Analysis of Dynamic Stress Intensity Factor of Finite Piezoelectric Compo–site Plate Under a Dynamic Load

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Piezoelectric materials are widely utilized in electronic device applications such as sensors. This paper investigates the dynamic response of a central crack in a finite piezoelectric layered composite plate under impact load. This novel project studies the crack effect on the finite geometries of a piezoelectric layered composite plate. A finite element method for transient dynamic crack analysis in two-dimensional, homogeneous, anisotropic and linear elastic composites was used for this investigation. Initially, the simulation method was verified and calibrated using a simple case study of a piezoelectric material ($BaTiO_3$) with a pre-crack. The results were in good agreement with previous studies in this field. Later, the verified model was used to simulate the piezoelectric layered composite plate. The results showed that the maximum dynamic stress intensity factor depends on the thickness and material properties of the piezoelectric layer and these are crucial parameters in the design and evaluation of a piezoelectric.

Keywords: *Fracture mechanics, Finite element method, Piezoelectric composite plate, Dynamic stress intensity factor, Dynamic load.*

1. INTRODUCTION

In the development of electromechanical systems and devices, smart structures with integrated functional ceramics are gaining growing interest. The electromechanical coupling behavior of piezo ceramics makes this class of materials useful in electronic device applications, particularly for transducers, sensors, etc. [1].

Piezoelectric ceramics, however, are very brittle and susceptible to fracture during service. Therefore, it is important to understand the fracture behavior of piezoelectric materials. In [2–4] anti-shear of cracked piezoelectric ceramics strip and in [5,6] cracked piezoelectric composites were investigated.

In the past years, extensive studies have presented valuable analytical solutions for the dynamic fracture problem in piezoelectric materials. For example, Shindo et al [7] dealt with the dynamic problem of an infinite orthotropic piezoelectric ceramic containing a Griffith crack under in-plane impact loading. Their results showed that for the piezoelectric materials, the dynamic stress intensity factor and the dynamic energy release rate exceed the corresponding static value by about 27-31% and 70- 88%, respectively. Chen et al. [8] studied the transient response of a piezoelectric strip with a vertical crack under electromechanical impact load. Their numerical computation of the resulting coupled dynamic equations revealed that the electric field retards or promotes the propagation of the crack in a piezoelectric strip, at different stages of the impact loading process, depending upon the crack length and the electric load. Wang and Noda [9] investigated the electro-elastic interaction and fracture behavior of a piezoelectric layer bonded to a dissimilar elastic layer. A novel time-domain BEM for transient dynamic crack analysis of linear piezoelectric solids was implemented by Garcia et al. [10,11]. The obtained results were more stable and less sensitive to the selected time-steps than the classical one.

Some differences were observed between experiments and analytical solutions in piezoelectric composite plates and one reason for the differences was the existence of defects such as crack which affect the response of piezoelectric sensors.

While many piezoelectric devices are constructed in laminated form, only a few studies have been made for the transient response of the piezoelectric layered composite plates. Ueda used an analytical method to study the dynamic behaviour of an infinite piezoelectric layered composite plate with a central crack under normal impact loading [12]. However, the piezoelectric sensors have finite dimensions, where no work has been done for the finite geometries.

The aim of this study is to investigate the effect of crack on the response of the piezoelectric layered composite plates with finite dimensions. The results would be useful to have an optimum sensor design and to consider the crack effects on the piezoeletric behaviour. In this work, the dynamic behviour of a piezoelectric layered composite plate with a central crack is examined under normal impact loading. The crack is oriented normally to the interfaces and ends at the interface between the orthotropic piezoelectric plate and the elastic layer. Finite element modelling is

Received: May 2016, Accepted: June 2016
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doi:10.5937/fmet1604348F

utilized to find dynamic stress intensity factor near the crack tip. The simulation method is checked and calibrated with a simple case study [13]. Later the numerical results for the dynamic stress intensity factor are obtained for several piezoelectric layered composite plates and are reported in the following sections.

2. MATERIALS AND METHODS

There are few studies in the simulation of the transient response of the piezoelectric layered composite plates. Therefore, at first, the simulation method is checked and calibrated using simple case studies, i.e. an anisotropic piezoelectric material ($BaTiO_3$) with a pre-crack. This case study is available in the previous study and it is possible to compare our results with the previously obtained results [13].

2.1 Characteristics of the sample plate

Characteristics of the case study that is used for validation of the simulation method are illustrated in Fig. 1. This case study is a piezoelectric ($BaTiO_3$) plate with a central through thickness crack of length 2a=4.8 mm in a rectangular plate with the dimensions $2b_1 \cdot 2b_2$= $(20 \cdot 40)mm^2$. Material properties and geometry of this case study are observable from table 2. As depicted in Fig. 2, mechanical load $\sigma_0(t)$ is applied at the opposite ends of the plate, following a ramp function with the time constant τ =1.e-6 s and σ_0 =5.0 MPa. Due to symmetries with respect to the plate in Fig. 2. is discretized by a two-dimensional finite element mesh, applying symmetry boundary conditions u_2=0 and on the ligament and u_1 =0 along the vertical symmetry line. A generalized state of plain strain ($u_{i,3}$=0) is assumed in the model.

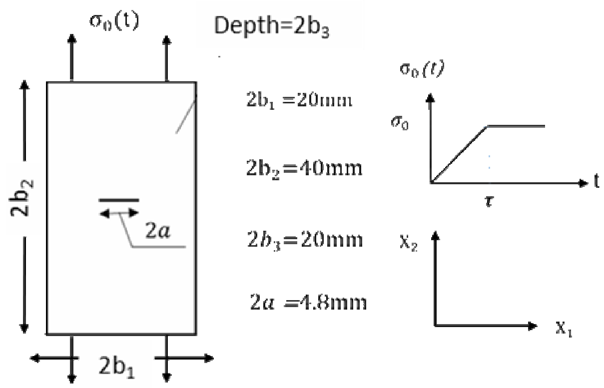


Figure 1. Geometry and characteristics of the piezoelectric ($BaTiO_3$) plate [13]

Two-dimensional FE models are used for simulation of this simulation. A sample view of the model with and without mesh elements is illustrated in fig. 2.

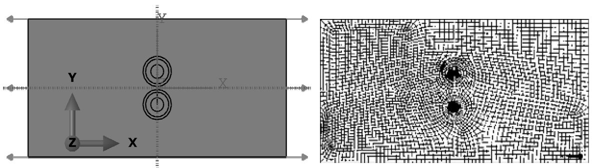


Figure 2. A sample view of the model with and without mesh elements

The results obtained from simulation of the piezoelectric ($BaTiO_3$) plate and a previous study by M. Enderlein [13], are indicated in fig. 3. Good agreement can be observed between our results and Enderlein's work.

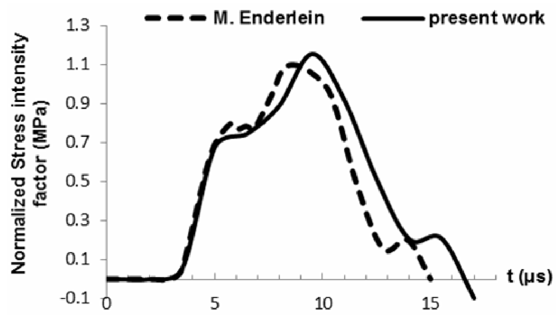


Figure 3. Comparison of the piezoelectric ($BaTiO_3$) plate and Enderlein's work

Percentage of errors of our results compared to Enderlein's work is listed in table 1. Where K_{max} is the maximum dynamic stress intensity factor and t_{max} is the required time to reach this stress intensity factor. The errors are small and acceptable.

Table 1. Percentage of errors of our results and Enderlein's work for the steel plate

	Kmax (MPa)	Tmax (µs)
Obtained results	1.1547	9.5
Enderlein's work	1.11	8.85
Percentage of the error (%)	4.027	7.344

After verification of the model using the simple case study, it is possible to apply it for the investigation of a finite piezoelectric layered composite plate. In the following, effect of the thickness and material properties of the piezoelectric layer on dynamic stress intensity factor and stress distribution are investigated.

2.2 Characteristics of the piezoelectric layered composite plate

The layered composite plate is made of a cracked piezoelectric layer of width $2h_l$ bonded between two elastic layers of width h_2 - h_l as shown in Fig 4. The ends of the crack length 2c ($c \leq h_l$) are situated at equal distances away from the interfaces which are directed normal to the crack plane. The system of rectangular Cartesian coordinates (x,y,z) is introduced in the composite in such a way that the crack is located centrally along the x-axis, and the z-axis is parallel to the interfaces. The piezoelectric layer is poled in the z-direction exhibiting transversely isotropic behaviour. A plane strain condition perpendicular to the y-axis is assumed [12].

Also, use of superposition reduces the problem to one of perturbation in which the crack surface impact tractions are the only nonzero external loads [14]. Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0 \leq x \leq h_2$ (h/2) and $0 \leq z \leq h$ only, therefore, we simulate the problem in $0 \leq x \leq h_2$. The piezoelectric composite plate is subjected to a tensile impact loading of the form $\bar{\sigma}_0(t)$, where $\bar{\sigma}_0(t)$ is 400 MPa and τ is 2 μs. The size of the cracked plate is defined by h=20 mm [15].

Table 2. Properties of materials [2]

	Elastic stiffnesses ($*10^{10}$ N/m^2)				Piezoelectric coefficients (C/m^2)			Dielectric constants ($*10^{-10}$ C/Vm)	
	c_{11}	c_{33}	c_{44}	c_{13}	e_{31}	e_{33}	e_{15}	ε_{11}	ε_{33}
PZT-4	13.9	11.3	2.56	7.43	-6.98	13.8	12.7	60	54.7
PZT-5H	12.6	11.7	2.3	8.41	-6.5	23.3	17	150.4	130
Al	8.84	8.84	2.7	3.43	0	0	0	-	-
$BaTiO_3$	15	14.6	4.4	6.6	-4.35	17.5	11.4	98.7	112

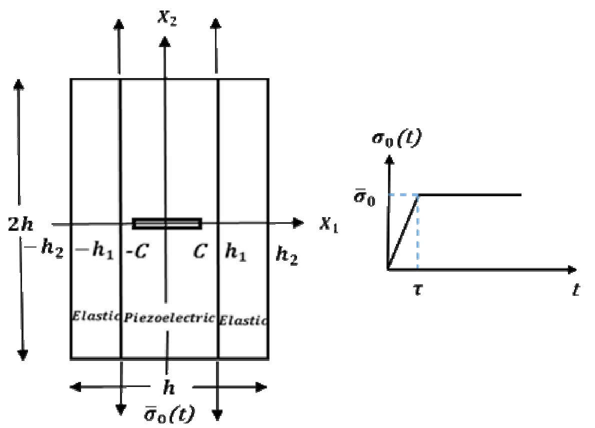


Figure 4. Geometry of the crack in the piezoelectric layered composite plate

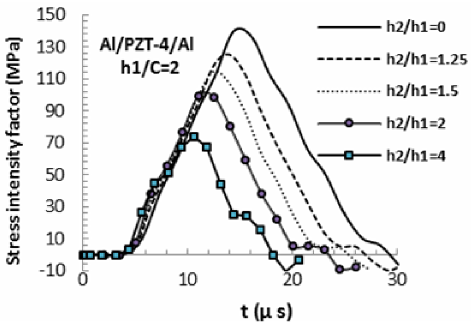
The elastic, piezoelectric and dielectric properties are listed in Table 2. The shear modulus and Poisson's ratio of aluminium are μ = c44 and ν = 0.28, respectively.

3. NUMERICAL RESULTS AND DISCUSSION

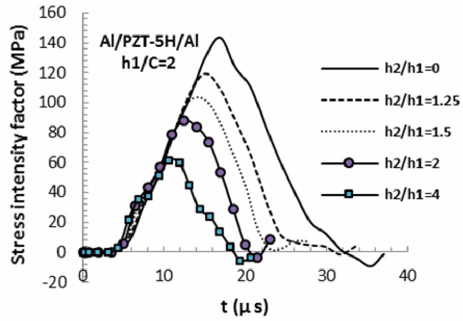
3.1 The thickness effect of the piezoelectric

The thickness effect of piezoelectric layer on stress intensity factor are considered for three cases: 1) Al/PZT-4/Al, 2) Al/PZT-5H/Al and 3) Al/$BaTiO_3$/Al laminates with $h_1 / c = 2$. The dynamic stress intensity factor of the piezoelectric composite plates versus time obtained from the simulation is shown in Figure 6.

In figures 5 and 6, stress intensity factor versus time, for infinite piezoelectric composite plates, are illustrated and are compared with the results of this paper. From the figures, it can be seen that at the start the infinite piezoelectric composite plate and finite composite plate, i.e. the results of this paper, has similar increasing trend. For both cases, after reaching a maximum value, their trends become descending, but, for the infinite plate, the stress intensity factor never takes zero value and after small decrease it becomes constant.



(a)



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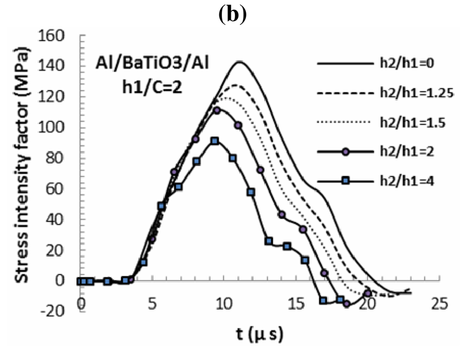

(c)

Fig. 5. Dynamic stress intensity factor of piezoelectric composite plates versus time for h_1/c=2

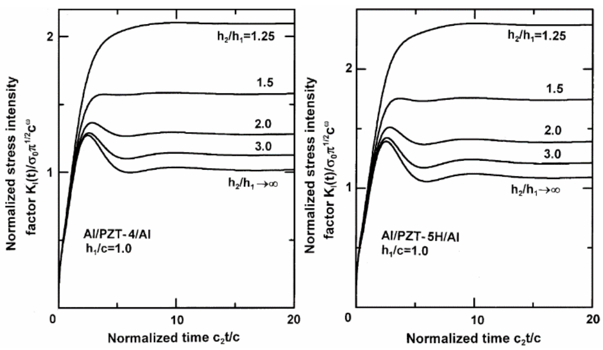

Fig. 6: Normalized stress intensity factor versus time for infinite piezoelectric composite plates with different thicknesses [12]

In the third section, the effect of h_2/h_1 on the stress intensity factor of the broken laminate is considered. It can be seen that the values of K_I^{max} and t_{max} increase with decreasing h_2/h_1.

In figures 5 and 6, stress intensity factor versus time, for infinite piezoelectric composite plates, are illustrated and are compared with the results of this paper. From the figures it can be seen that at the start for the infinite piezoelectric composite plate and finite composite plate, there is a similar increasing trend and for both cases, after reaching a maximum value, their trends decrease. But, in the infinite plate, the stress intensity factor never takes zero value and, after a small decrease, it becomes constant.

3.2 The material effect of the piezoelectric

The effect of piezoelectric properties on the dynamic stress intensity factor is investigated for three cases with $h_2/h_1=2$ and $h_1/c=2$ and the results are shown in Fig. 7.

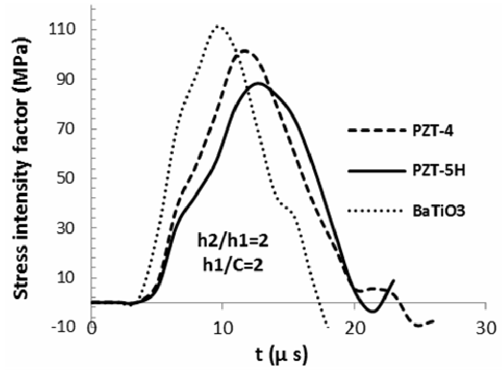

Fig. 7. The effect of the piezoelectric material properties on the dynamic stress intensity factor for h1/c=2 & h_2/h_1=2

It can be seen from the figure that the values of K_I^{max} and t_{max} are changing using different material properties in the piezoelectric layer. Note that the magnitudes of the dynamic stress intensity factor of the case studies decrease suddenly by increasing h_2/h_1 .

The sudden release of the uniaxial stress at the ends results in arrival of stress waves to the crack surface after a finite period of time. This period depends on the geometry of the problem and the material properties.

4. CONCLUSIONS

In this paper, the dynamic response of a piezoelectric layered composite plate having a central crack was investigated. The finite element method was used to simulate the dynamic response of infinite piezoelectric composite plates. The dynamic stress intensity factor versus time were evaluated and were compared with the finite piezoelectric composite results. The results show that dynamic stress intensity factor depends on the thickness and material properties of the piezoelectric layer. The results revealed that at the start of the loading, the infinite piezoelectric composite plate and the finite composite plate has similar increasing trend and, for both cases, after reaching a maximum value, their trends become decreasing. But for the infinite plate, the stress intensity factor never takes zero value and after a small decrease it becomes constant. Broadly speaking, the results indicated that the maximum dynamic stress intensity factor depends on the thickness and material properties of the piezoelectric layer under dynamic load. Therefore, these factors should be considered carefully in design and evaluation of the piezoelectric.

REFERENCES

[1] Rao, S.S. and Sunar, M.: Piezoelectricity and its use in disturbance sensing and control of flexible structures: a survey, Applied Mechanics Review, vol. 47, pp. 113–123, 1994.

[2] Shindo, Y., et al.: Electroelastic intensification near anti-plane shear crack in orthotropic piezoelectric ceramic strip, Theoretical and Applied Fracture Mechanics vol. 25, pp.65–71, 1996.

[3] Shindo, Y., Tanaka, K. and Narita, F.: Singular Stress and electric fields of a piezoelectric ceramic strip with a finite crack under longitudinal shear, Acta Mechanica, vol.120, pp.31–45, 1997.

[4] Shindo, Y., Watanabe, K. and Narita, F.: Electroelastic analysis of a piezoelectric ceramic strip with a central crack, International Journal of Engineering Science, vol. 38, pp.1–19, 2000.

[5] Narita, F. and Shindo, Y.: Layered piezoelectric medium with interface crack under anti-plane shear", Theoretical and Applied Fracture Mechanics, vol. 30, pp. 119–126, 1998.

[6] Narita F. and Shindo, Y.: The interface crack problem for bonded piezoelectric and orthotropic layers under antiplane shear loading, International Journal of Fracture vol. 98, 1999.

[7] Shindo, Y., Narita, F. and Ozawa. E.: Impact response of a finite crack in an orthotropic piezoelectric ceramic, Acta Mech, vol. 137, pp.99–107, 1999.

[8] Chen, Z.T. and Meguid. S.A.: The transient response of a piezoelectric strip with a vertical crack under electromechanical impact load, Int J Solids Struct vol. 37, pp. 6051–62, 2000.

[9] Wang, B.L. and Noda, N.: A crack in a piezoelectric layer bonded to a dissimilar elastic layer under transient load, Arch Appl Mech, vol. 71, pp. 487–94, 2001.

[10] Garcia-Sanchez, F., Zhang, Ch., Sla´dek, J. and Sla´dek, V.: 2D transient dynamic crack analysis in piezoelectric solids by BEM, Comput Mater Sci, vol. 39, pp. 179–86 2007.

[11] Garcia-Sanchez, F., Zhang, Ch. and Sa´ez Andre s.: 2-D transient dynamic analysis of cracked piezoelectric solids by a time-domain BEM. Comput Methods Appl Mech Eng, vol. 197, pp. 3108–21, 2008.

[12] Ueda, S.: Impact response of a piezoelectric layered composite plate with a crack, Theoretical and Applied Fracture Mechanics, vol. 38, pp. 221–242, 2002.

[13] Enderlein M., Ricoeur A. and Kuna M.: Finite element techniques for dynamic crack analysis in piezoelectrics, International Journal of Fracture, vol. 134, pp.191–208 2005.

[14] Lei, J., Garcia-Sanchez, F. and Zhang, Ch.: Transient response of an insulating crack near to the interface between two piezoelectric half-planes under electromechanical impacts by BEM, Engineering Analysis with Boundary Elements vol. 36, pp.1205–1212 2012.

[15] Garcia-Sanchez, F., Zhang, Ch. and Sa´ez Andre s.: A two-dimensional time-domain boundary element method for dynamic crack problems in anisotropic solids, Engineering Fracture Mechanics, vol. 75, pp. 1412–1430, 2008.

АНАЛИЗА ФАКТОРА ИНТЕЗИТЕТА ДИНАМИЧКОГ НАПОНА КОНАЧНЕ ПИЕЗО–ЕЛЕКТРИЧНЕ КОМПОЗИТНЕ ПЛОЧЕ ПОД УТИЦАЈЕМ ДИНАМИЧКОГ ОПТЕРЕЋЕЊА

С. Фотухи, С. Мохамед Реза Халили

Пиезоелектричне материјали се широко користе у применама електронских уређаја као што су сензори. Овај рад истражује динамички одговор централне пукотине у коначном пиезоелектричном слоју композитне плоче под утицајем оптерећења. Овај рад проучава утицај пукотине на коначну геометрију пиезоелектричних слојева композитне плоче. Метода коначних елемената за транзитну динамичку анализу пукотина у дводимензионалном, хомогеном, анизотропном и линеарно еластичном композиту је коришћена за ово истраживање.

У почетку, метод симулација је проверен и калибрисан коришћењем једноставаног случаја пиезоелектричног материјала (BaTiO3) са иницијалном прслином. Резултати су били у доброј сагласности са претходним студијама у овој области. Касније, потврђени модел је коришћен за симулацију пиезоелектричне слојевите композитне плоче.

Резултати су показали да максимални динамички фактор интензитета напона зависи од дебљине и материјалних својстава пиезоелектричног слоја и то су кључни параметри у дизајну и евалуације пиезоелектичних материјала.