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# Simulation and Motion Study of Mechanical Integrator 3D Model

*A Mechanical Integrator is a mechanical device which can be used to obtain the continual integration of the input variable value. This paper explains the operational principles and discloses the simulation and motion study results of the mechanical integrator 3D model assembly. Specifically, 3D models of the input function generator, the ball and disc integrator and the output x-y plotter are created and their operations are simulated. This work is significant mainly for education in the fields of solid modelling, mechanism synthesis and scientific visualization of the mechanism motion and operation. In particular, the motion analysis presented in this paper visualizes and discloses the mutual relationship between the integration calculus as the mathematical operation and the geometrical and kinematical characteristics of one special type of mechanical computing device.*

**Keywords:** function, integrator, mechanism, motion, simulation, visualization

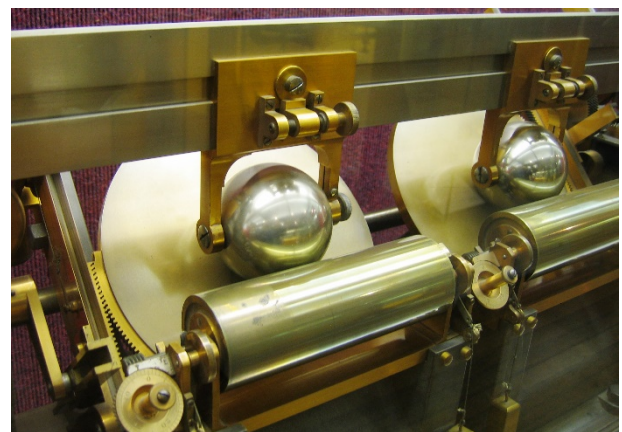
## 1. INTRODUCTION

A Mechanical Integrator is a mechanical device [1,2] used to obtain a continual integration of the input variable value. In mathematical terms, a mechanical integrator is a computing instrument [1,3] which generates, in accordance with initial parameters and arguments, a function as an integral of an input function. This mechanism was invented by William Thompson, 1st Baron Kelvin in 1886, and it was used as part of Harmonic Analyser [4], by which the coefficients of a Fourier series were calculated. During the 20th century, mechanical integrators had several important military applications [5,6] and played a key role in the development of analog computers [7]. They were essential parts of Differential Analysers [7] – advanced mechanical analog computers designed to solve differential equations by integration. Despite the fact that mechanical integrators have only historical significance in contemporary science and engineering, their operational principles are still important and can be used for education in the field of 3D modelling, mechanism synthesis, simulation and motion analysis.

## 2. THEORETICAL BACKGROUND

Let us consider the mechanism which is schematically shown in Fig.2. It consists of a smooth disk  $D$ , two balls (spheres)  $B$ , a balls carriage  $K$ , and a shaft (cylinder)  $C$  of radius  $r$ . The balls and the cylinder are spring or weight loaded against disk  $D$ . It is assumed that all the mentioned parts produce perfect non-slipping contacts between each other. Balls  $B$ , settled in the balls carriage  $K$ , can be radially displaced from the axis of rotation of

disc  $D$ . As is shown in Fig.2, the position of the balls is determined by the input variable  $y(t)$ . Disc  $D$  rotates with the constant angular velocity  $\omega_D$ , and this rotation is transferred to shaft  $C$  by balls  $B$ . Since the position of balls  $B$  depends on the variable  $y(t)$ , the angular velocities of the balls and the angular velocity  $\omega_C$  of the shaft  $C$  varies. For instance, if the balls  $B$  are in the centre of the disk  $D$ ,  $y(t)=0$ , and  $\omega_C=0$ . If the balls  $B$  are displaced out from the centre of the disk  $D$ ,  $y(t)\neq 0$ , and consequently  $\omega_C\neq 0$ .



**Figure 1. Lord Kelvin's mechanical integrator**

The angular velocity  $\omega_C$  of the shaft  $C$  is determined by the equation:

$$\omega_c(t) = \frac{d\phi_c(t)}{dt} = \frac{y(t)}{r} \omega_D \quad (1)$$

in which  $\phi_C$  is the angular displacement of the shaft  $C$ ,  $r$  is the shaft  $C$  radius and  $t$  is time. Regarding Equation (1), the angular displacement  $\phi_C$  of the shaft  $C$ , during the interval of time  $(t_0, t)$  can be determined from the following relation:

$$\phi_C(t) = \frac{\omega_D}{r} \int_{t_0}^t y(t) dt \quad (2)$$

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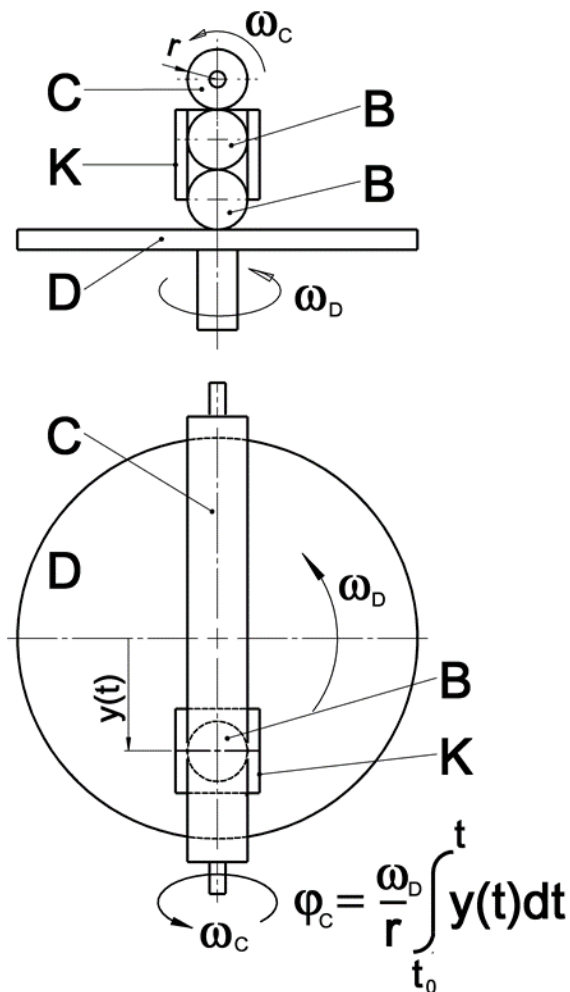


Figure 1. Ball and disk integrator

Equation (2) discloses that the angular displacement  $\varphi_C(t)$  of the shaft C, during the interval of time  $(t_0, t)$  is directly proportional to the integral of the variable  $y(t)$  as function of time. Thus, the mechanism shown in Fig. 2 performs the continual integration of the input function  $y(t)$  and generates the output function  $\varphi_C(t)$  proportional to the integral of the input function. In accordance to this fact and its constructive characteristics, the computing instrument schematically represented in Fig.2 is called the ball and disk integrator.

### 3. MECHANICAL INTEGRATOR ASSEMBLY

The complete assembly of the mechanical integrator, accomplished by the using of the Solidworks application [8,9], is shown in Fig.3. It consists of the following tree subassemblies: the input function generator (F), the ball and disk mechanical integrator (I) and the output  $x$ - $y$  plotter (P). The ball and disk integrator (I) is the main and the other two are auxiliary subassemblies.

The input function generator (Fig.3-F) has a rotary groove cam (Fig.3-1) whose groove (Fig.3-2) determines the position of the follower (Fig.3-4) by the pin as a function of the cam angular displacement. The rotary groove is driven by a motor (Fig.3-3) and rotates with the constant angular velocity  $\omega_F$ . Thus, in accordance with the shape of the groove (channel) and the rotary motion of the cam, the input function generator creates a function which will be integrated by the

ball and disk integrator subassembly (Fig.3-F). The shape of the groove is defined in polar coordinate system which rotates with the constant angular velocity  $\omega_F$  by the function

$$\rho = \rho(\omega_F t) \quad (3)$$

As it was already emphasized, the function given by the expression (3) determines the displacement of the follower (Fig.3-4) and thereby defines the function which is created by the function generator (Fig.3-F). In this paper, two different function generators are modeled. The first function generator generates the following linear function

$$\begin{aligned} \rho(\omega_F t) &= \rho_0 + k \cdot \omega_F t \\ k &= \text{const}, 0 \leq \omega_F t \leq 2\pi \end{aligned} \quad (4)$$

and the second one generates the following quadratic function

$$\begin{aligned} (\omega_F t) &= -\frac{\rho_0 - y_M}{\pi^2} (\omega_F t)^2 + \\ &+ 2\frac{\rho_0 - y_M}{\pi^2} \omega_F t + \rho_0, y_M = \text{const}, \\ \rho_0 &= \text{const}, y_0 = \text{const}, 0 \leq \omega_F t \leq 2\pi \end{aligned} \quad (5)$$

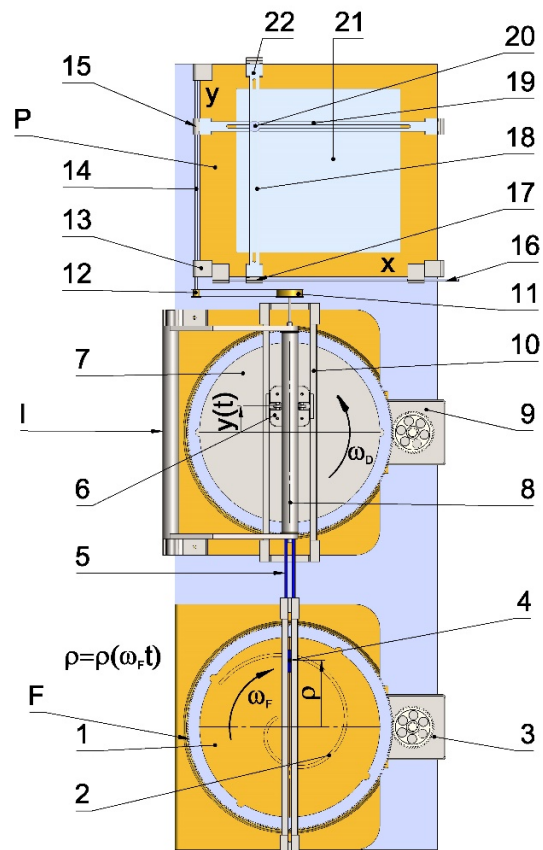
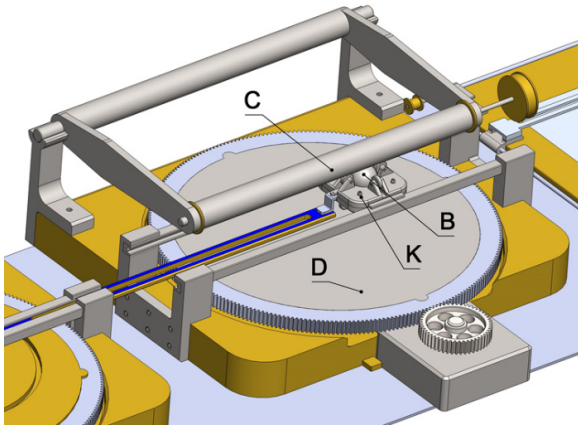


Figure 3. Mechanical integrator assembly

The second subassembly, the ball and disk integrator (Fig.3-I), performs the continual integration of the function generated by the input function generator (Fig.3-F) and thus creates a new function as the integral of the input one. It consists of a disk (Fig.3-7), two balls supported by a carriage (Fig.3-6) and an output rotating

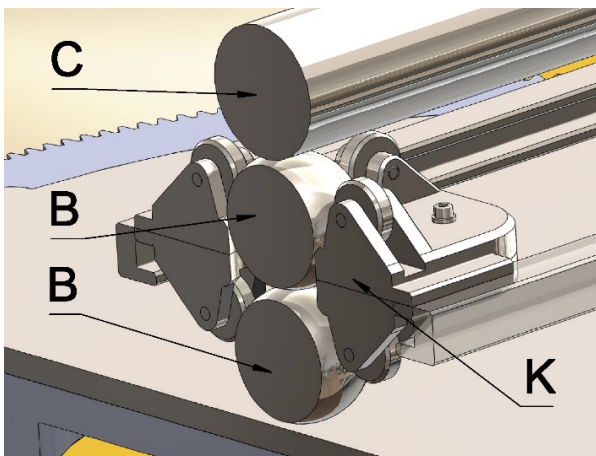
shaft (Fig.3-8). The disk (Fig.3-7) is driven by a motor (Fig.3-9) and rotates with the constant angular velocity  $\omega_D$ . The carriage is connected to the follower (Fig.3-4) of the cam mechanism by the sliding rod (Fig.3-5). Thus, the displacement of the carriage (Fig.3-6) is directly proportional to the value of the function generated by the first subassembly. The rotation of the disk is transmitted to the balls and the shaft, and from the shaft to the third subassembly – to the y component of the x-y plotter.



**Figure 4. Axonometric view of the ball and disk integrator; D – disk, K – balls carriage, B – ball, C - shaft**

The axonometric view of the ball and disk mechanical integrator is given in Fig.4. Balls B and the cylinder C are weight loaded against disk D and produce non-slipping contacts between each other. The cross section through the balls carriage K, balls B and shaft C is shown in Fig. 5. Since the rotation is transferred from disk D to shaft C by the frictional contact between disk D, balls B and shaft C, the slipping movement between the mentioned parts would jeopardize the operation of the entire integrator. Consequently, the slipping motion must be avoided by applying a sufficiently large load on shaft C against balls B and disk D.

The intensities and directions of the angular velocity  $\omega_F$  of the rotary groove cam and  $\omega_D$  of the integrator's disk can be adjusted before integration, but they must be constant during the integration.



**Figure 5. Cross section through balls carriage K, balls B and shaft C of the ball and disk integrator**

The third subassembly, the output x-y plotter (Fig.3-F), has two mechanical linear actuators which

accomplish the conversion of the rotary motion of the lead-screw into the linear motion of the nut. Two slotted crossbars, shown in Fig.3-18 and Fig.3-19, connected to the linear actuators nuts (Fig.3-15), (Fig.3-17), hold the pen (Fig.3-20) by which the output function is graphically presented on the plotter's paper (Fig.3-21). Slotted crossbars, one movable in x and another in y direction, are supported by the sliders (Fig.3-22). Linear actuators for both directions are supported by the bearings (Fig.3-13), and consist of leadscrews (Fig.3-14) (Fig.3-16) and nuts (Fig.3-15), (Fig.3-17). The angular displacement of the integrator shaft (Fig.3-8) is transferred to the leadscrew (Fig.3-14) by the belt drive (Fig.3-11), (Fig.3-12). The leadscrew in x direction (Fig.3-16) is driven by the rotary motor with the constant angular velocity. Thus, the displacement of the linear actuator in y direction is proportional to the output function generated by the integrator and the uniform displacement of the linear actuator in x direction is proportional to time. The pen (Fig.3-20) traces the graphic of the integrator output function.

#### 4. RESULTS OF THE SIMULATION

As an example, two functions are integrated by the mechanical integrator modelled in this paper. The first function is linear (4) and the second one is quadratic (5). The 3D models of linear and quadratic function generators are created and the computer simulations of the generated functions integrations are accomplished. The integration of the linear function is shown in Fig.6, and the integration of the quadratic function is shown in Fig.7.

The parameters for the integration of the linear function are: the angular velocity of the rotary groove cam (Fig.3-1) is  $\omega_F=3 \text{ rpm}$ , and the angular velocity of the integrator's disk (Fig.3-7) is  $\omega_D=30 \text{ rpm}$ . The integration lasts 16 seconds. Regarding the initial position  $y_0$  of the balls' carriage (Fig.3-6) relative to the centre of the disk (Fig.3-7), the integrator performs the integration of the following linear function:

$$y(t) = y_0 + k \cdot \omega_F t, k = \text{const}, \quad (6)$$

$$y_0 = \text{const}, 0 \leq \omega_F t \leq 2\pi$$

It is clearly visible on the plotter paper shown in Fig.6 that the integration of the linear function produces the graphic of the quadratic function.

The parameters for the integration of the quadratic function are: the angular velocity of the rotary groove cam (Fig.3-1) is  $\omega_F=3 \text{ rpm}$ , and the angular velocity of the integrator's disk (Fig.3-7) is  $\omega_D=40 \text{ rpm}$ . The integration also lasts 16 seconds. In accordance with the initial position  $y_0$  of the balls' carriage (Fig.3-6) relative to the centre of the disk (Fig.3-7), the integrator integrates the following quadratic function:

$$(\omega_F t) = -\frac{\rho_0 - y_M}{\pi^2} (\omega_F t)^2 +$$

$$+ 2 \frac{\rho_0 - y_M}{\pi^2} \omega_F t + \rho_0, y_M = \text{const}, \quad (7)$$

$$\rho_0 = \text{const}, y_0 = \text{const}, 0 \leq \omega_F t \leq 2\pi$$



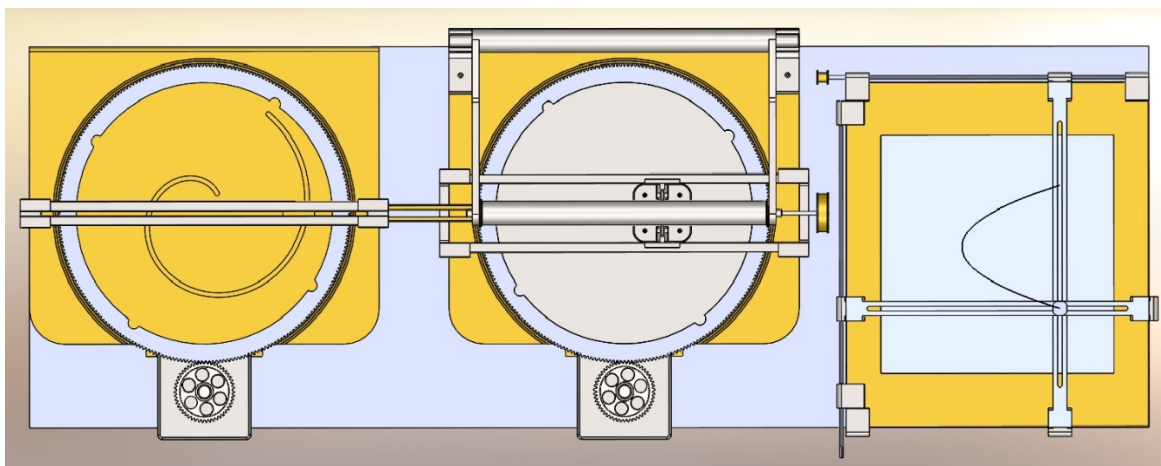


Figure 6. Integration of linear function

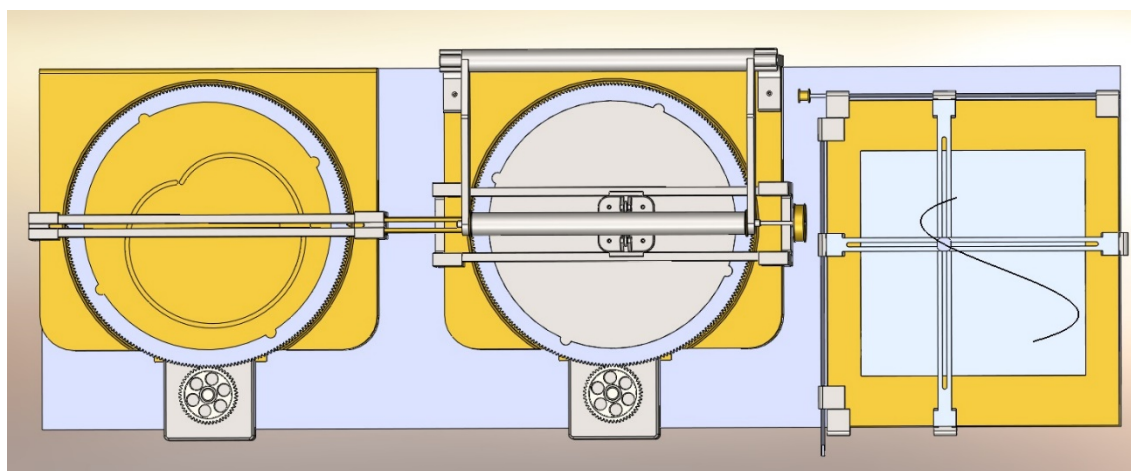


Figure 7. Integration of quadratic function

It is distinctly visible on the plotter paper shown in Fig. 7 that the integration of the quadratic function produces the graphic of the cubic function.

## 5. CONCLUSION RESULTS OF THE SIMULATION

This work is significant mainly for education in the fields of solid modelling, mechanism synthesis and scientific visualization of the mechanism motion and operation [10]. In particular, the motion analysis presented in this paper visualizes and discloses the mutual relationship between the integration calculus as the mathematical operation and the geometrical and kinematical characteristics of one special type of mechanical computing device.

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## СИМУЛАЦИЈА И АНАЛИЗА КРЕТАЊА 3Д МОДЕЛА МЕХАНИЧКОГ ИНТЕГРАТОРА

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З. Јели, М. Обрадовић, Д. Ј. Попа

Механички интегратор је механички уређај помоћу  
кога се врши континуална интеграција улазне про-

менљиве величине. Овај рад објашњава оперативне принципе и открива резултате симулације и кретања у склопу 3Д модела механичког интегратора. Конкретно, креирани су 3Д модели генератора улазних функција, интегратора типа „лопта са диском“ и  $x$ - $y$  излазног плотера, а извршена је и симулација њихових кретања и операција. Овај рад је значајан углавном за образовање у областима 3Д

солид моделирања, синтезе механизма и научне визуелизације кретања и рада механизма. Конкретно, анализа кретања представљена у овом раду визуелизује и открива међусобни однос између интегрисања као апстрактне математичке операције и геометријских и кинематичких карактеристика једног посебног типа механичког уређаја за рачунање.