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Kinematic Modeling of 5-axis Horizontal Milling Machine Emulated From Vertical Articulated Robot

Industrial robots are a proper alternative to 5-axis machine tools for multi-axis milling of parts from softer materials and lower tolerances. For these reasons, researchers develop software solutions to bring programming of machining robots closer to programming of machine tools. The paper presents the results of kinematic modeling of vertical articulated industrial robots to emulate 5-axis horizontal machine tools. The solution to the inverse kinematics problem was the basis for the analysis of the machining robot workspace. Starting from equations of direct and inverse kinematics the control and programming system in G-code was developed, based on EMC2 software system. Developed control and programming system enables use of the available CAD/CAM systems for programming 5-axis machine tools, i.e. it is directly applicable by programmers and operators experienced in CNC technologies and G-code programming. Verification of kinematic modeling and developed control and programming system was carried out by machining experiments on an available domestic 6-axis robot LOLA 50.

Keywords: *robotic machining, machinig tools, kinematic modeling, control system, programming, G-code.*

1. INTRODUCTION

Industrial robots, compared to 5-axis machine tools, are characterized by substantially lower cost, large workspace and high flexibility. Consequently, intensive research and development of robots for multi-axis milling, primarily of complex, large dimension parts from softer materials and lower tolerances, are under way. The application of industrial robots for this class of tasks is today widely recognized as an alternative to relatively costly 5-axis machine tools [1-4]. Intensive studies [2,5] conducted in the world are related to solving two groups of problems that represent major limiting factors for broader application of robotic machining. These problems embrace the complexity of programming [4] and unsatisfactory stiffness of the robot serial structure [6].

The complexity of programming the machining robot is basically the outcome of lack of cooperation of robot makers compared to machine tools makers who were willing to utilize CAD/CAM systems based on ISO 6983 standard i.e. G-code. It is for these reasons that robot and CAD/CAM software manufacturers today develop softwares for translating G-code generated from avalilable CAM software and appropriate postprocessors for 5-axis machine tools into the robot programming languages. Another direction of customizing the existing robot programming languages

for machining applications is the development of postprocessors for them. These approaches enable to some extent to bring programming of machining robot closer to programming of multi-axis machine tools [4,7].

This paper presents the approach of kinematic modeling of industrial robots with the aim of emulating 5-axis horizontal and vertical machine tools. Starting from the kinematic modeling, i.e. equations of direct and inverse kinematics, the control and programming system was developed in G-code based on PC real-time Linux platform and EMC2 software system for control of machine tools, robots and parallel kinematic machine tools [8]. The thus developed control and programming system enables use of the current CAD/CAM systems for programming 5-axis machine tools based on G-code, i.e. such system is directly applicable by programmers and operators experienced in CNC technology and programming in G-code.

2. EXAMPLES OF EMULATED MILLING MACHINE TOOLS

In addition to robots designed today by renowned robot makers for tasks of multi-axis machining of complex parts [9] these tasks are also performed by the existing vertical articulated robots of 5 and 6 DOF, with or without an auxiliary axis, equipped with a motor spindle.

Multi-axis robotic machining, i.e. 3- to 5-axis machining, performed on 5-axis and 6-axis vertical articulated robots, Figure 1, may include the following cases [7,10]:

- 3-to 5-axis machining with 5-axis vertical articulated robots,
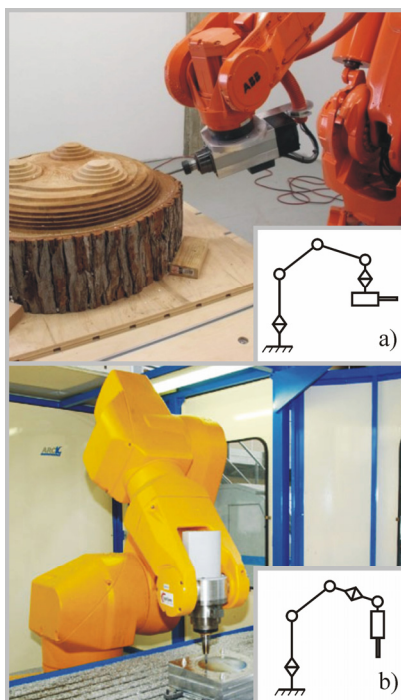
- 3-to 5-axis machining with 5-axis vertical articulated robots emulated from standardized 6-axis robots, where the axes of the fourth or sixth joint are blocked, and
- 3-to 5-axis machining with standardized 6-axis vertical articulated robots, where all robot's axes are active during machining and with one tool orientation angle, not included in 5-axis machining, being kept constant.
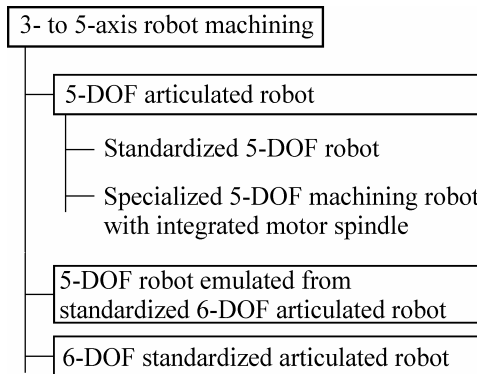

Figure 1. Three to 5-axis machining with industrial robots

Three to 5-axis machining with 5-axis vertical articulated robots can utilize standardized or specialized 5-axis machining robots, Figure 1. Examples of standardized and specialized 5-axis vertical articulated robot for 3- to 5-axis machining are shown in Figure 2. As can be seen, a standardized 5-axis robot, Figure 2a, due to common arrangement of orientation axes actually emulates 5-axis horizontal machine tool. A specialized 5-axis machining robot, Figure 2b, unlike the standardized one, has a specific arrangement of orientation axes so that it emulates a 5-axis vertical machine tool. Such emulated 5-axis machine tools belong to the class of machine tools in which tool orientation is realized by its rotations [11, 12].

Figure 2. Standardized and specialized 5-axis machining robots [7]

In this paper, emulation of standardized and specialized 5-axis robots, i.e. 5-axis vertical and horizontal machine tools, utilized available domestic 6-axis robot LOLA 50, Figure 3, which also shows appropriate interface plates for the interface of the motor spindle. Figures 3a and 3b show the possibilities of emulating these two types of 5-axis robots from a standardized 6-axis vertical articulated robot by blocking a corresponding robot's axis.
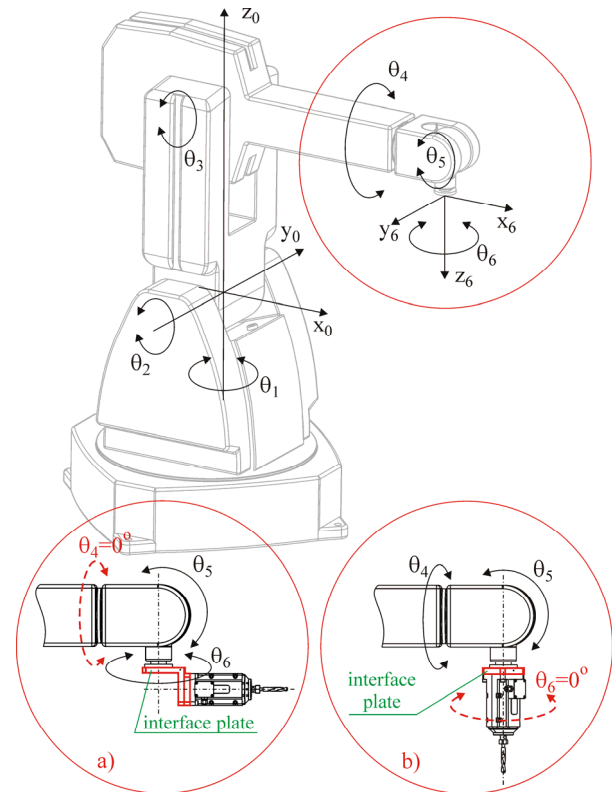
Figure 3. Six-axis vertical articulated robot with appropriate interface plates

Five-axis robot in Figure 3a, with blocked fourth axis (θ_4), represents an emulated standardized robot from Figure 2a. Five-axis robot in Figure 3b, with blocked sixth axis (θ_6) of a 6-axis robot, represents an emulated specialized machining robot from Figure 2b. Further, from a standardized 6-axis robot, Figure 3, or from emulated different types of 5-axis robots, Figures 3a and 3b, different types of 5-axis vertical and/or horizontal machine tools can be emulated, with the aim of programming them by using available CAD/CAM systems and G-code. Emulation of 5-axis horizontal and vertical machine tools, where tool orientation is realized by its rotations, from a standardized 6-axis robot included the following cases:

- 5-axis horizontal milling machine (X, Y, Z, A, B), by blocking robot's fourth axis ($\theta_4 = 0°$) and
- 5-axis vertical milling machine (X,Y,Z,A,B), by blocking robot's sixth axis ($\theta_6 = 0°$).

An example of emulating 5-axis vertical milling machine (X,Y,Z,A,B), Figure 3b, is given in [4], whereas this paper presents the kinematic modeling and development of control and programming system for 5-axis horizontal milling machine (X,Y,Z,A,B) emulated from the 6-axis robot.

3. KINEMATIC MODELING

As aforementioned, the available experimental 6-axis robot with blocked fourth axis was observed as 5-axis horizontal milling machine (X,Y,Z,A,B).

In order to develop the open architecture control system with programming in G-code, a kinematic model of 5-axis horizontal milling machine emulated from a 6-axis robot is required. In this paper, kinematic modeling includes solving the direct and inverse kinematics problem and analysis of workspace for the cases of 3- and 5-axis machining.

3.1 Joint and world coordinates

A geometric model of 5-axis horizontal milling machine emulated from 6-axis robot is shown in Figure 4. The reference frame {M} is adopted according to the standard for 5-axis horizontal milling machines (X,Y,Z,A,B) [11]. The tool frame {T} is adopted to have the origin coinciding with the milling tool tip, T, where axis z_T coincides with the tool axis. The workpiece frame {W} is arbitrarily adopted on the workpiece. Vectors **v** referenced in frames {M} and {W} are denoted as M_V and W_V.
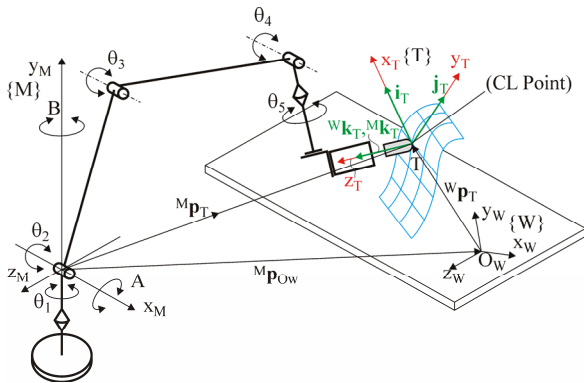
Figure 4. Tool position and orientation in the work piece frame {W} and robot reference frame {M}

The vector of joint coordinates is represented as

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \end{bmatrix}^T \quad (1)$$

where $\theta_i,\ i = 1,2,...,5$, are scalar variables actuated and controlled by actuators.

CAD/CAM systems calculate the tool path defined by successive tool positions and orientations in the workpiece frame, Figure 4. The calculated tool path is machine independent. Tool position is defined by the position vector of tool tip, T, in the workpiece frame as

$$^W\mathbf{p}_T = \begin{bmatrix} x_t & y_t & z_t \end{bmatrix}^T \quad (2)$$

while tool orientation is given by the unit vector of the tool axis as

$$^W\mathbf{k}_T = \begin{bmatrix} k_{tx} & k_{ty} & k_{tz} \end{bmatrix}^T \quad (3)$$

As the axes of reference frames {M} and {W} need not be parallel, the tool tip position vector and unit vector of tool axis can be expressed in the reference frame {M} as

$$^M\mathbf{p}_T = \begin{bmatrix} x_M & y_M & z_M \end{bmatrix}^T = {}^M\mathbf{p}_{Ow} + {}^M_W R \cdot {}^W\mathbf{p}_T \quad (4)$$

$$^M\mathbf{k}_T = \begin{bmatrix} k_{Tx} & k_{Ty} & k_{Tz} \end{bmatrix}^T = {}^M_W R \cdot {}^W\mathbf{k}_T \quad (5)$$

where

$$^M\mathbf{p}_{Ow} = \begin{bmatrix} x_{Ow} & y_{Ow} & z_{Ow} \end{bmatrix}^T \quad (6)$$

is the position vector of the origin of the workpiece frame {W}. The determination of the position vector $^M\boldsymbol{p}_{Ow}$, and orientation of the workpiece frame {W} are conducted according to the standard procedure for 5-axis machine tools and thereafter orientation matrix $^M_W R$, in equations (4) and (5), is calculated and executed in the control system.

Given that the robot has 5 DOF, only the direction of the axis z_T is controllable, while axes x_T and y_T have uncontrollable rotation about the axis z_T. The description and orientation of the frame {T} with respect to the frame {M} can be defined by homogeneous transformation matrices [13,14] as

$$^M_T T = \left[\begin{array}{ccc|c} & ^M_T R & & ^M\mathbf{p}_T \\ \hline 0 & 0 & 0 & 1 \end{array} \right] =$$

$$= \left[\begin{array}{ccc|c} i_{Tx} & j_{Tx} & k_{Tx} & x_M \\ i_{Ty} & j_{Ty} & k_{Ty} & y_M \\ i_{Tz} & j_{Tz} & k_{Tz} & z_M \\ \hline 0 & 0 & 0 & 1 \end{array} \right] \quad (7)$$

where rotation matrix $^M_T R$ represents the orientation, while vector $^M\boldsymbol{p}_T$ represents the position of the frame {T} with respect to the reference frame {M}. To bring the tool axis, i.e. the z_T axis of the frame {T}, to a desirable angular position relative to the frame {M}, the tool frame {T} must be first rotated about the axis y_M by the angle B, and then about the axis x_M by the angle A, as prescribed by the convention for 5-axis milling machine (X, Y, Z, A, B). The rotation matrix $^M_T R$ is derived as

$$^M_T R = R_{x_M, A} \cdot R_{y_M, B} =$$

$$= \begin{bmatrix} cB & 0 & sB \\ sA \cdot sB & cA & -sA \cdot cB \\ -cA \cdot sB & sA & cA \cdot cB \end{bmatrix} = \begin{bmatrix} i_{Tx} & j_{Tx} & k_{Tx} \\ i_{Ty} & j_{Ty} & k_{Ty} \\ i_{Tz} & j_{Tz} & k_{Tz} \end{bmatrix} \quad (8)$$

where matrices

$$R_{x_M, A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cA & -sA \\ 0 & sA & cA \end{bmatrix} \text{ i}$$

$$R_{y_M, B} = \begin{bmatrix} cB & 0 & sB \\ 0 & 1 & 0 \\ -sB & 0 & cB \end{bmatrix} \quad (9)$$

represent so-called basic rotation matrices [14]. In these matrices, "c" and "s" denote sine and cosine function of the angles A and B.

Starting from the fact that it is of significance only the tool axis direction z_T determined by the unit vector $^M\mathbf{k}_T$, equation (5), by equating the corresponding members of matrix ^M_TR from equation (8), the angles A and B can be determined as

$$B = A\tan 2(k_{Tx}, \sqrt{1-k_{Tx}^2}) \qquad (10)$$

$$A = A\tan 2(\frac{-k_{Ty}}{cB}, \frac{k_{Tz}}{cB}) \qquad (11)$$

Although there is another solution in equation (10), by using the positive square root the single solution for $-90° \leq B \leq 90°$ is always calculated [12]. This way, the world coordinate vector is completely determined as

$$\mathbf{x} = \begin{bmatrix} x_M & y_M & z_M & A & B \end{bmatrix}^T \qquad (12)$$

3.2 Direct and inverse kinematics

The Denavit-Hartenberg (D-H) approach [4,13-15] was used to model the industrial robot. To perform kinematic analysis, first, coordinate frames are rigidly attached to each link. Figure 5 shows D-H coordinate frames for the robot in the reference position.
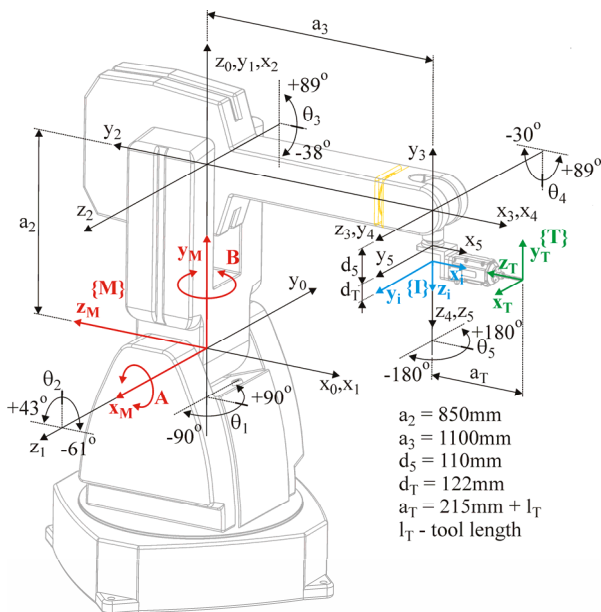
Figure 5. D-H coordinate frames for machining robot in the reference position

Relative position between coordinate frames can be described by homogeneous transformations. The homo–geneous transformation that describes the position of one relative to another segment is traditionally referred to as matrix A. The matrix $^{i-1}_iA$ designates D-H trans–formation matrix that describes the position of the frame {i} relative to the frame {i - 1} and can be represented as

$$^{i-1}_iA = \begin{bmatrix} c\theta_i & -c\alpha_i \cdot s\theta_i & s\alpha_i \cdot s\theta_i & a_i \cdot c\theta_i \\ s\theta_i & c\alpha_i \cdot c\theta_i & -s\alpha_i \cdot c\theta_i & a_i \cdot s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (13)$$

where θ_i, d_i, a_i and α_i are D-H kinematic parameters for describing relative transformations between segments i and i - 1. D-H kinematic parameters for the robot from Figure 5 are given in Table 1.

Table 1. D-H kinematic parameters

Link i	a_i[mm]	α_i[°]	d_i[mm]	θ_i[°]
1	0	90	0	θ_1
2	a_2	0	0	θ_2+90
3	a_3	0	0	θ_3-90
4	0	90	0	θ_4
5	0	0	d_5	θ_5

Constraints in motion range of robot joints, shown in Figure 5, are specified with respect to adopted robot's reference position and represent smaller motion ranges than are the real robot's ranges of motion. Thus adopted constraints of motion range of robot joint axes, as above mentioned, provide: (i) conveniences related to robot stiffness, (ii) taking into account only one solution of inverse kinematics and (iii) avoiding singularity inside the adopted workspace.

Substituting D-H parameters from Table 1 into matrix $^{i-1}_iA$, equation (13), transformation matrices were generated, $^{i-1}_iA$ where i = 1,2,...,5.

Given that the robot is observed as a 5-axis horizontal milling machine (X, Y, Z, A, B), the tool axis must coincide with the axis z_T of the frame {T}. As evident from Figure 5, the position of the frame {T} can be described with respect to the frame (x_5, y_5, z_5) by the homogeneous transformation matrix ^5_TT as

$$^5_TT = {^5_IT} \cdot {^I_TT} =$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_t \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & -1 & a_t \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \qquad (14)$$
$$= \begin{bmatrix} 0 & 0 & -1 & a_t \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_t \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In this case, it should be noted that the coordinate frame of the interface plate {I} was introduced to simplify solving the inverse kinematics problem of the 5-axis horizontal machine tool.

Also, from Figure 5 the position of the coordinate frame {0} can be described with respect to the co–ordinate frame {M} by the homogeneous transfor–mation matrix ^M_0T as

$$^M_0T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (15)$$

Direct kinematics problem. As it is well-known [13,14], tool position and orientation, i.e. frame {T}

position and orientation, relative to the robot's reference frame {M} for the given joint coordinate vector $\boldsymbol{\theta}$ and specified parameters for segments can be determined as

$$
{}^M_T T = {}^M_0 T \cdot {}^0_1 A \cdot {}^1_2 A \cdot {}^2_3 A \cdot {}^3_4 A \cdot {}^4_5 A \cdot {}^5_T T =
$$

$$
= \begin{bmatrix} {}^M_T R & \vdots & {}^M\mathbf{p}_T \\ \hline 0\ \ 0\ \ 0 & \vdots & 1 \end{bmatrix} = \begin{bmatrix} i_{Tx} & j_{Tx} & k_{Tx} & \vdots & x_M \\ i_{Ty} & j_{Ty} & k_{Ty} & \vdots & y_M \\ i_{Tz} & j_{Tz} & k_{Tz} & \vdots & z_M \\ \hline 0 & 0 & 0 & \vdots & 1 \end{bmatrix} \quad (16)
$$

where

$$
\begin{aligned}
i_{Tx} &= s_1 \cdot c_{234} \cdot s_5 + c_1 \cdot c_5 \\
i_{Ty} &= -s_{234} \cdot s_5 \\
i_{Tz} &= c_1 \cdot c_{234} \cdot s_5 - s_1 \cdot c_5 \\
j_{Tx} &= s_1 \cdot s_{234} \\
j_{Ty} &= c_{234} \\
j_{Tz} &= c_1 \cdot s_{234} \\
k_{Tx} &= s_1 \cdot c_{234} \cdot c_5 - c_1 \cdot s_5 \\
k_{Ty} &= -s_{234} \cdot c_5 \\
k_{Tz} &= c_1 \cdot c_{234} \cdot c_5 + s_1 \cdot s_5 \\
x_M &= -a_t \cdot k_{Tx} - \\
&\quad - s_1 \cdot ((d_t + d_5) \cdot s_{234} + a_3 \cdot c_{23} - a_2 \cdot s_2) \\
y_M &= -a_t \cdot k_{Ty} - \\
&\quad - (d_t + d_5) \cdot c_{234} + a_3 \cdot s_{23} + a_2 \cdot c_2 \\
z_M &= -a_t \cdot k_{Tz} - \\
&\quad - c_1 \cdot ((d_t + d_5) \cdot s_{234} + a_3 \cdot c_{23} - a_2 \cdot s_2)
\end{aligned} \quad (17)
$$

and where $\theta_{ij} = \theta_i + \theta_j$.

In equation (17), in addition to the position of tool tip in the reference frame {M}, ${}^M\mathbf{p}_T = [x_M\ y_M\ z_M]^T$ the third column of the rotation matrix ${}^M_T R$ is also cal–culated. As previously mentioned, given that the robot has 5 DOF, only the direction of the axis z_T is controllable, while axes x_T and y_T have uncontrollable rotation about the axis z_T. This means that from the vector ${}^M\mathbf{k}_T = [x_M\ y_M\ z_M]^T$, equation (17), angles A and B can be calculated using equations (10) and (11). This way, the world coordinate vector, equations (12), is completed and direct kinematics problem solved.

Inverse kinematics problem. Robot inverse kine–matics problem will be solved partly in geometric terms and partly in algebraic terms.

As above mentioned, since the robot has 5 DOF, only the direction of the axis z_T is controllable, i.e. the tool vector ${}^M\mathbf{k}_T$, which means that the inverse kine–matics problem can be solved only by virtue of vectors ${}^M\mathbf{p}_T$ and ${}^M\mathbf{k}_T$. For this reason, in order to make solving the inverse kinematics problem less complex, the coordinate frame of the interface plate {I} was introduced, because it is reachable using vectors ${}^M\mathbf{p}_T$ and ${}^M\mathbf{k}_T$. It is evident from Figure 6 that the position of point I is influenced by joint coordinates θ_1, θ_2, θ_3 and θ_4. For the specified position vector ${}^M\mathbf{p}_T$ and specified tool orientation, i.e. angles A and B, the rotation matrix ${}^M_T R$ is first calculated using equation (8). Thereafter,

using vector ${}^M\mathbf{k}_T$ from calculated rotation matrix ${}^M_T R$, the position vector of the point I is calculated as

$$
{}^M\mathbf{p}_I = \begin{bmatrix} p_{xi} \\ p_{yi} \\ p_{zi} \end{bmatrix} = {}^M\mathbf{p}_T + a_t \cdot {}^M\mathbf{k}_T =
$$

$$
= \begin{bmatrix} x_M + a_t \cdot sB \\ y_M - a_t \cdot sA \cdot cB \\ x_M + a_t \cdot cA \cdot cB \end{bmatrix} \quad (18)
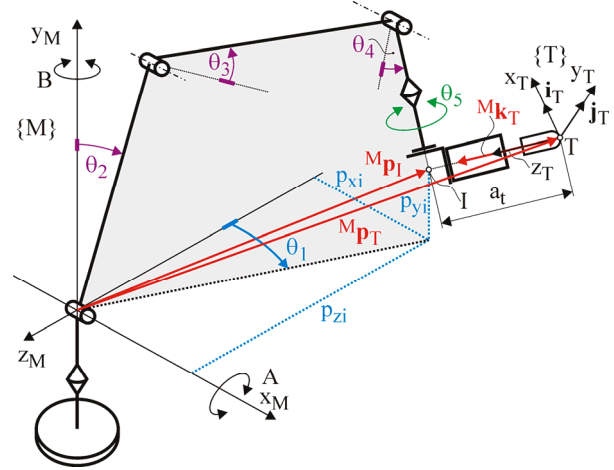$$

Figure 6. Projection of the vector ${}^M\mathbf{p}_I$ onto $(x_M, -z_M)$ plane

Observing the projection of point I, Figure 6, in the plane $(x_M, -z_M)$ and given the robot reference position, angle θ_1 is geometrically calculated as

$$
\theta_1 = A\tan 2(-p_{xi}, -p_{zi}) \quad (19)
$$

Angles θ_2, θ_3, θ_4, and θ_5 are solved in algebraic terms applying the matrix inversion method of matrices A. Given that the angle θ_1 is determined, equation (16) can be written as

$$
{}^M_T T = {}^M_1 T \cdot {}^1_T T = \begin{bmatrix} i_{Tx} & j_{Tx} & k_{Tx} & \vdots & x_M \\ i_{Ty} & j_{Ty} & k_{Ty} & \vdots & y_M \\ i_{Tz} & j_{Tz} & k_{Tz} & \vdots & z_M \\ \hline 0 & 0 & 0 & \vdots & 1 \end{bmatrix} \quad (20)
$$

where matrix ${}^M_1 T$ can be calculated as

$$
{}^M_1 T = {}^M_0 T \cdot {}^0_1 A = \begin{bmatrix} -s_1 & 0 & c_1 & \vdots & 0 \\ 0 & 1 & 0 & \vdots & 0 \\ -c_1 & 0 & -s_1 & \vdots & 0 \\ \hline 0 & 0 & 0 & \vdots & 1 \end{bmatrix} \quad (21)
$$

and matrix ${}^1_T T$ as

$$
{}^1_T T = {}^1_2 A \cdot {}^2_3 A \cdot {}^3_4 A \cdot {}^4_5 A \cdot {}^5_T T = \begin{bmatrix} * & * & -c_5 \cdot c_{234} & \vdots & * \\ * & * & -c_5 \cdot s_{234} & \vdots & * \\ * & * & -s_5 & \vdots & * \\ \hline 0 & 0 & 0 & \vdots & 1 \end{bmatrix} \quad (22)
$$

using the joint coordinate vector $\boldsymbol{\theta}$.

Since angle θ_1 is geometrically solved and in further procedure of solving the inverse kinematics problem it

represents known quantity, using equations (20), matrix $_T^1T$ can be also expressed via the world coordinate vector and angle θ_1 as

$$_T^1T = {}_1^MT^{-1}(\theta_1) \cdot {}_T^MT(\mathbf{x}) \tag{23}$$

that is, as

$$_T^1T = \begin{bmatrix} * & * & -s_1 \cdot k_{Tx} - c_1 \cdot k_{Tz} & | & * \\ * & * & k_{Ty} & | & * \\ * & * & c_1 \cdot k_{Tx} - s_1 \cdot k_{Tz} & | & * \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \tag{24}$$

Elements of matrices in previous equations desig–nated with asterisks are not shown because they are not needed for solving the inverse kinematics problem.

Equating the corresponding members of matrices from equations (22) and (24) and taking into account that $-30° \le \theta_5 \le 89°$, Figure 5, angle θ_5 can be calculated as

$$\theta_5 = A\tan 2(s_5, c_5) \tag{25}$$

where

$$s_5 = -c_1 \cdot k_{Tx} + s_1 \cdot k_{Tz} \tag{26}$$

and

$$c_5 = \sqrt{1 - s_5^2} \tag{27}$$

Also, equating the corresponding members of mat–rices from equations (22) and (24), it is obtained that

$$s_{234} = -\frac{k_{Ty}}{c_5} \tag{28}$$

and

$$c_{234} = \frac{s_1 \cdot k_{Tx} + c_1 \cdot k_{Tz}}{c_5} \tag{29}$$

that is, the sum of angles θ_2, θ_3 and θ_4 can be calculated as

$$\theta_{234} = A\tan 2(s_{234}, c_{234}) \tag{30}$$

Starting again from equation (16), given that angles θ_2 i θ_5 are now solved, it can be written as

$$_T^MT = {}_1^MT \cdot {}_I^1T \cdot {}_T^IT = \begin{bmatrix} i_{Tx} & j_{Tx} & k_{Tx} & | & x_M \\ i_{Ty} & j_{Ty} & k_{Ty} & | & y_M \\ i_{Tz} & j_{Tz} & k_{Tz} & | & z_M \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \tag{31}$$

where matrix $_1^MT$ is calculated according to equation (21). Matrix $_T^IT$ is defined when deriving equation (14), while matrix $_I^1T$ is calculated as

$$_I^1T = {}_2^1A \cdot {}_3^2A \cdot {}_4^3A \cdot {}_5^4A \cdot {}_I^5T =$$
$$= \begin{bmatrix} * & * & * & | & s_{234} \cdot (d_5 + d_t) + a_3 \cdot c_{23} - a_2 \cdot s_2 \\ * & * & * & | & -c_{234} \cdot (d_5 + d_t) + a_3 \cdot s_{23} + a_2 \cdot s_2 \\ * & * & * & | & 0 \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \tag{32}$$

In this case, it should be noted that all matrices are generated via the joint coordinate vector $\boldsymbol{\theta}$.

Using equation (31), matrix $_I^1T$ can be expressed via the world coordinate vector and angles θ_1 and θ_5 as

$$_I^1T = {}_1^MT^{-1}(\theta_1) \cdot {}_T^MT(\mathbf{x}) \cdot {}_T^IT^{-1} \tag{33}$$

that is, as

$$_I^1T = \begin{bmatrix} * & * & * & | & a_t \cdot (-s_1 \cdot k_{Tx} - c_1 \cdot k_{Tz}) - \\ & & & | & -s_1 \cdot x_M - c_1 \cdot z_M \\ * & * & * & | & a_t \cdot k_{Ty} + y_M \\ * & * & * & | & a_t \cdot (c_1 \cdot k_{Tx} - s_1 \cdot k_{Tz}) + \\ & & & | & + c_1 \cdot x_M - s_1 \cdot z_M \\ \hline 0 & 0 & 0 & | & 1 \end{bmatrix} \tag{34}$$

Like in previous case, when solving angles θ_1 and θ_5, the elements of matrices in previous equations designated with asterisks are not shown because they are not needed for solving the inverse kinematics problem.

Equating the corresponding members of matrices from equations (32) and (34), one obtains equations

$$\begin{aligned} s_{234} \cdot (d_5 + d_t) + a_3 \cdot c_{23} - a_2 \cdot s_2 = \\ = a_t \cdot (-s_1 \cdot k_{Tx} - c_1 \cdot k_{Tz}) - s_1 \cdot x_M - c_1 \cdot z_M \end{aligned} \tag{35}$$

and

$$\begin{aligned} -c_{234} \cdot (d_5 + d_t) + a_3 \cdot s_{23} + a_2 \cdot s_2 = \\ = a_t \cdot k_{Ty} + y_M \end{aligned} \tag{36}$$

Squaring equations (35) and (36) and then summing them up, it is obtained that

$$s_3 = \frac{v_1^2 + v_2^2 - a_2^2 - a_3^2}{2 \cdot a_2 \cdot a_3} \tag{37}$$

where

$$\begin{aligned} v_1 = -a_t \cdot (s_1 \cdot k_{Tx} + c_1 \cdot k_{Tz}) - \\ - s_1 \cdot x_M - c_1 \cdot z_M - s_{234} \cdot (d_5 + d_t) \end{aligned} \tag{38}$$

and

$$v_2 = a_t \cdot k_{Ty} + y_M + c_{234} \cdot (d_5 + d_t) \tag{39}$$

Taking into account that $-30° \le \theta_3 \le 89°$, Figure 5, now based on equations (37) - (39), angle θ_3 can be calculated as

$$\theta_3 = A\tan 2(s_3, c_3) \tag{40}$$

where

$$c_3 = \sqrt{1 - s_3^2} \tag{41}$$

If equations (35) and (36) are divided and shift is introduced that

$$tg(\beta) = \frac{a_3 \cdot c_3}{a_3 \cdot s_3 + a_2} \tag{42}$$

the angle θ_2 can be calculated as

$$\theta_2 = A\tan 2(a_3 \cdot c_3, a_3 \cdot s_3 + a_2) - A\tan 2(v_1, v_2) \tag{43}$$

Angle θ_4, based on equations (30), (40) and (43), is calculated as

$$\theta_4 = \theta_{234} - \theta_2 - \theta_3 \qquad (44)$$

whereby the inverse kinematics problem is solved.

3.3 Workspace analysis

Based on equations of inverse kinematics, it is possible to determine the position and orientation workspace of the robot. As mentioned above, here the robot is observed as a 5-axis horizontal milling machine (X, Y, Z, A, B). The approach applied in determining of the workspace is very useful, despite of being used in the literature to determine the position and orientation workspace of parallel machines [16].

For the case of a robot from Figure 3 the position and orientation workspace can be expressed by the Boolean function

$$W_s(x_M, y_M, z_M, A, B) = \{0,1\} \qquad (45)$$

whose value equals "1" if the tool position and orientation defined by the world coordinate vector **x** is achievable without exceeding the limited motion range of the joints. Starting from a selected point in the workspace volume, the determination of the workspace is done by a specific step-by-step strategy that locates tool in a given position and checks if that position is achievable or not by taking into account the constrained motion range of the joints.
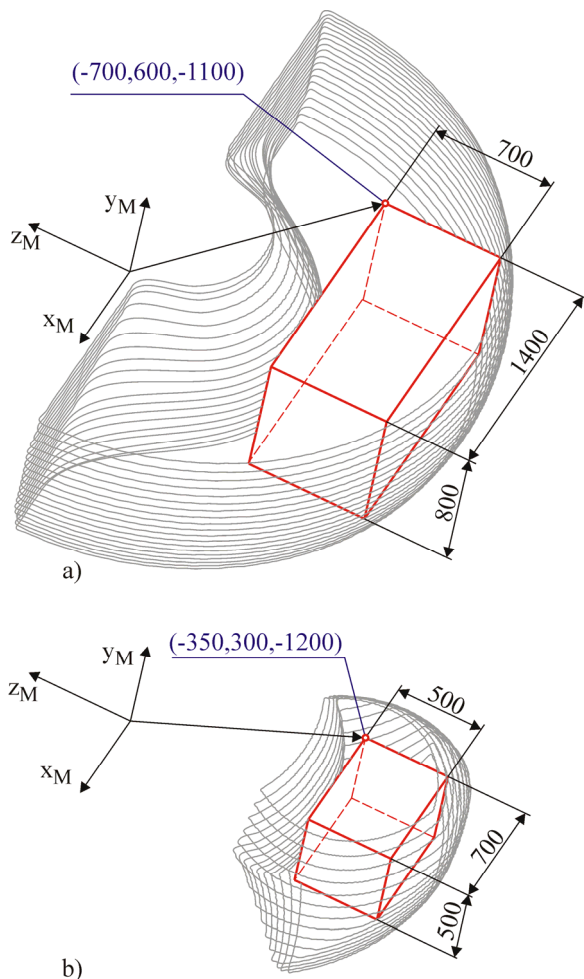

a)

b)

Figure 7. Workspaces in the cases of 3- and 5-axis machining

The first step involves determination of the robot workspace for the case of 3-axis machining (tool orien–tation is fixed, i.e. $A = 0°$ i $A = 0°$). Part of adopted workspace with boundaries convenient for machining (y_{min} = -200mm, y_{max} = 600mm) for the case of 3-axis machining is shown in Figure 7a.

Thereafter, for the adopted range of orientation angles A[-25°, 25°] and B[-25°, 25°] the workspace was determined for the case of 5-axis machining. Part of adopted workspace with boundaries convenient for machining (y_{min} = -200mm, y_{max} = 300mm) for the case of 5-axis machining is shown in Figure 7b. As it is known from practice, this workspace can be extended for the case when machining requires narrower boun–daries of orientation angles A and B. For programmers and operators of CNC machine tools both of these workspaces can be reduced to the parallelepipeds, as indicated in Figures 7a and 7b.

4. CONTROL AND PROGRAMMING SYSTEM

The development of the control and programming system prototype for 5-axis horizontal milling machine emulated from vertical articulated robot is based on EMC2 software. EMC2 was initially created by the NIST (National Institute of Standards and Technology) and is a free software released under the terms of the GPL (General Public License) [8,10].

Figure 8 shows a simplified structure of the control and programming system. As it is obvious, the programming part is very conventional, with the use of appropriate postprocessor to convert Cutter Location File (CLF) into G-code, according to ISO 6983. This means that the programmer starts from the workpiece CAD model in a common way, in this case in CAD/CAM system, generating CLF. The generated tool path is tested through the NC check as an animated display of tool path and material removal simulation.

In programming CAD/CAM environment virtual robot is configured, which is used for the simulation of tool path. Virtual robot machining by running the program is possible thanks to the applied modeling of the robot mechanism with all kinematic connections between the components, which allows the motion of a virtual robot as a system of rigid bodies [10]. The virtual robot in the CAD/CAM environment contains all relevant components of the robot mechanism, including kinematic relations between moving segments. In-between the robot segments are rotary axes (Pin). This virtual robot enables the motion of segments in ranges defined in the real robot, which is of particular importance for the identification of possible collisions during the simulation process.

Virtual robot machining allows the motion of movable segments and spindle head with tool at the end. The tool path is a result of the execution program obtained by programming using the CAD/CAM system.

Using the configured postprocessor for the hori–zontal 5-axis milling machines (X,Y,Z,A,B) spindle-tilting type, postprocessing of CLF is done to obtain the program in G-code, which is transferred to the robot control system.

Figure 8 also shows the internal software structure of the EMC2 [8], which contains four basic software

modules: motion controller (EMCMOT), discrete input/output controller (EMCIO), processes controller for coordination (EMCTASK) and a collection of text or graphic user inerfaces (GUI).

EMCTSK (Task coordinating module) distributes commands in the machine. It interpretes the programs, G-code, according to the ISO6983 standard.

EMCIO (Discrete I/O Controller) performs all communications not related to motion control. It has subordinate modules for the main spindle, tool change, accessory M functions, emergency stop, lubrication, etc.

EMCMOT (Motion Controller) is a module that works periodically, in real time and performs trajectory planning, calculations for inverse and direct kinematics needs and generation of control signals for machine drives. It closes the feedback loops, generates the next positions, performs trajectory interpolation between the programmed positions, controls boundary positions and axes reference positions, etc. In this module, solutions to inverse and direct kinematics are integrated for 5-axis horizontal milling machine emulated from vertical articulated robot.
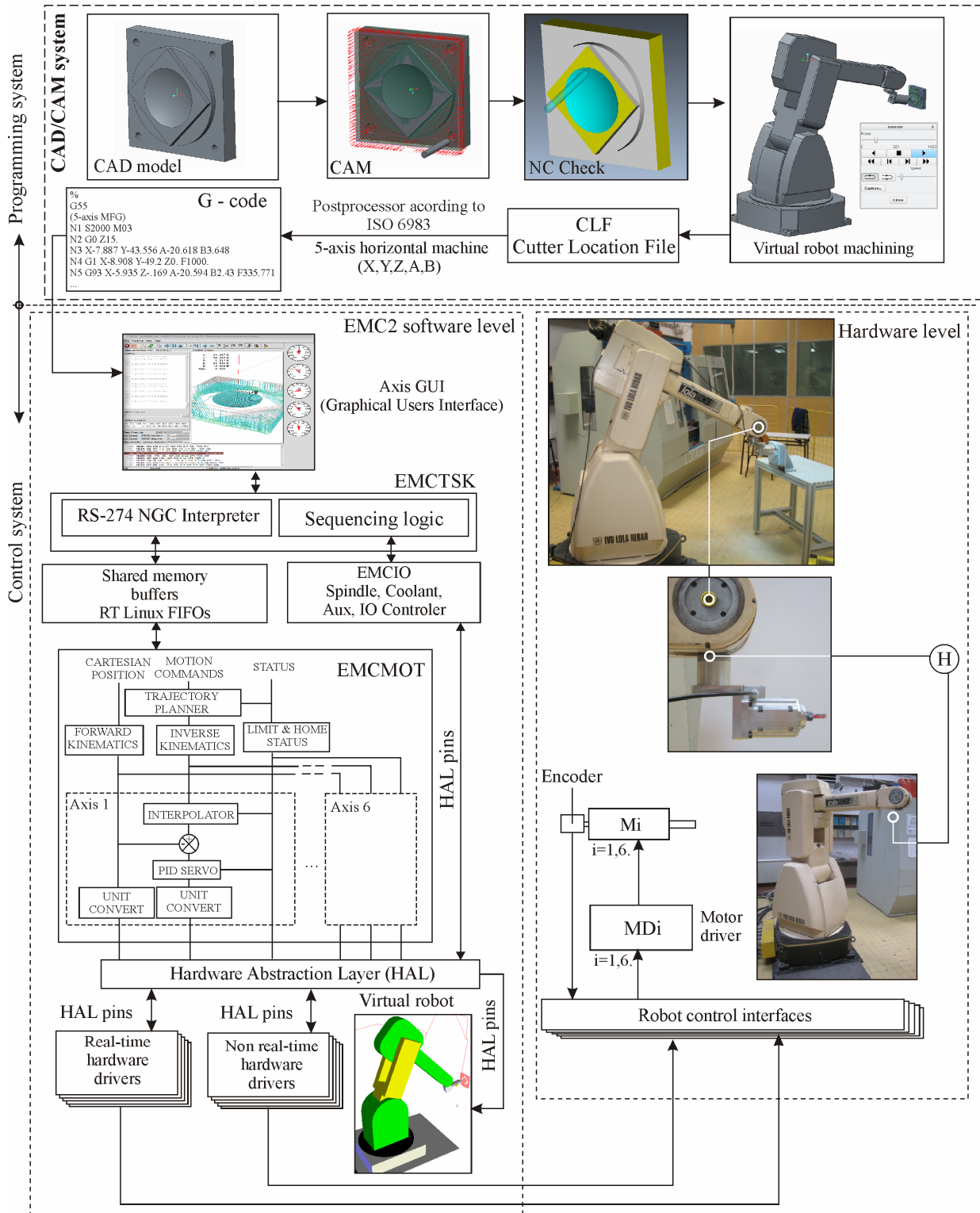

Figure 8. Control and programming system

EMC2 modular structure has contributed to its flexibility in both its application in machines with complex kinematics and linking to different hardware and software accessories. This is primarily owing to HAL (Hardware Abstraction Layer). The concept of HAL is to be a flexible interface between the motion controller, on one hand, and the user and machine interface, on the other hand. This implies multiple hardware interfaces with machines or robots that enable coupling of the motion controller to the actuators and measurement systems. Here, the interface with a configured virtual robot, integrated with control, is also realized over HAL.

GUI (Graphical user interface) is an external program which communicates with EMC2 by sending commands such as: machine turn on, switch to automatic mode, start the program, machine shut down. GUI can also send manual commands, initiated by the operator, such as: moving machines axes in manual mode (JOG) or sending all axes in a reference position. Different GUI can be used, while Axis is the most commonly used user interface.

For testing the functions of inverse and direct kinematics and collision detection, a virtual robot is configured. The virtual robot is configured using several predefined Python classes [10] and OpenGL in EMC2. Based on inverse and direct kinematics equations, kinematic module is programmed in C language and is integrated in EMC2 software system, Figure 8.

The virtual robot makes possible simulation of the real machining robot for the user, i.e., verification of the generated program in workspace [17]. Virtual robot is characterized by:

• validation of the part program generated by a commercial CAM software (PTC Creo 3.0),

• on-line visualization,

• off-line programming of the machining robot.

The hardware part of the system uses vertical articulated robot LOLA 50, with appropriate interface plate, which allows to set up a tilting spindle when the robot works as a 5-axis horizontal milling machine. For such machining, depending on the workpiece, if necessary, an angle plate is set up on a worktable for locating and clamping needs of the workpiece. Experimental verification of the control and program–ming system is demonstrated in Section 5 using the example of machining a test workpiece.

5. MACHINING EXPERIMENT

The aim of machining experiments was to test the kinematic modeling as well as developed control and programming system.

The programming system used was CAD/CAM sys–tem PTC Creo 3.0, with the idea to program robotic mac–hining in an identical way as machining with a 5-axis hori–zontal milling machine (X, Y, Z, A, B) spindle-tilting type.

The test workpiece, Figure 9a, was designed similar to the test pieces suggested by various standards to enable a simple test of robot programming ability to produce standard features. Figure 9b shows experimental setup for machining this test workpiece, with robot LOLA 50.
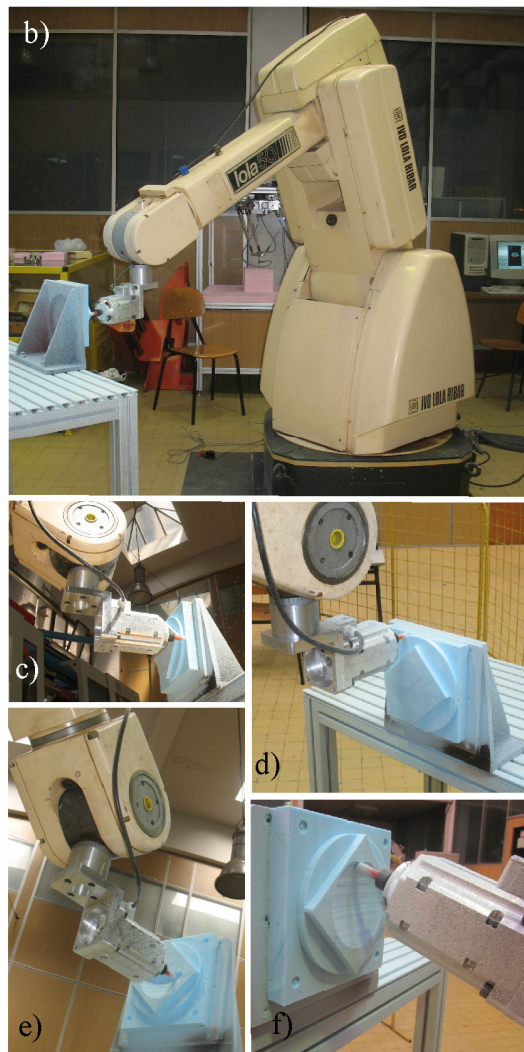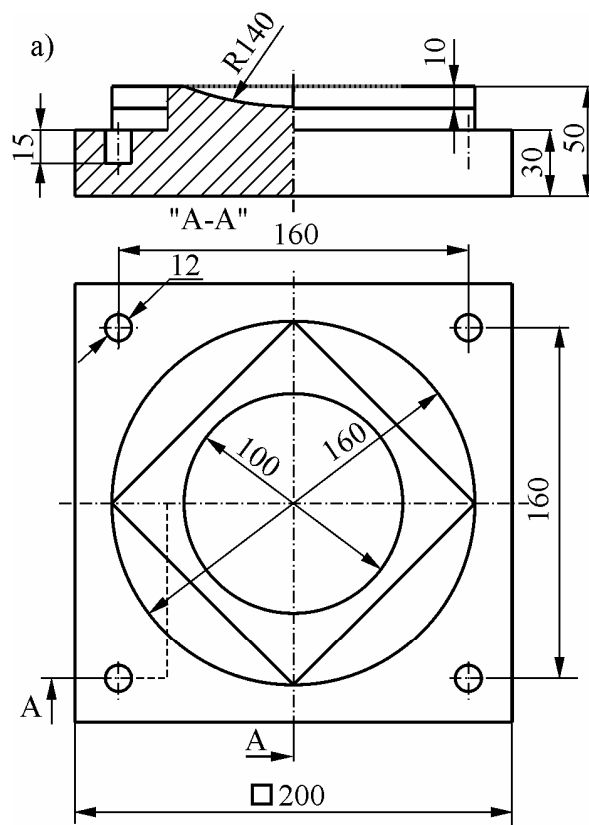

Figure 9. Machining experiment

Figure 9c and 9d shows three-axis pre-machining and finishing. Figure 9e and 9f shows 5-axis machining of concave calotte. For robot machining flat endmill (tool/flute length 60/30 mm, diameter 12 mm) was used. Prior to machining the programs are tested in EMC2 on a virtual machining robot in order to perform the final verification of the program. Figure 10 shows a comparative view of machining, Figure 10a, and machining simulation on a virtual robot, Figure 10b, integrated with the EMC2 control system, Figure 10c for the example of 5-axis machining of a concave calotte. These experiments confirmed that it is possible to realize the low-cost robotic machining system that emulates 5-axis horizontal milling machine spindle-tilting type, for the machining parts of light materials and lower tolerance, which can be directly used by CNC machine tools programmers and operators.
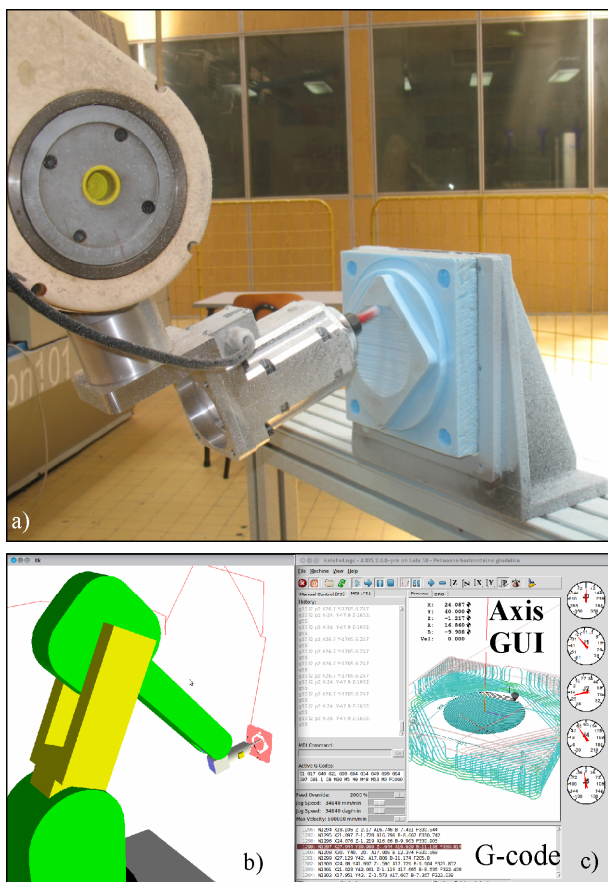

Figure 10. Real and virtual robot machining

6. CONCLUSION

The paper presents the concept of kinematic modeling of 5-axis horizontal milling machine emulated from vertical articulated robot for machining parts of light materials with lower tolerances. The proposed concept is based on avalilable 6-axis vertical articulated robot LOLA 50 considered as a specific configuration of 5-axis horizontal milling machine (X, Y, Z, A, B) spindle tilting type and possibilities of programming in G-code.

Kinematic modeling approach is shown in detail as well as the prototype of the developed low-cost control and programming system based on EMC2 software system.

Verification of the experimental robotic machining system is presented using the example of 3- to 5-axis machining of test workpiece of light material.

The developed and investigated 5-axis horizontal milling machine emulated from vertical articulated robot indicates that such commercial system may be superior to the compatible robotic machining solutions, considering the G-code is still very widely used in industry.

REFERENCES

[1] Alici, G. and Shirinzadeh, B.: Enhanced stiffness modeling, identification and characterization for robot manipulators, IEEE Trasanctions on Robotics, Vol. 21, No. 4, pp. 554–564, 2005.

[2] Pan, Z. and Zhang, H.: Robotic machining from programming to process control: a complete solu–tion by force control, Industrial Robot: An Inter–national Journal, Vol. 35, No. 5, pp. 400–409, 2008.

[3] DePree, J. and Gesswein, C.: Robotic machining white paper project-Halcyon Development, 2008, https://www.robotics.org/robotic-content.cfm/ Robotics/Halcyon-Development-RIA/id/43

[4] Milutinovic, D., Glavonjic, M., Slavkovic, N., Dimic, Z., Zivanovic, S., Kokotovic, B. and Tanovic, Lj.: Reconfigurable robotic machining system cont–rolled and programmed in a machine tool manner, International Journal of Advanced Manufacturing Technology, Vol. 53, No. 9-12, pp. 1217-1229, 2011.

[5] Lehmann, C. et al.: Machining with Industrial Robots: The COMET Project Approach, in: Neto, P. and Moreira, A.P. (Ed.): *Robotics in Smart Manufacturing*, Springer Berlin Heidelberg, Berlin, pp. 27-36, 2013.

[6] Slavkovic N., Milutinovic D., Kokotovic B., Glavonjic M., Zivanovic S. and Ehmann K.: Cartesian Compliance Identification and Analysis of an Articulated Machining Robot, FME Transactions Vol. 41, No. 2, pp. 83-95, 2013.

[7] Slavkovic, N.: *Identification, modelling and com–pensation of errors due to machining robot static compliance*, PhD thesis, University of Belgrade, Faculty of Mechanical Engineering, Belgrade, 2015.

[8] LinuxCNC, http://www.linuxcnc.org/

[9] High speed machining (hsm) robots, www.staubli. com/en/robotics/robot-solution-application/high-speed-machining-robots/

[10] Dimic, Z. et al.: Virtual environment in control and programming system for reconfigurable machining robot, Tehnički vjesnik / Technical Gazette, Vol. 23, No. 6, pp. 1821-1829, 2016.

[11] ISO 841:2001 Industrial automation systems and integration - Numerical control of machines - Coordinate system and motion nomenclature.

[12] Lee, R.-S. and She, C.-H.: Developing a postpro–cessor for three types of five-axis machine tools, The International Journal of Advanced Manufacturing Technology, Vol. 13, No. 9, pp. 658–665, 1997.

[13] Craig, J.J.: *Introduction to Robotics: Mechanics and Control, 2nd ed.,* Addison-Wesley, Reading, Massachusetts, 1989.

[14] Fu, K.S., Gonzalez, R.C. and Lee, C.S.G.: *Intro–duction to Robotics: Control, Sensing, Vision, and Intelligence*, McGraw-Hill, New York, 1987.

[15] Milutinovic M., Slavkovic N. and Milutinovic D.: Kinematic Modeling of Hybrid Parallel-Serial Five-Axis Machine Tool, - FME Transactions, Vol.41, No.1, pp. 1-10, 2013.

[16] Innocenti, C. et al.: Exhaustive enumera–tion of fully parallel kinematic chains, Dynamic System and Control, Vol. 55, No. 2, pp. 1135–1141, 1994.

[17] Zivanovic, S. et al.: Methodology for Configuring Desktop 3-axis Parallel Kinematic Machine, Faculty of Mechanical Engineering, Belgrade, FME Transactions, Vol.37, No. 3, pp. 107-115, 2009.

КИНЕМАТИЧКО МОДЕЛИРАЊЕ ХОРИЗОНТАЛНЕ 5-ОСНЕ ГЛОДАЛИЦЕ ЕМУЛИРАНЕ РОБОТОМ ВЕРТИКАЛНЕ ЗГЛОБНЕ КОНФИГУРАЦИЈЕ

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Индустријски роботи представљају успешну алтер–нативу 5-осним машинама алаткама за вишеосну обраду глодањем делова од мекших материјала ниже класе тачности. Из ових разлога се развијају софтверска решења која би приближила програ–мирање робота за обраду програмирању машина алатки.

У раду су представљени резултати кинема–тичког моделирања индустријских робота верти–калне зглобне конфигурације у циљу емулирања хори–зонталних 5-осних машина алатки. На основу решења инверзног кинематичког проблема анали–зиран је радни простор робота за обраду.

Полазећи од једначина директне и инверзне кинематике развијен је и систем управљања и програмирања у G-коду базиран на EMC2 софт–верском систему. Развијени систем управљања и програ–мирања омогућава употребу постојећих CAD/CAM система за програмирање 5-осних машина алатки, односно директно је применљив од стране програмера и оператера који имају искуства у CNC технологијама и програмирању у G-коду. Верификација кинематичког моделирања и разви–јеног система управљања и програмирања је остварена експериментима обраде на распложивом 6-оосном роботу домаће производње ЛОЛА 50.