters are calculated by minimizing the cost function in every iteration of the controller.

The algorithm for model predictive control is presented in the block diagram of figure 3, where instead of fixed feedback gain, the control output is calculated based on the optimization following every iteration based on the measured values.

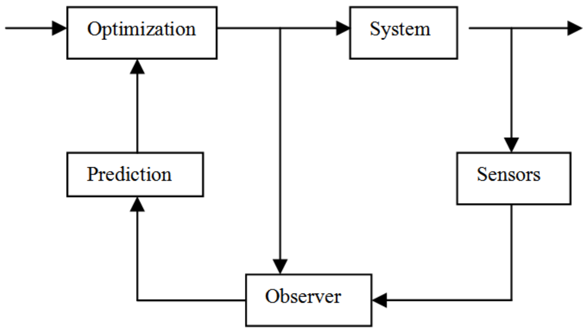

Figure 3. Model predictive control block diagram

If the system is modeled with discrete state space model:

$$x_{k+1} = Ax_k + Bu_k$$
$$y_k = Cx_k + Du_k \tag{17}$$

where again A is the state matrix, B is the input matrix, C is the output matrix, and the D in real systems is usually 0.

The state variables and the control output are calculated with the next steps given for each state variable and control output:

$$y_{k+1} = Cx_k \tag{18}$$

The predictive controller calculates the system model in few future steps in order to estimate the system behavior and calculate the optimal control output for the next iteration. This algorithm updates the measurements and calculates the optimization task in each iteration. The calculation complexity and resources needed to complete it depends on the number of future steps to be taken in consideration.

The state vector is:

$$x_k = Mx_k \tag{19}$$

where M is the predictive matrix, and n_p is the length of the predictive horizon. The quadratic cost function is:

$$j_k = x_k^T C^T C_{xk} + u_k^T u_k \tag{20}$$

Input penalization matrix is introduced R and is tuned according to the system characteristics:

$$j_k = x_k^T C^T C_{xk} + u_k^T R u_k \tag{21}$$

Another penalization matrix is Q that introduces weighting factor in the states:

$$j_k = x_k^T Q x_k + u_k^T R u_k \tag{22}$$

It is necessary to calculate the cost function for the future steps on the predictive horizon which breaks the problem of generation control signal in this few basic steps in every iteration:

- Measure actual system state at sample x_k ;
- Minimize the cost function;
- Apply the first element of the vector of optimal control to the system.

This calculates the optimal control output each step. Although a lot of attention has been brought in developing algorithms to efficiently solve the quadratic cost function, calculating it in each iteration requires high power controller when high system dynamics is considered.

3. EXPERIMENTAL SETUP DESCTRIPTION

Setting up an experiment for active vibration control of a flexible beam with piezoelectric patch actuators requires analyses in all development phases and many technical aspects have to be considered in order to obtain a controllable device for research purposes. A support construction enabling positioning the flexible beam vertically in various support configurations was designed and hereafter the hinged-hinged configuration is observed. Analyses were derived for the beam dimensions, as well as the sensor and actuator positioning. In general, non-collocated systems suffer from a lack of robustness and should not be used if the uncertainty of the system is large [4], but the controller performance may be better than for collocated systems if a sufficiently accurate mathematical model is available [8].

The four pairs of piezoelectric transducers are placed in collocated way at the positions 75 mm, 816 mm, 1554 mm and 1890 mm from the upper end according analyses carried out using norms. Figure 4 shows the laboratory experiment as it is setup. The position of the electrodynamic shaker is between pair 3 and 4.

This experimental setup has been build for fundamental research vibration control concepts of smart beams with different boundary conditions. Having a shaker mounted changes the system dynamics from ideal conditions, and introduces controlled high energy level disturbance in the system.

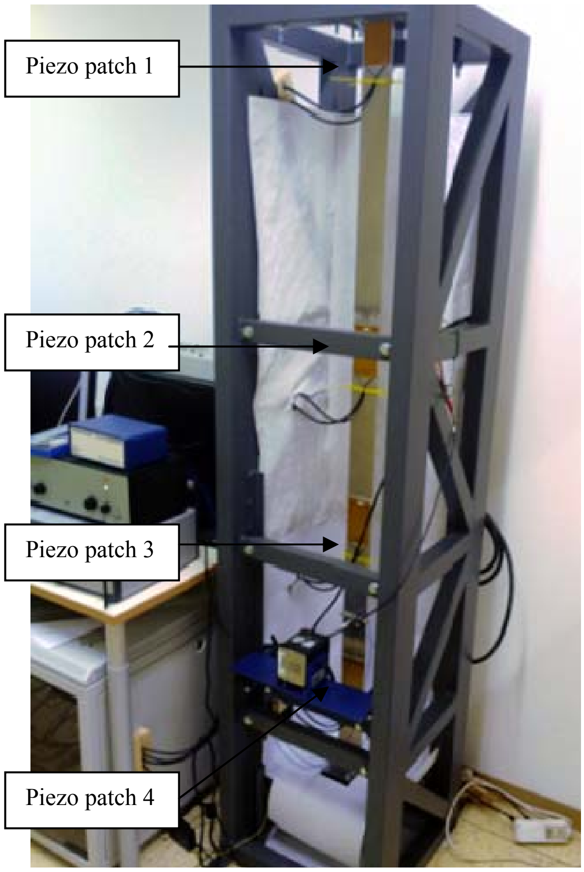

Figure 4. Experimental setup for active vibration

The geometry and material parameters of the beam were chosen as given in Table 1.

Table 1. Beam properties

Dimension	*Value*
Beam length L	2000mm
Beam width b	75mm
Beam height h	3mm
Material: aluminum	ρ =2700kg/m³
Module of elasticity	E=70GPa

The sensing and actuation are done by 4 pairs of piezoelectric patch sensors and actuators applied in a collocated way. The optimal placement of piezo patches is an optimization problam and there are may mehotdologies to be applied [10, 11]. Different collocation stratedgies are presented in [9]. The piezo actuators require more power for active vibration control [1], but compared to traditional actuators (motors, hydraulics) offer faster response time and higher reliability [8]. They are driven by dedicated, custom built high-voltage power amplifiers. Connected to high-resistance voltage analog/digital converters (ADCs), the MFC patches used as sensors yielded sufficient signal level for direct acquisition.

A lateral disturbance force can be introduced by an eletrodynamic shaker acting on the beam. An integrated impedance head at the disturbance location allows direct measurement of the disturbance force and the acceleration at that beam position.

The equipment needed for active vibration control is given in Table 2.

Table 2. Equipment specification

Equipment	*Description*
Piezo-Patches Actuators, 4 pcs	Smart-material MFC-M-8557-P1 Range: -500V to +1500V
Piezo-Patches Sensors, 4 pcs	Smart-material MFC-M-2814-P2, (Range: -60V to +360V)
Shaker	Modal Shop, Model 2007E; deflection (max): 13 mm pk-pk
High voltage amplifiers	Custom-built Range: -2.5..7.5 V / -500..1500V
Impedance head (force and acceleration sensor)	PCB Piezotronics, Model 288D01 Range: ±220N force, ±50g accel. Signal conditioner PCB 482C15

Proper measurement signal conditioning (exploitation of full signal range, not clipping the signal) is achieved by careful matching of physical signal magnitudes, conversion factors, and acquisition setup. The physical magnitudes have been estimated by analytic onsets and simulation.

4. EXPERIMENTAL RESULTS

The beam model was experimentally obtained using system identification. System identification is a methodology that can be defined as the mathematical modeling of dynamical systems based on measurement data and statistical approaches for finding models and to adequately describe the system behavior [8]. Using tools such as the System Identification Toolkit from LabVIEW different types of models were created. For the purpose of the research presented in this paper, the beam was excited by noise on the 4 piezo actuators as well as on the shaker. The data was recorded with a sampling frequency of 1000Hz, but then it was downsampled to 100Hz and a low pass filter was applied in the process of data preparation. The data was split into two parts, where 50% of the data was used for building a model and 50% for model cross-validation.

Based on these analyses, a state space model with optimal number of 10 states was obtained and the model shows the 5 first eigen frequencies in the state matrix, 4,37 Hz; 13,12 Hz; 17,22 Hz; 29,78 Hz; and 45,06 Hz.

The singular value plot of the state space model, showing the dominant picks in the first eigen frequencies is presented in figure 5.

Model based optimal control was developed using LabVIEW Control Design and Simulation Toolkit and then was deployed and implemented on the NI cRIO real-time and FPGA platform. Both algorithms showed improvement in the vibration level of the smart beam on different input disturbances.

The idea was to focus on constraints handling on both algorithms. Piezoelectric actuators work in predefined range and a control algorithm that takes in the consideration the voltage limits is expected to be more effective. The LQR handles constraints with direct saturation of the output signal before it reaches the actuators in order not to exceed the physical limits of the actuator. To compare the algorithm performance a pulse signal was applied to the shaker and then the control effort was compared on piezo actuator 3. The working range of the piezoelectric actuators has been scaled to work on ±5 V input voltage, which means that if the optimal gain exceeds these limits the control signal will be directly saturated before reaching the actuator.

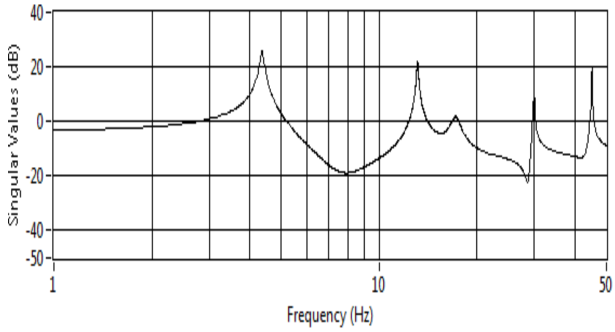

Figure 5. Singular value plot

Comparison of actuator effect in a LQR and MPC implementation is given in figure 6. From figure 6 could be noticed that the LQR algorithm generates signal that exceeds the limits that introduces clipping of the signal before reaching the actuator, and the MPC algorithm works in the limit range. This concludes the efficiency of the MPC algorithm and its possibility in implementation in smart beams.

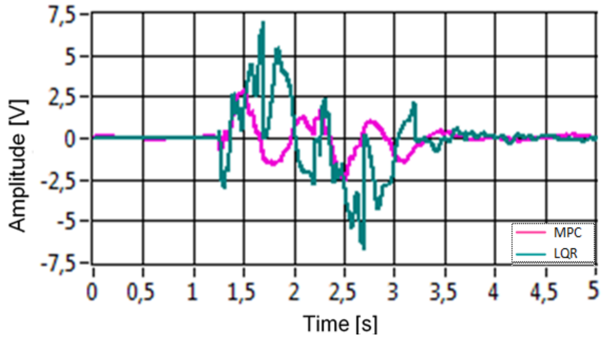

Figure 6. Comparison of actuator effect in a LQR and MPC

In figure 7 a bode plot of measured sensor signal of controlled vibration responses of the LQR and MPC optimal feedback algorithms is presented. The measurements are done when the system is disturbed with noise signal brought to the shaker. The first and the third vibration modes of the beam are considerable damped, and the second, the forth and the fifth are almost completely damped.

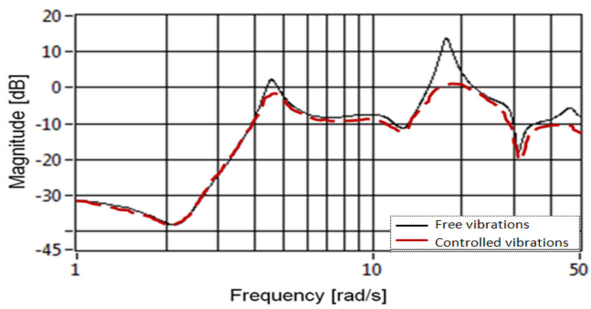

Figure 7. Comparison of actuator effect in a LQR and MPC

5. CONCLUSIONS

This paper's contribution is in showing application of piezoelectric elements in vibration control systems such as smart structures. Smart structures have been in focus of research in the last decade pushing boundaries of their potential commercial application. Smart structures characteristics could be found applicable and advantageous in different engineering fields, from airplane wings and helicopter blades, over car chassis, to micropositioning, and different applications to noise cancellation.

Technology developments in piezoelectric active elements took smart structures developments to a completely different level, giving researchers possibilities to complete their ideas.

Power controllers allow model based algorithms to run for fast dynamics systems such as the smart structures. Optimal model based control algorithms were developed, tested and analyzed in this paper and both show good results in vibration damping on the flexible beam experimental setup. The discussion showed that MPC algorithm takes in consideration constraints of the actuators in every iteration which makes it more effective in real application. The real time and FPGA controller commercially available are capable of running advanced control algorithms which makes it possible for future implementation of model predictive control in fast dynamics applications.

This research continues in design and developments of different smart structures applying different control strategies, in order to test possibilities of smart structures application in real engineering problems.

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МОДЕЛ ЗАСНОВАН НА КОНТРОЛИ ВИБРАЦИЈА ПАМЕТНИХ ФЛЕКСИБИЛНИХ СТРУКТУРА КОРИШЋЕЊЕМ ПИЕЗОЕЛЕКТРИЧНИХ ПРЕТВАРАЧА

Јована Јованова, Виктор Гаврилоски, Марјан Ђидров, Гоце Тасевски

У овом раду говори се о развоју и имплементацији оптималних алгоритама за контролу вибрација код флексибилних греда са пиезоелектричним актуаторима. Пизоелектрични трансдуктори су због својих карактеристика и поузданости постали водећи елементи у структурама на основу њихових карактеристика и поузданости. Модел структура са пиезоелектричним сензорима и актуаторима је направљен применом методологије идентификације система. Добијен модел је имплементиран у оптималном алгоритму за контролу вибрација. Линеарни квадратни регулатор и модел предвидивог управљања развијени су и тестирани коришћењем програма LabVIEW и NI cRIO. Алгоритам MPC показује боље особине услед ограничења актуатора и захтева вишу снагу контролера у реалном времену.