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# A Numerical Model for Mechanical Interaction of Rough Surfaces of the "Piston-Cylinder Liner" Tribosystem

*The paper analyzes the numerical model of interaction of rough surfaces of the piston-cylinder liner tribosystem in the form of a Markov chain. It is based on a model that takes into account elastic and plastic deformation and fatigue failure under load. We have given a system of equations for the distribution of time-varying heights of protrusions. For contact interface of the piston-cylinder liner tribosystem of the diesel engine we analyzed the evolution of distributions.*

**Keywords:** piston-cylinder liner tribosystem, surface roughness, film thickness.

## 1. INTRODUCTION

Existing methods for calculating friction characteristics are approximate and often inadequate for the engineer. Friction characteristics determined by the factors such as surface micro geometry, physical characteristics of materials, velocity and the applied load. There is an approach based on the representation of the relief function for the random arguments [1,2]. Such models can estimate the area of contact, the friction force, coefficient of friction, wear, etc. However, this approach ignores the surfaces change and, consequently, contact and friction characteristics over time. In this paper a model of random change of surfaces heights over time for "piston-cylinder liner" tribosystem is developed.

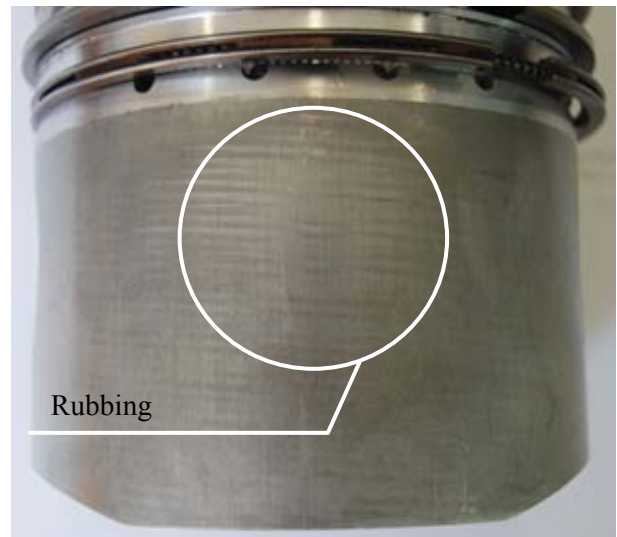
The "piston-cylinder liner" tribosystem is mostly in the hydrodynamic lubrication regime. At high loading levels, near the TDC, tribosystem is in the mixed or boundary lubrication regimes. The latter is important for the determination of the service life of the tribosystem as a whole.

## 2. DETERMINATION OF THE HYDROMECHANICAL CHARACTERISTICS OF THE "PISTON-CYLINDER LINER" TRIBOSYSTEM

The main hydromechanical characteristics (HMC) of the "piston-cylinder" tribosystem are:  $h_{\min}(\tau)$  – instantaneous values of the minimum oil film thickness;  $p_{\max}(\tau)$  – instantaneous values of the maximal hydrodynamic pressure;  $\bar{h}_{\min}^*$  – average value of  $h_{\min}(\tau)$ ;  $\bar{p}_{\max}^*$  – average value of  $p_{\max}(\tau)$ ;  $N(\tau)$  and  $N^*$  – instantaneous and average power loss of friction;  $Q^*$  – the average flow rate of oil in the direction of the

combustion chamber;  $T_{\text{eff}}^*$  – the average effective temperature of the lubricating layer.

Since the contact interaction of the elements is experimentally confirmed by the formation of "rubbing" on the piston skirt of diesel engine after certain hours (Fig. 1), the calculated characteristics of tribosystem complemented the areas, where minimal thickness of a lubricant film less than the permissible value  $h_{\text{lim}}$  –  $\alpha h_{\text{lim}}$  [%].



**Figure 1. The result of contact interaction of the piston-cylinder liner interface**

The reactions of the lubricating layer were determined based on the results of the numerical integration of the modified Elrod equation for pressure in the lubricating layer and the degree of filling of clearance [3]. The modified Elrod equation contains the function  $\Omega(\varphi, \bar{z})$  and takes the form of:

$$\frac{\partial}{\partial \varphi} \left[ \frac{\bar{h}^3}{12\mu_{\text{eff}}^*} \frac{\partial}{\partial \varphi} (g\Omega) \right] + \frac{1}{a^2} \frac{\partial}{\partial \bar{z}} \left[ \frac{\bar{h}^3}{12\mu_{\text{eff}}^*} \frac{\partial}{\partial \bar{z}} (g\Omega) \right] =$$

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$$= \frac{\bar{w}}{2} \frac{\partial}{\partial \bar{z}} \left\{ \bar{h} [1 + (1-g)\Omega] \right\} + \frac{\partial}{\partial \tau} \left\{ \bar{h} [1 + (1-g)\Omega] \right\}, \quad (1)$$

where:  $\bar{h} = h/h_0$ ;  $\bar{\mu}_{\text{eff}}^* = \mu_{\text{eff}}^*/\mu_0$ ;  $-a \leq \bar{z} \leq a$ ;  $\bar{z} = z/R$ ;  $\varphi = xR$ ;  $a = B/2R$ ;  $\tau = \omega_0 t$ ;  $\bar{w} = w/(\omega_0 R)$ ;  $\bar{h}$ ,  $\bar{\mu}_{\text{eff}}$  – dimensionless film thickness and the effective viscosity of the lubricant;  $B$  and  $R$  – the width and radius of the bearing;  $\bar{\mu}_{\text{eff}}^*$  – effective viscosity of the lubricant, the corresponding temperature to  $T_{\text{eff}}^*$ ;  $\mu_0$ ,  $h_0$ ,  $\omega_0$  – respectively lubricant viscosity, typical film thickness at the center position of the piston in cylinder and rotation speed of crankshaft;  $\bar{w}$  – the dimensionless linear velocity of the piston;  $g$  – the switching function.

The function  $\Omega(\varphi, \bar{z})$  is related to the degree of filling of the clearance  $\theta(\varphi, \bar{z})$  and is characterized by a function that determines the mass content of the liquid phase (oil) in the volume of the clearance between the piston and cylinder using the relationship  $\theta = 1 + (1-g)\Omega$ .

For calculating the trajectory of the piston on the lubricating layer in the cylinder, the system of coordinates is fixed to a stationary cylinder. At the start, the origin of this moving coordinate system is at the center of mass (point C) of the moving piston (Fig. 2).

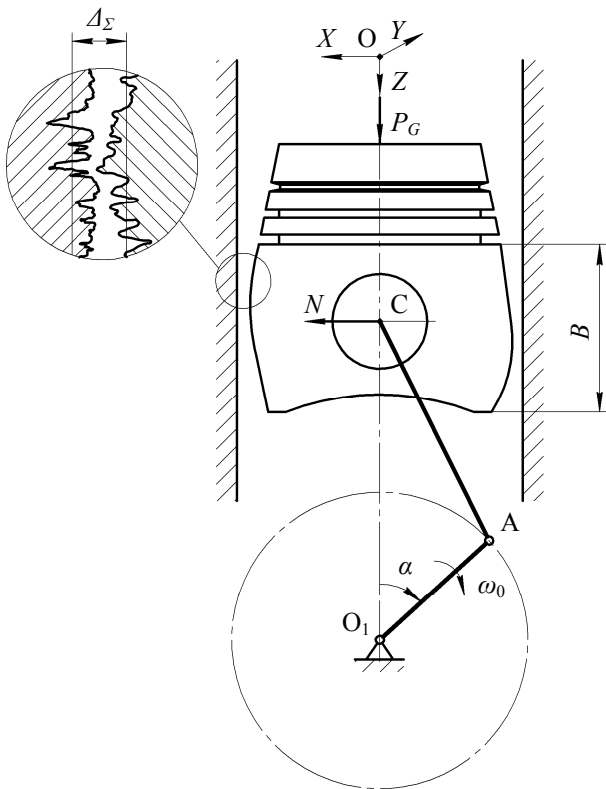


Figure 2. The scheme of the piston-cylinder liner interface

According to [4], it is assumed that movement of the piston in the cylinder is only in the plane perpendicular to the axis of the piston pin.

Given the initial data and the methodology for calculating the trajectory of piston's motion on the lubricating layer, as described in [5,6], we got the dependence of the minimum film thickness  $h_{\text{min}}$  in function of a crank angle  $\alpha$  for a diesel engine (Fig. 3).

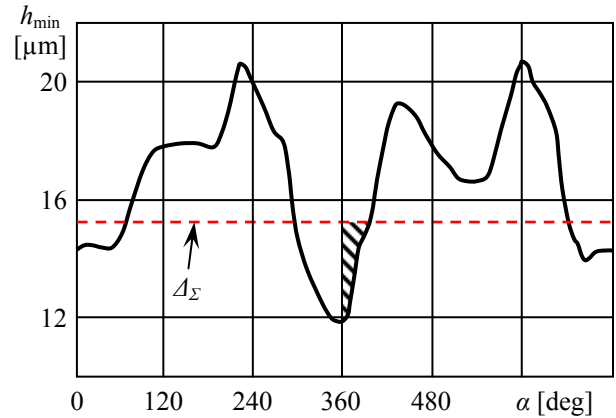


Figure 3. The minimal lubricant film thickness in function of the crank angle

The Figure 3 presents a dotted line area corresponding to working stroke, where the values of the  $h_{\text{min}}$  reach the values less than the total value of the sum of the maximum heights of microasperities ( $\Delta_\Sigma$ ) for the contact surfaces. In this moment of time the acting load (side force) increases and reaches 5000 – 7000 N [6]. The value of  $\Delta_\Sigma$  was determined by recording profile traces of specimens of the piston and cylinder sleeve a Leica DCM 3D electron microscope and special software, which yielded the value of 15  $\mu\text{m}$ .

For only the period of tribosystem's working time, where  $h_{\text{min}}$  less  $\Delta_\Sigma$  (contact area, see Fig. 3), we analyzed characteristics of the “piston-cylinder liner” tribosystem using Markov chains. Description of the used approach and model is below.

### 3. MARKOV CHAINS

To analyze the process of friction in the contact area we used the discontinuous model where surfaces are represented by asperities of random height (Fig. 4). Pairs of contacting asperities are changed when the shift occurs. Asperities in contact deform and destruct each other, that is why heights of contacting asperities may change. Total height change of all surface asperities leads to roughness transformations and variation of friction characteristics (real contact area, friction coefficient, wear).

Let's consider  $K$  pairs of asperities positioned in one row. Time is discontinuous and is measured in shift counts. After  $n$  shifts heights of asperities of one surface are represented by vector  $\xi_k(n)$ , ( $k = 1, \dots, K$ ), and heights of asperities of the other surface are represented by vector  $\eta_k(n)$ . The asperity height change of the  $k^{\text{th}}$  pair in one shift is described by general equations:

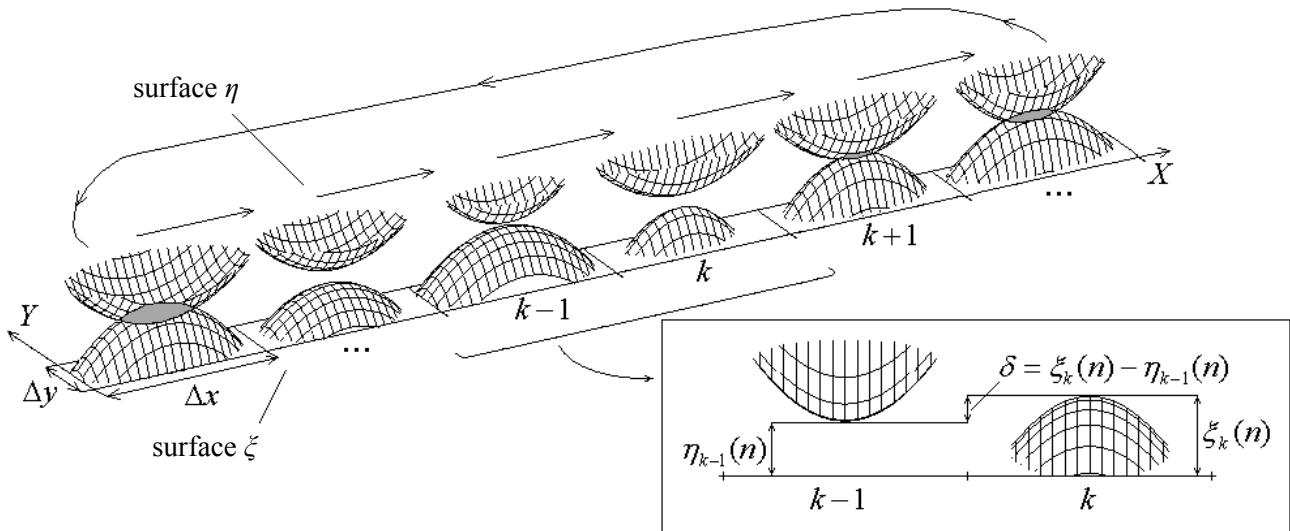
$$\xi_k(n+1) = \Psi(\xi_k(n), \eta_{k-1}(n)), \quad (2)$$

$$\eta_k(n+1) = \Phi(\xi_k(n), \eta_{k-1}(n)), \quad (3)$$

where:  $k = \overline{1, K}$ ,  $\eta_0(n) = \eta_K(n)$  and  $\Phi$  and  $\Psi$  determine stochastic mechanism of asperity interaction as a sphere interaction [7], with random radiuses.

Presented model of asperity interaction takes into account elastic-plastic deformation and wear.

The model is realized on a computer. After  $N$  shifts are performed, asperity height histogram is depicted



**Figure 4. Model of surfaces and sketch of cyclical shifts**

real contact area; friction force and wear are measured. Presented results agree with well known observations, such as independence of “equilibrium roughness” from initial surface roughness.

It is proposed to describe the trajectory of height changes of single asperity by means of Markov series [8]. The height  $\zeta$  of a single asperity changes when it interacts with asperities of the other surface, having heights  $\eta(t)$  at the moment  $t$ . Time step  $\Delta t$  is selected in such a way that values  $\eta(t)$  and  $\eta(t + \Delta t)$  are close to uncorrelated. It is supposed that asperity with the height  $\zeta$  interacts with the series of asperities of the other surfaces with heights  $\eta_1, \dots, \eta_n, \dots$ , distributed with density  $q_n(y)$ . Thus a series of heights  $\zeta_1, \dots, \zeta_n, \dots$  is obtained, height value  $\zeta_{n+1}$  depends on  $\zeta_n$  and  $\eta_{n+1}$ :

$$\zeta_{n+1} = \Psi(\zeta_n, \eta_{n+1}), \quad (4)$$

where;  $\Psi$  determines accepted mechanism of asperity contact interaction.

A series  $\zeta_1, \dots, \zeta_n, \dots$  is Markov series. Moreover, the heights  $\zeta_1, \dots, \zeta_n, \dots$  are discontinuous, the series  $\zeta_1, \dots, \zeta_n, \dots$  becomes Markov chain. Similarly,  $\zeta$  is distributed with density  $p_\zeta(x)$  and next equation is obtained:

$$\eta_{n+1} = \Psi(\zeta_n, \eta_{n+1}). \quad (5)$$

Once two functions  $\Psi$  and  $\Phi$  are elaborated the system of equations for stationary distributions  $p^*$  and  $q^*$  is obtained:

$$\begin{cases} p^* = p^* \cdot P_\zeta(q^*, \Psi) \\ q^* = q^* \cdot P_\eta(p^*, \Phi) \end{cases}, \quad (6)$$

where:  $P_\zeta(q^*, \Psi)$  and  $P_\eta(p^*, \Phi)$  are matrices of transition probabilities for series of  $\zeta_n$  and  $\eta_n$ .

Elements of matrices are determined by distributions  $p^*$  and  $q^*$  – respectively and by two functions  $\Psi$  and  $\Phi$ . For example, elements of matrix  $P_\zeta(q^*, \Psi)$  are obtained:

$$\|P_\zeta\|_{ij} = P\{\zeta_{n+1} = j \mid \zeta_n = i\} = \sum_{l: \Psi(i, l) = j} q^*(l). \quad (7)$$

Elements  $P_\eta(p^*, \Phi)$  are determined similarly.

#### 4. RESULTS AND DISCUSSION

Initial data were determined on the basis of recording profile traces of specimens of the piston skirt and cylinder liner [6]: the surface  $\zeta$  is the surface of the cylinder liner, surface  $\eta$  is the surface of the piston skirt.

For a period of time, where  $h_{\min}$  less  $\Delta_\Sigma$  we estimated next parameters: the distribution of the heights of roughness of surfaces ( $p_\zeta, q_\eta$ ); the root mean square deviations (rms) of surface roughness ( $\sigma_\zeta, \sigma_\eta$ ); the average contact area; the friction force  $F_{fr}$  and friction coefficient  $f$ ; the linear wear  $I_h$ .

Figures 5a and b present the height distribution of surfaces in function of the friction distance. Changes of the surface (and consequently changes of the height distribution of the projections of the contact surfaces) due to fatigue failure is extremely slow, due to a low probability of this event. At the initial process of friction mostly high surface elements come in contact. The destruction of the high elements come into contact with less high surface elements.

Under the influence of fatigue failure is a redistribution of the elements of heights, and in particular in the area of contact.

Figures 6a and 6b confirm the existence of “equilibrium roughness”.

Figure 7 shows the dependence of wear on friction distance. Wear increased slowly.

The increase of wear is connected with a change of the height of the surfaces roughness under the influence of fatigue failure, as well as with the convergence of the interacting surfaces.

#### 5. CONCLUSION

The HMC of the “piston-cylinder liner” tribosystem were calculated for diesel engine, which allowed us to estimate the duration of the surfaces contact interaction.

We proposed a Markov model of the interaction of rough surfaces, which allows estimating the friction characteristics over the time for contact area of “piston-cylinder liner” tribosystem.

Estimations agree with experimental observations.

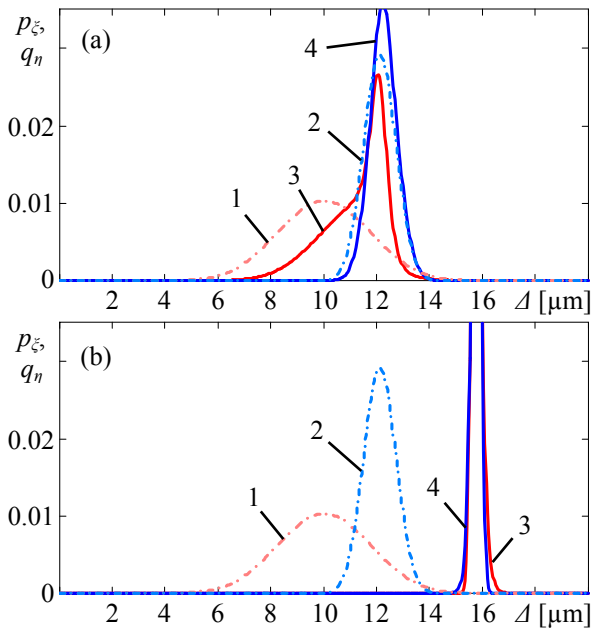


Figure 5. The distribution of the heights of roughness of contact surfaces for friction distances: (a) 70 km and (b) 700 km; 1, 2 – initial curves for cylinder liner and piston respectively; 2, 3 – curves after interaction

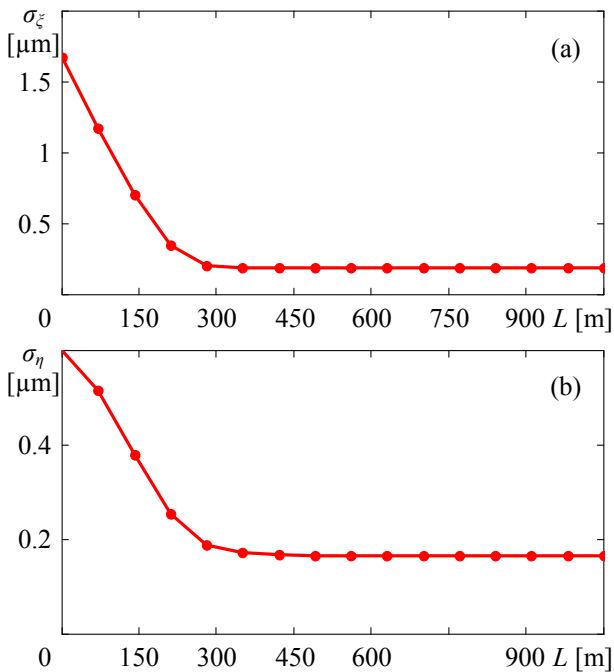


Figure 6. The rms of the heights of surface roughness of: (a) cylinder liner and (b) piston in function of friction distance

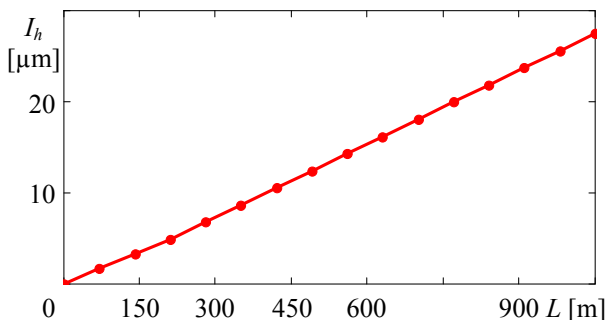


Figure 7. The linear wear in function of friction distance

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## НУМЕРИЧКИ МОДЕЛ МЕХАНИЧКЕ ИНТЕРАКЦИЈЕ ХРАПАВИХ ПОВРШИНА ТРИБОЛОШКОГ СИСТЕМА „КЛИП-ЦИЛИНДАР“

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Алексеј Дојкин

У раду се анализира нумерички модел интеракције храпавих површина триболошког система „клип-

цилиндар“ у облику ланца Маркова. Овај модел је заснован на принципу који узима у обзир еластичне и пластичне деформације, које настају при оптерећењу, и оштећења услед замора. Дат је систем

једначина којим се описује расподела висина врхова неравнина током времена. На примеру триболошког система „клип-цилиндар“ Дизел мотора је анализирана промена ове расподеле током рада.