Bulk Operations Modeling: The Case of the Seaport Automobile Terminal

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This paper discusses the loading processes at seaside link of seaport automobile terminal (SAT) as a queuing model with bulk arrivals. We explain in detail the performances of the reported SAT in the Port of Bar, and propose the related queue model regarding the considered loading process. This model allows use to obtain related numerical and graphical results for the steady-state probabilities of the defined queue model. Using these results, we are able to discuss the values of these probabilities in dependence of the basic performances to the modeling processes at the SAT involving in the expression for the utilisation factor (service utilisation). For possible further comparison analysis, we give numerical and graphical results related to the mentioned probabilities concerning four different values of the size of arriving group of automobiles in the considered queue model.

Keywords: *Bulk operations modeling, seaport automobile terminal, $M^X/D/1$ queue model, steady-state probabilities, Port of Bar.*

1. INTRODUCTION

Bulk arrival queues have wide range of applications (see more in [1-3]). These queues have widespread applications in the port systems ([4-7]). Many results in bulk operations modeling have been obtained by considering models, where customers arrive one by one and are served individually. It is frequently observed in river and sea ports that the customers arrive in groups. In this study bulk operations modeling of seaport automobile terminal (SAT) in the Port of Bar (Montenegro) is presented ([8-12]). The defined analytical model corresponds to the bulk queue with one ship ramp (server) in which the size of arrival group at the ramp is a constant random variable.

An integral optimization model and estimated manpower planning at the Bremerhaven port represented a very important study that was done in [13] and [14] where authors derived an integral decision model as a complex combinatorial problem. In [15] and [16] authors discussed about the planning of transshipment of vehicles based on a multi-agent system (MAS). The allocation of drivers to the vehicles have been considered and discussed. The MAS is tested using randomly generated problem instances with different distributions of the manufacturer shares in the vehicle streams. The tests verify a certain robustness of the MAS with regard to the number of permanently employed drivers and the cost surcharge for hired drivers.

This paper deals with the traffic modeling of the SAT in the port of Bar, which faced a fast growth in the past few years (see more in [8], [17] and [18]). The main aim of the paper is related to the operational policies at the terminal because the statistical analysis shows the difference in ships' size. Moreover, during last three years is presented a bigger number of ships going at Ro-Ro berth ([9], [11], [17] and [18]).

During 2013, at the SAT in the Port of Bar were serviced 99 ships. All those ships exported exactly 97491 automobiles that mostly came by rail transport while a few percentages were distributed by road transport. This trend continued also in 2014 and 2015, and therefore, made a base for the construction of adequate analytical model for the operating activities at the terminal ([17] and [18]). For more information on investigations of queuing systems in port see [19].

The paper is organized as follows. In Section 2 we propose the modeling methodology for considered loading process of the SAT in the Port of Bar. In particular, we deduce the system of infinite number of linear equations for the steady-state probabilities of the corresponding defined queue model. Using this system, in Section 3 we give some numerical and graphical results for the considered model. For further comparison analysis, these results are obtained for four different values of the size of arriving group of automobiles (i.e., for $g = 2,4,6,8$). Namely, these numerical and graphical results are extensions of those obtained in [10], where it is considered only the "Port of Bar" case $g = 6$. Section 4 gives concluding remarks.

2. MATHEMATICAL MODEL

As noticed above, this paper discusses the loading process at seaside link of the SAT as a queuing model with bulk arrivals. The stochastic characteristics and assumptions of loading operations are as follows (see more in [10]):

- time of arrival of a single automobile or in bulk (automobiles' group) at the ship ramp cannot be precisely given;

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- the loading time through the ship ramp is a constant service time;
- the ship's ramp is not always occupied; in some periods there are no automobiles (the capacity is under utilized) and there are the time intervals of high utilisation when the queue is formed;
- the loading operation at the SAT via ship ramp includes the following: automobiles are moved from storage yard to the ship ramp; waiting in front of ship ramp if the ramp is occupied; loading at the ship ramp, parking on the assigned slots inside ship; returning of drivers by accompanying cars to storage yards to take another group of automobiles. This cycle is called the turnaround time for the loading process of automobiles onto ships, $\bar{t}_c$.

The seaside operations at the SAT, i.e. the loading process of ships may be considered as a bulk queuing model. In this case, customers are groups of automobiles, and the service channels are ships' ramps operating for the loading/unloading of automobiles. In the seaside link of SAT it is assumed as follows [10]:

- the applied queuing model is a stationary with infinite waiting area at ship ramp;
- the sources of arriving pattern are not integral parts of loading process of ships via ramps;
- the service channel is the ship ramp with similar or identical and independent handling capacities;
- the units arrivals can be single automobile or automobile groups;
- automobiles are loaded via ship ramp or waiting to be loaded and in this case none can be rejected (accordingly, the queue length is assumed to be infinite);
- the size of an arriving group of automobiles is a random variable;
- the queue discipline is first come first served by group's bulk and random within the group's bulk.

More formally, the described model is known as a $M^X/D/1$ queue. The automobiles allocated on storage areas arrive in groups of size g at the ramp according to a time-homogeneous Poisson process with (the mean arrival) rate λ in a considered unit time (in view of the facts that the arrivals of groups of automobiles in all cycles are mutually independent). The size of every arrival group at the ramp is a constant random variable X (with the distribution $P\{X=g\}=1$). The ramp is in fact a single service that is loading of automobiles via the ship ramp. We assume that a related service time is determined, that is, the mean loading rate per ship ramp is μ. Since in each considered case the mean arrival rate λ with respect to the considered unit time (of a group) is relatively small with respect to the mean loading rate per ship ramp, we can assume that each automobile will find a waiting place available upon arrival; so that, we can suppose that the related queue model possesses an infinite capacity.

Accordingly, the arrivals of automobiles from storage areas in groups at the ramp and loading stages via ship ramp can be viewed as the $M^{X\equiv g}/D/1/\infty/s$ queue. Unfortunately, in our knowledge does not exist formulae in closed form for performances of such a type of bulk queue with a finite population. However, since the size of group g is usually small at the SAT with respect to the size of the population (total number of automobiles) s (in fact, $g/s<1/100$), our $M^{X\equiv g}/D/1/\infty/s$ queue may be well approximated by the $M^{X\equiv g}/D/1$ queue with infinite population.

Then, in accordance to the above notations, we have (see more in [10]):

- The arrival rate of every group in a particular gang is $\bar{t}_c$ and it is actually equal to the previously defined turnaround time; hence, the mean arrival rate λ of a considered queue model is $\lambda=h/\bar{t}_c$. In order to ensure sustainable operations, g drivers are grouped into h gangs. In every cycle are involved h gangs, where in each of these gangs are engaged g drivers. (Here it is used the fact that the arrivals of groups of cars are independent, and the well known fact that if X_1 and X_2 are two independent Poisson variables with mean λ_1 and λ_2, respectively, then $X=X_1+X_2$ is also a Poisson variable with mean $\lambda_1+\lambda_2$).
- The related service time is a constant $1/\mu=5s$, that is, the service rate (of passages of cars over the ramp) is $\mu=12$ per one minute.
- The utilisation factor (the server utilisation) ρ is $\rho=\lambda g/\mu=hg/(\bar{t}_c\mu)=\alpha hg$ with $\alpha=1/(\bar{t}_c\mu)$ for a particular embarkation process)
- Total number of operational gangs that are particularly engaged in each cycle, h, is 4. Number of cars that are transported in each operational gang, g, is 6.

Notice that in recent papers ([8-12]), the authors analyzed the traffic modeling of operations at the SAT in the Port of Bar. In particular, using some analytical results for the batch queue system $M^X/D/c$ established in [3] (also see [1] and [2]), authors in [10] and [12] deduced some suitable formulae for certain basic stochastic characteristics (performances) related to the loading operations at terminal over ship ramp at seaside link of the SAT in the Port of Bar. These performances are derived without the use of notion of the state probabilities of related queue model. Here we focus our attention to the determination of steady-state probabilities P_n, $n=0,1,2,....$ (P_n denotes the proba– bility that n customers are in the system).

Now consider the above described stationary queue $M^{X\equiv g}/D/1$ with related parameters. Furthermore, let $Y(t)$ denote the total number of arrivals during the period $(0,t)$, and let

$$\pi_n(t)=P\{Y(t)=n\},\ n=0,1,2,.... \tag{1}$$

Then under a general assumption that the size of arrival group at the system is a random variable X (with $P\{X=n\}=a_n$, $n=0,1,2,....$, the following recurrence formulae are satisfied (see [1] and [3]):

$$\pi_0(t) = e^{-\lambda t}, \tag{2}$$

$$\pi_{n+1}(t) = \frac{\lambda t}{n+1} \sum_{j=0}^{n} (n-j+1) a_{n-j+1} \pi_j(t), \; n = 0,1,2,.... \tag{3}$$

Since in our case we have $a_g = 1$ and $a_i = 0$ for each $i \neq g$, by using mathematical induction, the formulae (2) and (3) easily yield

$$\pi_{kg}(t) = e^{-\lambda t} \frac{(\lambda t)^k}{k!}, \; k = 0,1,2,..., \tag{4}$$

and

$$\pi_n(t) = 0, \text{ otherwise.} \tag{5}$$

For our queue model the steady-state Chapman-Kolmogorov equations for the distribution $\{P_j,\, j=0,1,2,...\}$ are given by (see Equations (1) and (2) in [1])

$$P_0 = \pi_0\left(\frac{1}{\mu}\right)(P_0 + P_1) \tag{6}$$

$$P_n = \pi_n\left(\frac{1}{\mu}\right)(P_0 + P_1) + \sum_{m=2}^{n+1} P_m \pi_{n+1-m}\left(\frac{1}{\mu}\right), \tag{7}$$

$n = 1, 2, ...$

Then substituting (2), (4) and (5) into (6) and (7), and using the fact that after a routine calculation, in view of the fact that $\lambda / \mu = \rho / g$, the equations (6) and (7) respectively becomes

$$P_0 = e^{-\rho/g}(P_0 + P_1), \tag{8}$$

$$P_n = \pi_n\left(\frac{1}{\mu}\right)(P_0 + P_1) + \sum_{i=0}^{[(n-1)/g]} P_{n+1-ig} e^{-\rho/g} \frac{(\rho/g)^i}{i!}, \tag{9}$$

$n = 1, 2, ...$

where, as usual, $[a]$ denotes the greatest integer not exceeding a, and in (9) $\pi_n(1/\mu)$ may be replaced by a suitable expression given by (4) or (5).

Furthermore, it is known that in the case of any single-server queueing system (see e.g., [2])

$$P_0 = 1 - \rho. \tag{10}$$

Notice that the expressions (4) - (10) yield the following ones:

$$P_n = P_{n+1} e^{-\rho/g} \text{ for } n = 1,2,..., g-1, \tag{11}$$

$$P_g = \frac{\rho}{g}(1-\rho) + P_{g+1} e^{-\rho/g}, \tag{12}$$

and

$$P_{g+1} = P_{g+2} e^{-\rho/g} + P_2 e^{-\rho/g} \frac{\rho}{g}. \tag{13}$$

3. NUMERICAL AND GRAPHICAL RESULTS

Given here are some numerical and graphical results for the steady-state probabilities P_n with $n = 0,1,2,3,4$ for the queue model $M^{X \equiv g}/D/1$ described in the previous section whose size of the arriving group is $g = 6$. for comparison analysis related to the certain values of steady-state probabilities, here we also consider the cases when $g = 4$, $g = 2$ and $g = 8$.

Firstly, consider the case when $g = 6$ (see [10]). taking $g = 6$ and $P_0 = 1 - \rho$ given by (10) into (8) we obtain:

$$P_1 = (1-\rho)(e^{\rho/6} - 1), \tag{14}$$

which substituting in (9) with $n = 1$ gives

$$P_2 = P_1 e^{\rho/6} = (1-\rho) e^{\rho/6} (e^{\rho/6} - 1) \tag{15}$$

Taking (10), (14) and (15) into (9) with $n = 2$, and using (5), yields

$$P_3 = P_2 e^{\rho/6} = (1-\rho) e^{\rho/3} (e^{\rho/6} - 1), \tag{16}$$

which substituting in (9) with $n = 3$, and using (5), immediately gives:

$$P_4 = P_3 e^{\rho/6} = (1-\rho) e^{\rho/2} (e^{\rho/6} - 1) \tag{17}$$

By using the expressions (10) and (14) - (17), we obtain the numerical and graphical results given in Table 1, and Figures 1 and 2, respectively.

Table 1. The values of P_n ($n = 0,1,2,3,4$) for $g = 6$ as a function of utilisation factor ρ

ρ	P_0	P_1	P_2	P_3	P_4	$\sum_{i=0}^{4} P_i$
0.4	0.6	0.041	0.044	0.047	0.051	0.783
0.5	0.5	0.043	0.047	0.051	0.058	0.699
0.6	0.4	0.042	0.047	0.051	0.057	0.597
0.7	0.3	0.037	0.042	0.047	0.053	0.479
0.8	0.2	0.029	0.033	0.037	0.043	0.342
0.9	0.1	0.016	0.019	0.022	0.025	0.182

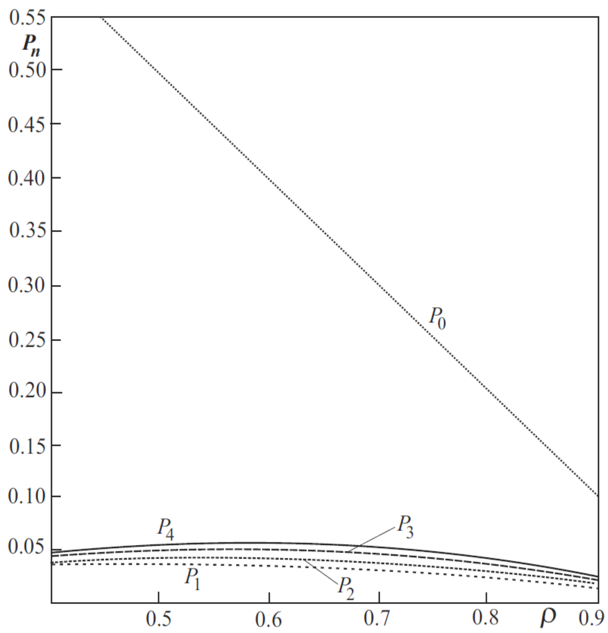

Figure 1. Steady-state probabilities P_n ($n = 0,1,2,3,4$) for $g = 6$ as a function on ρ $(0.4 \leq \rho \leq 0.9)$

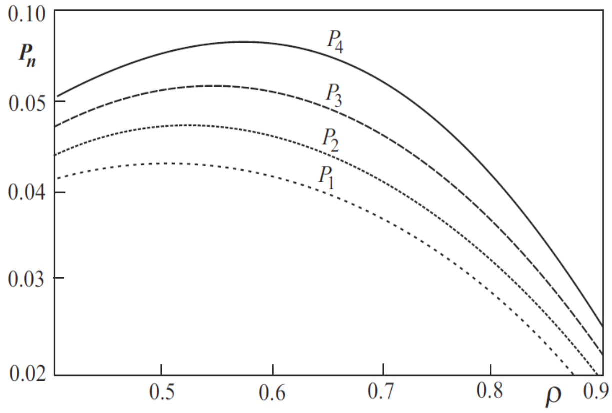

Figure 2. Steady-state probabilities P_n ($n = 1,2,3,4$) for $g = 6$ as a function on ρ $(0.4 \le \rho \le 0.9)$

From Table 1 and Figures 1 and 2 we see that the all values P_n ($n = 1,2,3,4$) are less than 0.06 for all considered values of ρ. Moreover, the values of P_0 decrease from 0.6 to 0.1 when the related utilisation factors increase from 0.4 to 0.9. Thus, in order to decrease the value of P_0 and other values P_n with "small" indices n, it is necessary to increase related values of ρ. As noticed in Section 2, the utilisation factor (the server utilisation) ρ is $\rho = \lambda g / \mu = = hg / (\bar{t}_c \mu) = \alpha hg$ with $\alpha = 1/(\bar{t}_c \mu)$ (for a particular embarkation process). From this we see that for a fixed service time $1/\mu$ (which is in our considered case equal to 5s) and the arrival rate of a group of automobiles, the value of ρ is proportional with the related product hg (the total number of drivers/accompanying cars for operational gangs which are engaged in considered embarkation process in the Port of Bar). We believe that this fact should be useful for port authority to develop strategies and directions in order to improve some basic/important Ro-Ro automobile terminal's performances.

Now consider the case when $g = 4$. Taking $g = 4$ and $P_0 = 1 - \rho$ given by (10) into (8) we obtain

$$P_1 = (1-\rho)(e^{\rho/4} - 1), \tag{18}$$

which substituting in (11) with $n = 1$ gives

$$P_2 = P_1 e^{\rho/4} = (1-\rho) e^{\rho/4} (e^{\rho/4} - 1) \tag{19}$$

Taking (10), (18) and (19) in (11) with $n = 2$ yields

$$P_3 = P_2 e^{\rho/4} = (1-\rho) e^{\rho/2} (e^{\rho/4} - 1), \tag{20}$$

which substituting in (11) with $n = 3$ immediately gives

$$P_4 = P_3 e^{\rho/4} = (1-\rho) e^{3\rho/4} (e^{\rho/4} - 1) \tag{21}$$

By using the expressions (10) and (18) - (21), we obtain the numerical results given in Table 2.

Table 2. The values of P_n ($n = 0,1,2,3,4$) for $g = 4$ as a function of utilisation factor ρ

ρ	P_0	P_1	P_2	P_3	P_4	$\sum_{i=0}^{4} P_i$
0.4	0.6	0.063	0.070	0.077	0.085	0.895
0.5	0.5	0.067	0.075	0.085	0.097	0.824
0.6	0.4	0.065	0.075	0.087	0.102	0.729
0.7	0.3	0.057	0.068	0.081	0.097	0.603
0.8	0.2	0.044	0.054	0.066	0.081	0.445
0.9	0.1	0.025	0.032	0.040	0.050	0.247

Now consider the case when $g = 2$. Taking $g = 2$ and $P_0 = 1 - \rho$ given by (10) into (8) we obtain

$$P_1 = (1-\rho)(e^{\rho/2} - 1), \tag{22}$$

which substituting in (11) with $n = 1$ gives

$$P_2 = P_1 e^{\rho/2} = (1-\rho) e^{\rho/2} (e^{\rho/2} - 1) \tag{23}$$

The equality (12) for $g = 2$ can be written as

$$P_2 = \frac{\rho}{2}(1-\rho) + P_3 e^{-\rho/2}, \tag{24}$$

whence replacing (23) we find that

$$P_3 = \frac{(1-\rho)e^{\rho/2}}{2}(2e^{\rho} - 2e^{\rho/2} - \rho) \tag{25}$$

The equality (13) for *g* = *2* can be written as

$$P_3 = P_4 e^{-\rho/2} + P_2 e^{-\rho/2} \frac{\rho}{2} \tag{26}$$

By substituting (23) and (25) into (26), the following is obtained

$$P_4 = \frac{(1-\rho)e^{\rho/2}}{2}(2e^{3\rho/2} - 2e^{\rho} - 2\rho e^{\rho/2} + \rho) \tag{27}$$

By using the expressions (10), (22), (23), (25) and (27), the numerical results given in Table 3 are obtained.

Table 3. The values of P_n ($n = 0,1,2,3,4$) for $g = 2$ as a function of utilisation factor ρ

ρ	P_0	P_1	P_2	P_3	P_4	$\sum_{i=0}^{4} P_i$
0.4	0.6	0.133	0.162	0.052	0.031	0.978
0.5	0.5	0.142	0.182	0.074	0.049	0.947
0.6	0.4	0.140	0.189	0.093	0.069	0.891
0.7	0.3	0.126	0.178	0.104	0.085	0.793
0.8	0.2	0.098	0.147	0.100	0.090	0.635
0.9	0.1	0.057	0.089	0.069	0.068	0.383

Finally, consider the case when *g* = *8* (see more in [10]). Then by (10), $P_0 = 1 - \rho$, and proceeding in the same manner as in the case when *g* = *6*, by using the expressions (4), (5), (8) and (9), we obtain

$$P_1 = (1-\rho)(e^{\rho/8} - 1), \tag{28}$$

$$P_2 = (1-\rho) e^{\rho/8} (e^{\rho/8} - 1), \tag{29}$$

$$P_3 = (1-\rho)e^{\rho/4}(e^{\rho/8}-1) \tag{30}$$

and

$$P_4 = P_3 e^{\rho/8} = (1-\rho)e^{3\rho/8}(e^{\rho/8}-1) \tag{31}$$

By using the expressions (10), (28) - (31), numerical results given in Table 4 are obtained.

Table 4. The values of P_n ($n = 0,1,2,3,4$) for $g = 8$ as a function of utilisation factor ρ

ρ	P_0	P_1	P_2	P_3	P_4	$\sum_{i=0}^{4} P_i$
0.4	0.6	0.031	0.032	0.034	0.073	0.770
0.5	0.5	0.032	0.034	0.037	0.080	0.683
0.6	0.4	0.031	0.034	0.036	0.081	0.582
0.7	0.3	0.027	0.030	0.033	0.075	0.465
0.8	0.2	0.021	0.023	0.026	0.060	0.330
0.9	0.1	0.012	0.013	0.015	0.035	0.175

Notice that the values in the last columns of Tables 1, 2, 3 and 4 denote the related probabilities that at most four automobiles are present at the considered queue system. These probabilities for the cases *g = 4* and *g = 2* are graphically presented as a function on the utilisation factor ρ $(0.4 \le \rho \le 0.9)$ in Figures 3 and 4, respectively.

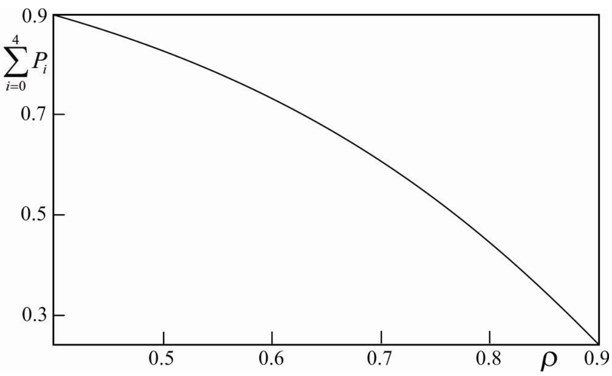

Figure 3. The sum $\sum_{i=0}^{4} P_i$ for the size $g = 4$ of arriving group as a function on ρ $(0.4 \le \rho \le 0.9)$

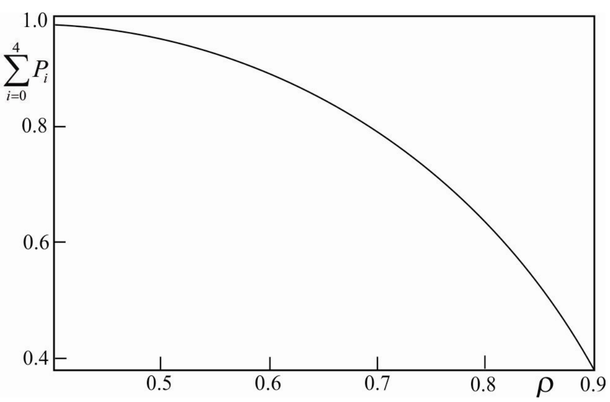

Figure 4. The sum $\sum_{i=0}^{4} P_i$ for the size $g = 2$ of arriving group as a function on ρ $(0.4 \le \rho \le 0.9)$

From Tables 1-4 we immediately obtain the numerical results given in Table 5.

Table 5. The values of $\sum_{i=1}^{4} P_i$ for $g = 2,4,6,8$ as a function of utilisation factor ρ

(ρ)	$g=2$	$g=4$	$g=6$	$g=8$
0.4	0.378	0.295	0.183	0.170
0.5	0.447	0.324	0.199	0.183
0.6	0.491	0.329	0.197	0.182
0.7	0.493	0.303	0.179	0.165
0.8	0.435	0.245	0.142	0.130
0.9	0.283	0.147	0.082	0.075

Notice that the values from Table 5 present the related probabilities that at least one and at most four automobiles are present at considered queue system.

4. CONCLUSION

This paper together with the recent papers [8-12] refers to the introductory research of the modeling processes at the SAT in the Port of Bar. In particular, we study the loading processes at seaside link of SAT as a queuing model with bulk arrivals. Analyzing some stochastic and deterministic characteristics of related loading operations, we propose a suitable queue model for describing arrivals and services of automobiles at the ship ramp (service). In this paper, we deduce an infinite system of linear algebraic equations for related steady-state probabilites. A suitable recurrence form of this system allows use to explicitly express these probabilities as a function of utilisation factor (service utilisation).

Notice that for possible further comparison analysis, in this paper we have obtained numerical and graphical results related to the mentioned steady-state probabilities of the considered queue model concerning four different values of the size of arriving group of automobiles in the considered queue model. In particular, our analytical and numerical results should be useful for determining the size of arriving group of automobiles *g* (which is in "port of Bar" case equal to 6) in the sense of optimizations of several important performances of modeling processes at the SAT in the Port of Bar.

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МОДЕЛИРАЊЕ ОПЕРАТИВНИХ ПРОЦЕСА СА ГРУПНИМ ДОЛАСЦИМА: АУТОМОБИЛСКИ ТЕРМИНАЛИ У ЛУКАМА

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У овом раду описан је процес укрцаја аутомобила на брод као модел редова чекања са групним доласцима аутомобила на бродској рампи. Овде су описане оперативне перформансе аутомобилског терминала у луци Бар у односу на разматрани модел реда чекања са групним одласцима. Коришћењем тог модела добијени су нумерички и графички резултати за вероватноће стања разматраног модела реда чекања. На основу добијених резултата разматрају се вредности вероватноћа стања система у зависности од основних перформанси моделрианог процеса на аутомобилском термниалу укључених у изразу за фактор искоришћења. За могућу даљу упоредну анализу добијени су нумерички и графички резултати за вероватноће стања моделираног система које се односе на четири различите вредности величине групе аутомобила који долазе на опслуживање, тј. процес укрцаја на брод преко бродске рампе.