N. Nafia
Laboratory of Electric Systems and Telecommunications (LSET)
Cadi Ayad University
Faculty of Sciences and Techniques
Marrakech,
Morocco

A. El Kari
Laboratory of Electric Systems and Telecommunications (LSET)
Cadi Ayad University
Faculty of Sciences and Techniques
Marrakech,
Morocco

H. Ayyad
Laboratory of Electric Systems and Telecommunications (LSET)
Cadi Ayad University
Faculty of Sciences and Techniques
Marrakech,
Morocco

M. Mjahed
Ecole Royale de l'Air
Maths & Systems Dep
Marrakech,
Morocco

Design of a Robust Interval Type-2 Fuzzy Adaptive Super Twisting Control for a Given Class of Disturbed MIMO Nonlinear Systems

This study deals with the tracking control problem of a large class of multi input multi output (MIMO) nonlinear systems with unknown dynamics and subject to unknown disturbances. First, two interval type-2 adaptive fuzzy systems (IT2-AFSs) are constructed to efficiently estimate the unknown nonlinear dynamics. Then, based on IT2-AFSs and super-twisting algorithm (STA), a new robust adaptive fuzzy-reaching STC law (AF-RSTCL) has been added to the global control law to improve the robustness of the studied systems in the presence of approximation errors and unknown disturbances. In order to avoid the chattering phenomenon and guarantee simultaneously the best tracking performance, the gains of the designed AF-RSTCL are optimally online estimated. The adaptive parameters of the global synthesized control law are deduced from the stability analysis in the sense of Lyapunov. Finally, an example of simulation is used to confirm the effectiveness of the developed method in achieving the predetermined objectives of the tracking control.

Keywords: *Tracking control, Nonlinear systems, Type-2 fuzzy systems, super twisting control, adaptive control.*

1. INTRODUCTION

The design of robust control algorithms for disturbed multi input multi output (MIMO) uncertain systems with highly coupled nonlinearities is one of the biggest control problems encountered in the system control field. Therefore, in recent decades, several sophisticated control approaches have been developed for controlling such complex processes. Among them, those using fuzzy systems (FSs), adaptive control and sliding mode control (SMC) have been credited in many different real applications as effective and robust methods of control [1-8].

SMC is an efficient approach of robust control techniques widely utilized in many practical applications due to its capability to handle any uncertainties or disturbances influencing the system dynamics. However, the chattering phenomenon caused by the discontinuous term of the SMC law can be harmful to the actuators [9, 10]. To eliminate or at least attenuate the chattering phenomenon, several control techniques have been employed in various technical and technological applications [11-17]. And in particular, those using super twisting algorithm (STA) algorithm, have recently been extensively used in the literature [18-22]. The STA has been introduced by Levant [23] to attenuate the chattering phenomenon while simultaneously ensuring a finite time convergence and the stability of the closed loop control system. However, the design of good values of the super-twisting control (STC) law gains remains one of the major disadvantages of such kind of algorithm. Indeed, the large gains values generate a big chattering in the control system and the small ones damage the system robustness, and can even lead to the instability of the closed loop control system. In [24], two terms are added to the conventional STC algorithm to guarantee the desired performance for uncertain nonlinear systems. And most recently, in [25], based on STC algorithm and a high order sliding mode observer, a robust controller-observer is proposed for an uncertain unmanned aerial system to ensure the desired tracking performance.

On the other hand, and due to the universal approximation property of FSs [26], fuzzy control methods have been largely studied and successfully implemented in control problem by many researchers [27-32]. However, type-1 FSs (T1-FSs), in the presence of linguistic uncertainties and measurement errors, are not able to process excellently such perturbations. Therefore, to overcome this constraint, type-2 FSs (T2-FSs), which constitute a generalization of T1-FSs, show appreciable and attractive features in taking into account such uncertainties [33-36]. The key feature is the so called footprint of uncertainty (FOU), which allows incorporating directly the uncertainties in the membership functions (MFs). In [35], to show the superiority of a reactive proposed control approach based on T2-FSs to drive a robot mobile towards a mobile target, it is compared against its counterpart using T1-FSs instead of T2-FSs. And, in [36], the proposed method, which is used to describe and control an underactuated mobile two wheeled inverted pendulum in the presence of modelling errors and external disturbances, shows that when it

Received: November 2018, Accepted: July 2019
Correspondence to: Nafia Nabil, Laboratory of Electric Systems and Telecommunications (LSET), Cadi Ayad University, Faculty of Sciences and Techniques Marrakech, Morocco, E-mail: bil.nafia@gmail.com
doi:10.5937/fmet2001063N

is based on T2-FSs rather than T1-FSs, it is more effective in achieving the desired performance of control.

Compared with the reported researches in the literature, the main contributions of this study are presented as follows:

(1) A new effective tracking control design approach is developed for a general class of MIMO nonlinear systems characterized by the following features:
- All dynamics are entirely unknown and suffer from time varying disturbances.
- The disturbances that influence the studied systems dynamics are completely unknown, which means that no information of the upper bounds of these perturbations is available.
- The physical parameters are unknown and present parametric variations.

(2) Two IT2-adaptive FSs (IT2-AFSs) are constructed to perfectly estimate the unknown nonlinear dynamics.

(3) In order to avoid the chattering while simultaneously improving the tracking control performance in the presence of approximation errors and unknown disturbances, a new synthesized adaptive fuzzy-reaching STC law (AF-RSTCL) based on IT2-AFSs and STA has been added to the proposed control law. The AF-RSTCL gains are optimally online estimated in order to provide the good gains values that guarantee the system robustness and the best tracking performance with a smooth continuous control law.

(4) The adaptation laws of both the IT2-AFSs and AF-RSTCL gains are deduced using the stability analysis theorem of Lyapunov. The closed loop stability and the tracking error convergence are guaranteed despite all the constraints discussed above.

This paper is organized as follows. Section 2 describes the IT2-FSs. In section 3, the problem formulation is presented, and in section 4, we explain the proposed tracking control approach designed for a large class of disturbed MIMO nonlinear systems with unknown dynamics. Section 5 presents the simulation results of a two-link robot manipulator to confirm the effectiveness of the proposed tracking control algorithm. The conclusion is given in section 6.

2. INTERVAL TYPE-2 FUZZY SYSTEMS

IT2-FSs are used due to their excellent efficiency to handle directly measurement uncertainties and inaccurate linguistic information used to synthesize T2 fuzzy rules. Thus, the fuzzy sets for an IT2-FS are implemented in such a way that their associated MFs can easily incorporate the above discussed uncertainties through their FOU property.

The jth IT2 fuzzy rule of an IT2-FS, having q inputs and one output, can be formulated as follows [37]:

$$\text{Rule}(j): \text{If } x_1 \text{ is } \tilde{E}_1^j \text{ and } x_2 \text{ is } \tilde{E}_2^j \dots \text{ and } x_q \text{ is } \tilde{E}_q^j \text{ Then } y \text{ is } \tilde{\theta}^j,\ j=1,\dots,M \quad (1)$$

Where $\tilde{E}_i^j$ are antecedent IT2 fuzzy sets ($i=1,\dots,q$) and $\tilde{\theta}^j$ are consequent IT2 fuzzy sets; $\underline{x}=\begin{bmatrix} x_1 & x_2 & \dots & x_k \end{bmatrix}^T \in \mathbb{R}^q$ is the state vector; $y \in \mathbb{R}$ is the system (1) output; and M denotes the number of IT2 fuzzy rules.

For the system (1), if the product is used as an inference engine then the firing set associated to the jth IT2 fuzzy rule can be given as follows:

$$W^j(\underline{x}) = \left[w_l^j(\underline{x}), w_r^j(\underline{x}) \right] \quad (2)$$

Where $w_l^j(\underline{x}) = \pi_{i=1}^q \mu^L_{\tilde{E}_i^j}(x_i)$ and $w_r^j(\underline{x}) = \pi_{i=1}^q \mu^R_{\tilde{E}_i^j}(x_i)$, such that $\mu^L_{\tilde{E}_i^j}(x_i)$ and $\mu^R_{\tilde{E}_i^j}(x_i)$ represent respectively the left and right most values of the MFs associated to the IT2 fuzzy sets $\tilde{E}_i^j$.

By using the center of sets technique, the output sets of the inference engine, which are IT2 fuzzy sets, are reduced to an IT2 fuzzy set called type reduced set. Then, based on the center of gravity method, the defuzzified output of the system (1) can be obtained as follows [38]:

$$y = \frac{y_l + y_r}{2} \quad (3)$$

where y_l and y_r can be expressed as:

$$\begin{cases} y_l = \min\limits_{w^j} \dfrac{\sum_{j=1}^{M} \theta_l^j w^j}{\sum_{j=1}^{M} w^j} = \theta_l^T \xi_l \\ y_r = Max_{w^j} \dfrac{\sum_{j=1}^{M} \theta_r^j w^j}{\sum_{j=1}^{M} w^j} = \theta_r^T \xi_r \end{cases} \quad (4)$$

where $\xi_l = \begin{bmatrix} \xi_l^1 & \xi_l^2 & \dots & \xi_l^M \end{bmatrix}^T$ and $\xi_r = \begin{bmatrix} \xi_r^1 & \xi_r^2 & \dots & \xi_r^M \end{bmatrix}^T$ are respectively the lower and upper vectors of fuzzy basis functions; $\theta_l = \begin{bmatrix} \theta_l^1 & \theta_l^2 & \dots & \theta_l^M \end{bmatrix}^T$ and $\theta_r = \begin{bmatrix} \theta_r^1 & \theta_r^2 & \dots & \theta_r^M \end{bmatrix}^T$ are the lower and upper parameter vectors, respectively; w^j is an element of the firing set W^j $(w^j \in W^j)$.

y_l and y_r can be obtained easily using the Karnik-Mendel algorithm [39].

For more details of IT2-FSs, see [40].

3. PROBLEM STATEMENT

Consider a general class of MIMO n th order nonlinear systems, having k inputs and p outputs, described by the following equation:

$$\begin{cases} x^{(n)} = F(\underline{x}) + G(\underline{x})u + D \\ y = x \end{cases} \tag{5}$$

where $u = \begin{bmatrix} u_1 & u_2 & \dots & u_k \end{bmatrix}^T \in \mathbb{R}^k$ and $y = \begin{bmatrix} y_1 & y_2 & \dots & y_p \end{bmatrix}^T \in \mathbb{R}^p$ are the input and the output of the system (5), respectively, such that $p \le k$; $F(\underline{x}) = \begin{bmatrix} f_1 & f_2 & \dots & f_p \end{bmatrix}^T \in \mathbb{R}^p$ is a vector of bounded unknown nonlinear continuous functions, and $G(\underline{x}) = \begin{bmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,k} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,k} \\ \vdots & \vdots & \vdots & \vdots \\ g_{p,1} & g_{p,2} & \cdots & g_{p,k} \end{bmatrix} \in \mathbb{R}^{p\times k}$ is a matrix of bounded unknown nonlinear continuous functions; $\underline{x} = \begin{bmatrix} x^T, \dot{x}^T, \dots, x^{(n-1)T} \end{bmatrix}^T \in \mathbb{R}^q$ represent the system state vector, with $x = \begin{bmatrix} x_1 & x_2 & \dots & x_p \end{bmatrix}^T$ is the first element of $\underline{x}$, and $q = n \times p$;
$D = \begin{bmatrix} d_1 & d_2 & \dots & d_p \end{bmatrix} \in \mathbb{R}^p$ represents unknown bounded disturbances.

Assume that $G(\underline{x})$ is a non null matrix, and let $G^+(\underline{x})$ denote the Moore-Penrose pseudo-inverse Matrix of $G(\underline{x})$.

4. TRACKING CONTROL DESIGN METHOD

The control problem is to steer the state x of the system (5) to a desired state $x_d = \begin{bmatrix} x_d^1 & x_d^2 & \dots & x_d^p \end{bmatrix}^T$. Hence the first step in designing a robust tracking control despite unknown dynamics and unknown disturbances that affect the system (5) is to effectively estimate the unknown dynamics. Thus, in this paper, $F(\underline{x})$ and $G(\underline{x})$ are substituted by their IT2-AFSs.

Based on the IT2-FS defined in (1) and characterized by its output expressed in (3), the IT2-AFS outputs used to estimate the unknown nonlinear functions f_i and $g_{i,j}$ can be given as:

$$\begin{aligned} \hat{f}_i\left(\underline{x}, \theta_f(i)\right) &= \xi_f^T(i)\theta_f(i) \\ \hat{g}_{i,j}\left(\underline{x}, \theta_g(i,j)\right) &= \xi_g^T(i,j)\theta_g(i,j) \end{aligned} , \quad \begin{aligned} i &= 1,\dots,p \\ j &= 1,\dots,k \end{aligned} \tag{6}$$

where $\xi_f^T(i) = \frac{1}{2}\left(\xi_f(i)_l^T + \xi_f(i)_r^T\right)$ and $\xi_g^T(i,j) = \frac{1}{2}\left(\xi_g(i,j)_l^T + \xi_g(i,j)_r^T\right)$, such that:

$\xi_f(i)_l = \begin{bmatrix} \xi_f(i)_l^1 & \xi_f(i)_l^2 & \dots & \xi_f(i)_l^M \end{bmatrix}^T$ and $\xi_f(i)_r = \begin{bmatrix} \xi_f(i)_r^1 & \xi_f(i)_r^2 & \dots & \xi_f(i)_r^M \end{bmatrix}^T$ are respectively the lower and upper vectors of fuzzy basis functions, as described by equation (4), and they are used to estimate $F(\underline{x})$;
likewise,
$\xi_g(i,j)_l = \begin{bmatrix} \xi_g(i,j)_l^1 & \xi_g(i,j)_l^2 & \dots & \xi_g(i,j)_l^M \end{bmatrix}^T$ and $\xi_g(i,j)_r = \begin{bmatrix} \xi_g(i,j)_r^1 & \xi_g(i,j)_r^2 & \dots & \xi_g(i,j)_r^M \end{bmatrix}^T$ are the vectors of fuzzy basis functions used to estimate $G(\underline{x})$; $\theta_f(i) = \begin{bmatrix} \theta_f^1(i) & \theta_f^2(i) & \dots & \theta_f^M(i) \end{bmatrix}^T$ and $\theta_g(i,j) = \begin{bmatrix} \theta_g^1(i,j) & \theta_g^2(i,j) & \dots & \theta_g^M(i,j) \end{bmatrix}^T$ are the online tuned parameter vectors of the corresponding IT2-AFS defined by equation (6).

4.1 Interval Type-2 Adaptive Fuzzy Super Twisting Control Design: Step 1

STC is an effective robust second order SMC capable to drive the system state trajectories with a continuous control law to the desired dynamics in finite time despite the existence of uncertainties and disturbances.

Let $e = \begin{bmatrix} e_1 & e_2 & \dots & e_p \end{bmatrix}^T = x_d - x$ be the tracking error. Then, the sliding surface for the system (5) is designed as [41]:

$$\begin{aligned} s(\underline{x},t) &= \begin{bmatrix} s_1 & s_2 & \dots & s_p \end{bmatrix}^T \\ &= \left(\frac{\partial}{\partial t} + \lambda\right)^{(n-1)} e \\ &= \sum_{j=0}^{n-1} \frac{(n-1)!}{j!(n-j-1)!} \lambda^j \left(\frac{\partial}{\partial t}\right)^{(n-j-1)} e \end{aligned} \tag{7}$$

where $\lambda = diag\left(\lambda_i\right)_{1 \le i \le p} \in \mathbb{R}^{(p\times p)}$, is a diagonal matrix, with λ_i is a positive constant associated to the sliding surface s_i.

The time derivative of the sliding surface $s(\underline{x},t)$ is given as:

$$\dot{s}(\underline{x},t) = x_d^{(n)} - F(\underline{x}) - G(\underline{x})u - D + \delta \tag{8}$$

where $\delta = \sum_{j=1}^{n-1} \frac{(n-1)!}{j!(n-j-1)!} \lambda^j \left(\frac{\partial}{\partial t}\right)^{(k-j-1)} \dot{e}$

The main objective of the tracking control using STC is to ensure that the condition $s_i = \dot{s}_i = 0$ $(i = 1,\dots,p)$ is verified in finite time.

The optimal parameters of $\hat{f}_i\left(\underline{x}, \theta_f(i)\right)$ and $\hat{g}_{i,j}\left(\underline{x}, \theta_g(i,j)\right)$ can be given by:

$$\theta_f^*(i) = \arg\min_{\theta_f(i)} \left(\sup_{\underline{x}} \left| \hat{f}_i - f_i \right| \right) \qquad i = 1,\dots,p$$
$$\theta_g^*(i,j) = \arg\min_{\theta_g(i,j)} \left(\sup_{\underline{x}} \left| \hat{g}_{i,j} - g_{i,j} \right| \right), \quad j = 1,\dots,k \tag{9}$$

Then, The minimum approximation error of f_i and $g_{i,j}$ can be expressed as:

$$\varepsilon_i = f_i^* - f_i + \sum_{j=1}^{k} \left(g_{i,j}^* - g_{i,j} \right) u_j \tag{10}$$

where $f_i^* = \xi_f^T(i)\theta_f^*(i)$ and $g_{i,j}^* = \xi_g^T(i,j)\theta_g^*(i,j)$ are the optimal approximations of f_i and $g_{i,j}$, respectively.

Based on STA and IT2-AFSs, the synthesized control law that ensures the predetermined objective of control is given as:

$$u = \begin{bmatrix} u_1 & u_2 & \dots & u_k \end{bmatrix}^T = \hat{G}^+ \left(x_d^{(n)} - \hat{F} + \delta - u_{ST} \right) \tag{11}$$

where $\hat{F} = \begin{bmatrix} \hat{f}_1 & \hat{f}_2 & \dots & \hat{f}_p \end{bmatrix}^T$ and $\hat{G}^+$ denote the Moore-Penrose pseudo-inverse Matrix of

$$\hat{G} = \begin{bmatrix} \hat{g}_{1,1} & \hat{g}_{1,2} & \cdots & \hat{g}_{1,k} \\ \hat{g}_{2,1} & \hat{g}_{2,2} & \cdots & \hat{g}_{2,k} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{g}_{p,1} & \hat{g}_{p,2} & \cdots & \hat{g}_{p,k} \end{bmatrix};$$

$u_{ST} = \begin{bmatrix} u_{ST}^1 & u_{ST}^2 & \dots & u_{ST}^p \end{bmatrix}^T$ is the reaching STC law (RSTCL), and it is given as [23]:

$$u_{ST}^i = -a_i \int_0^{t_i^r} sign(s_i)dt - b_i \left| s_i \right|^{0.5} sign(s_i), \; i = 1,\dots,p \tag{12}$$

where t_i^r denotes the reaching time of the sliding surface s_i to $s_i = 0$; a_i and b_i are the gains of u_{ST}^i.

The adaptation laws of the parameter vectors $\theta_f(i)$ and $\theta_g(i,j)$ are given as:

$$\dot{\theta}_f(i) = -\gamma_f(i)s(i)\xi_f(i) \qquad i = 1,\dots,p$$
$$\dot{\theta}_g(i,j) = -\gamma_g(i,j)s_i u_j \xi_g(i,j), \quad j = 1,\dots,k \tag{13}$$

where $\gamma_f(i)$ and $\gamma_g(i,j)$ are positive learning parameters.

Theorem 1. Consider the general class of MIMO nonlinear systems expressed in (5), the IT2-AFSs presented in (6) and the adaptation laws given in (13). Then, the control law defined in (11), it guarantees the stability of the closed loop system and ensures the desired tracking performance despite approximation errors, unknown disturbances and unknown dynamics

Proof 1. The following augmented Lyapunov function candidate is used to demonstrate the stability of the closed loop system:

$$v = \frac{1}{2}s^T s + \frac{1}{2}\sum_{i=1}^{p} \frac{\tilde{\theta}_f^T(i)\tilde{\theta}_f(i)}{\gamma_f(i)} + \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{k} \frac{\tilde{\theta}_g^T(i,j)\tilde{\theta}_g(i,j)}{\gamma_g(i,j)} \tag{14}$$

where

$\tilde{\theta}_f(i) = \theta_f(i) - \theta_f^*(i)$ and $\tilde{\theta}_g(i,j) = \theta_g(i,j) - \theta_g^*(i,j)$

The time derivative of v gives:

$$\dot{v} = s^T \dot{s} + \sum_{i=1}^{p} \frac{\dot{\theta}_f^T(i)\tilde{\theta}_f(i)}{\gamma_f(i)} + \sum_{i=1}^{p}\sum_{j=1}^{k} \frac{\dot{\theta}_g^T(i,j)\tilde{\theta}_g(i,j)}{\gamma_g(i,j)}$$
$$= s^T \left(x_d^{(n)} - F(\underline{x}) - G(\underline{x})u - D + \delta \right) + \tag{15}$$
$$\sum_{i=1}^{p} \frac{\dot{\theta}_f^T(i)\tilde{\theta}_f(i)}{\gamma_f(i)} + \sum_{i=1}^{p}\sum_{j=1}^{k} \frac{\dot{\theta}_g^T(i,j)\tilde{\theta}_g(i,j)}{\gamma_g(i,j)}$$

From equation 11, one gets:

$$x_d^{(n)} = \hat{F} + \hat{G}u + u_{ST} - \delta \tag{16}$$

Then, substituting the above expression in (15) gives:

$$\dot{v} = s^T \left(\hat{F} - F + (\hat{G} - G)u + u_{ST} - D \right) +$$
$$\sum_{i=1}^{p} \frac{\dot{\theta}_f^T(i)\tilde{\theta}_f(i)}{\gamma_f(i)} + \sum_{i=1}^{p}\sum_{j=1}^{k} \frac{\dot{\theta}_g^T(i,j)\tilde{\theta}_g(i,j)}{\gamma_g(i,j)} \tag{17}$$

By subtracting and adding the optimal approximations $F^* = \begin{bmatrix} f_1^* & f_2^* & \dots & f_p^* \end{bmatrix}^T$ and

$$G^* = \begin{bmatrix} g_{1,1}^* & g_{1,2}^* & \cdots & g_{1,k}^* \\ g_{2,1}^* & g_{2,2}^* & \cdots & g_{2,k}^* \\ \vdots & \vdots & \vdots & \vdots \\ g_{p,1}^* & g_{p,2}^* & \cdots & g_{p,k}^* \end{bmatrix}$$

into equation (17), one gets:

$$\dot{v} = s^T \left((\hat{F} - F^*) + (F^* - F) + (\hat{G} - G^*)u + (G^* - G)u \right) +$$
$$s^T(u_{ST} - D) + \sum_{i=1}^{p} \frac{\dot{\theta}_f^T(i)\tilde{\theta}_f(i)}{\gamma_f(i)} + \sum_{i=1}^{p}\sum_{j=1}^{k} \frac{\dot{\theta}_g^T(i,j)\tilde{\theta}_g(i,j)}{\gamma_g(i,j)}$$
$$= \sum_{i=1}^{p} s_i \xi_f^T(i)\tilde{\theta}_f(i) + \sum_{i=1}^{p}\sum_{j=1}^{k} s_i \xi_g^T(i,j)\tilde{\theta}_g(i,j)u_j + \sum_{i=1}^{p} s_i \varepsilon_i$$
$$+ \sum_{i=1}^{p} s_i(u_{ST}^i - d_i) + \sum_{i=1}^{p} \frac{\dot{\theta}_f^T(i)\tilde{\theta}_f(i)}{\gamma_f(i)} + \sum_{i=1}^{p}\sum_{j=1}^{k} \frac{\dot{\theta}_g^T(i,j)\tilde{\theta}_g(i,j)}{\gamma_g(i,j)}$$
$$= \sum_{i=1}^{p} \left(s_i \xi_f^T(i) + \frac{1}{\gamma_f(i)} \dot{\theta}_f^T(i) \right) \tilde{\theta}_f(i) +$$
$$\sum_{i=1}^{p}\sum_{j=1}^{k} \left(s_i u_j \xi_g^T(i,j) + \frac{1}{\gamma_g(i,j)} \dot{\theta}_g^T(i,j) \right) \tilde{\theta}_g(i,j) + \tag{18}$$
$$\sum_{i=1}^{p} s_i(u_{ST}^i - \varphi_i)$$

where $\varphi_i = d_i - \varepsilon_i$ such that $\left| \varphi_i \right| \le \phi_i$, with ϕ_i is a positive constant.

By considering the adaptation laws defined in (13) and using equation (12), one gets:

$$\dot{v} = \sum_{i=1}^{p} s_i \left(-a_i \int_0^{t_i^r} sign(s_i)dt - b_i |s_i|^{0.5} sign(s_i) - \varphi_i \right) \quad (19)$$

The gains a_i and b_i are chosen to satisfy the following condition:

$$a_i t_i^r + b_i |s_i|^{0.5} \geq \phi_i \ , \ i = 1,..,p \quad (20)$$

The inequality below guarantees that $\dot{v}$ becomes negative. And therefore, the proof 1 is completed.

The selection of good values of the gains a_i and b_i, which ensures the desired tracking performance while avoiding the chattering, remains among the most complicated tasks in real applications. The selection of large gains values can improve the tracking performance, but this cause much chattering in the control system, which may harm the system actuators. On the contrary, the small gains deteriorate the tracking performance and can even cause the system instability. In this study, to overcome this constraint of control, the optimal gains of the RSTCL u_{ST}^i are efficiently online estimated using IT2-AFSs, which guarantees both the desired tracking performance and a smooth continuous control input.

2.2 Interval Type-2 Adaptive Fuzzy Super Twisting Control Design: Step 2

Based on the IT2-FS defined in (1), which is characterized by its output expressed in (3), the terms $u_a(i) = -a_i \int_0^{t_i^r} sign(s_i)dt$ and $u_b(i) = -b_i |s_i|^{0.5} sign(s_i)$ of the RSTCL u_{ST}^i are substituted by their corresponding IT2-AFSs to excellently estimate the optimal gains that guarantee the desired tracking performance by ensuring the condition (20) while simultaneously avoiding the chattering.

The designed IT2-AFSs of $u_a(i)$ and $u_b(i)$, are respectively given as:

$$\begin{aligned} \hat{u}_a(i) &= \xi_a^T(i)\theta_a(i)t_i^r \\ \hat{u}_b(i) &= \xi_b^T(i)\theta_b(i)|s_i|^{0.5} \end{aligned} , \ i = 1,...,p \quad (21)$$

where

$$\xi_a(i) = \frac{1}{2}\left(\xi_a(i)_l + \xi_a(i)_r\right) = \left[\xi_a^1(i) \quad \xi_a^2(i) \quad \ldots \quad \xi_a^N(i)\right]^T$$

and

$$\xi_b(i) = \frac{1}{2}\left(\xi_b(i)_l + \xi_b(i)_r\right) = \left[\xi_b^1(i) \quad \xi_b^2(i) \quad \ldots \quad \xi_b^N(i)\right]^T$$

, such that $\xi_a(i)_l$ and $\xi_b(i)_l$ are the lower vectors of fuzzy basis functions, as described by equation (4); Likewise, $\xi_a(i)_r$ and $\xi_b(i)_r$ are the upper vectors of fuzzy basis functions;

$\theta_a(i) = \left[\theta_a^1(i) \quad \theta_a^2(i) \quad \ldots \quad \theta_a^N(i)\right]^T$ and

$\theta_b(i) = \left[\theta_b^1(i) \quad \theta_b^2(i) \quad \ldots \quad \theta_b^N(i)\right]^T$ are the online tuned parameter vectors of the corresponding IT2-AFS defined by the above equation (21); N denotes the fuzzy rules number.

The optimal parameters of the IT2-AFSs $\hat{u}_a(i)$ and $\hat{u}_b(i)$ can be given by:

$$\begin{aligned} \theta_a^*(i) &= \arg\min_{\theta_a(i)} \left(\sup_S |\hat{u}_a(i) - u_a(i)| \right) \\ \theta_b^*(i) &= \arg\min_{\theta_b(i)} \left(\sup_S |\hat{u}_b(i) - u_b(i)| \right) \end{aligned} \quad (22)$$

The interval type-2 adaptive fuzzy super twisting control (IT2-AFSTC) law proposed in this paper is given as:

$$\begin{aligned} u &= \left[u_1 \quad u_2 \quad \ldots \quad u_k\right]^T \\ &= \hat{G}^+ \left(x_d^{(n)} - \hat{F} + \delta - \hat{u}_{ST}\right) \end{aligned} \quad (23)$$

where

$\hat{u}_{ST} = \left[\hat{u}_{ST}(1) \quad \hat{u}_{ST}(2) \quad \ldots \quad \hat{u}_{ST}(p)\right]^T = \hat{u}_a + \hat{u}_b$ is the designed AF-RSTCL, such that

$\hat{u}_a = \left[\hat{u}_a(1) \quad \hat{u}_a(2) \quad \ldots \quad \hat{u}_a(p)\right]^T$ and

$\hat{u}_b = \left[\hat{u}_b(1) \quad \hat{u}_b(2) \quad \ldots \quad \hat{u}_b(p)\right]^T$.

The adaptation laws of the parameter vectors $\theta_a(i)$ and $\theta_b(i)$ are given as:

$$\begin{aligned} \dot{\theta}_a(i) &= -\gamma_a(i) s_i t_i^r \xi_a(i) \\ \dot{\theta}_b(i) &= -\gamma_b(i) |s_i|^{\frac{3}{2}} sign(s_i) \xi_b(i) \end{aligned} \quad (24)$$

where $\gamma_a(i)$ and $\gamma_b(i)$ are positive learning parameters.

Theorem 2. By considering the general class of MIMO nonlinear systems expressed in (5), the IT2-AFSs described by equations (6) and (21), and the adaptation laws given by equations (13) and (24), then, the global control law presented by equation (23) ensures the stability of the closed loop system with the best tracking performance while avoiding the chattering, despite all the constraints that influence the control system, such as unknown dynamics, approximation errors and unknown disturbances.

Proof 2. To demonstrate the stability of the closed loop system, the following augmented Lyapunov function is used:

$$\begin{aligned} v = &\frac{1}{2}s^T s + \frac{1}{2}\sum_{i=1}^{p} \frac{\tilde{\theta}_f^T(i)\tilde{\theta}_f(i)}{\gamma_f(i)} + \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{k} \frac{\tilde{\theta}_g^T(i,j)\tilde{\theta}_g(i,j)}{\gamma_g(i,j)} + \\ &\frac{1}{2}\sum_{i=1}^{p} \frac{\tilde{\theta}_a^T(i)\tilde{\theta}_a(i)}{\gamma_a(i)} + \frac{1}{2}\sum_{i=1}^{p} \frac{\tilde{\theta}_b^T(i)\tilde{\theta}_b(i)}{\gamma_b(i)} \end{aligned} \quad (25)$$

where $\tilde{\theta}_a(i) = \theta_a(i) - \theta_a^*(i)$ and $\tilde{\theta}_b(i) = \theta_b(i) - \theta_b^*(i)$

By considering equations (14), (19) and (23), the time derivative of equation (25) can be given as:

$$\dot{v} = \sum_{i=1}^{p} s_i\left(\hat{u}_a(i)+\hat{u}_b(i)-\varphi_i\right)+ \sum_{i=1}^{p}\left(\frac{1}{\gamma_a(i)}\dot{\theta}_a^T(i)\tilde{\theta}_a(i)+\frac{1}{\gamma_b(i)}\dot{\theta}_b^T(i)\tilde{\theta}_b(i)\right) \tag{26}$$

Let $\hat{u}_a^*(i)=\xi_a^T(i)\theta_a^*(i)t_i^r=-a_i^*\int_0^{t_i^r} sign(s_i)dt$ and $\hat{u}_b^*(i)=\xi_b^T(i)\theta_b^*(i)\left|s_i\right|^{0.5}=-b^*(i)\left|s_i\right|^{0.5} sign(s_i)$ be respectively the optimal estimation of $u_a(i)$ and $u_b(i)$ that guarantees both the condition (20) and chattering avoidance. Indeed, $\hat{u}_a^*(i)$ and $\hat{u}_b^*(i)$ provide the optimal smooth gains a_i^* and b_i^* for the RSTCL u_{ST}^i, which allows to handle efficaciously the perturbations φ_i by ensuring the inequality (20) with a smooth continuous control input.

Since a_i^* and b_i^* are the optimal gains of a_i and b_i, then, one gets:

$$a_i^* t_i^r + b_i^*\left|s_i\right|^{0.5} \ge \phi_i \ , \ i=1,..,p \tag{27}$$

By subtracting and adding $\hat{u}_a^*(i)$ and $\hat{u}_b^*(i)$ into equation (26), one gets:

$$\begin{aligned}\dot{v} &= \sum_{i=1}^{p} s_i\left(\hat{u}_a(i)-u_a^*(i)+u_a^*(i)+\hat{u}_b(i)-u_b^*(i)+u_b^*(i)-\varphi_i\right)+ \sum_{i=1}^{p}\left(\frac{1}{\gamma_a(i)}\dot{\theta}_a^T(i)\tilde{\theta}_a(i)+\frac{1}{\gamma_b(i)}\dot{\theta}_b^T(i)\tilde{\theta}_b(i)\right)\\ &= \sum_{i=1}^{p} s_i\left(\xi_a^T(i)\tilde{\theta}_a(i)t_i^r+\xi_b^T(i)\tilde{\theta}_b(i)\left|s_i\right|^{0.5}\right)+ \sum_{i=1}^{p} s_i\left(u_a^*(i)+u_b^*(i)-\varphi_i\right)+\sum_{i=1}^{p}\left(\frac{1}{\gamma_a(i)}\dot{\theta}_a^T(i)\tilde{\theta}_a(i)+\frac{1}{\gamma_b(i)}\dot{\theta}_b^T(i)\tilde{\theta}_b(i)\right)\\ &= \sum_{i=1}^{p}\left(s_i t_i^r \xi_a^T(i)+\frac{1}{\gamma_a(i)}\dot{\theta}_a^T(i)\right)\tilde{\theta}_a(i)+ \sum_{i=1}^{p}\left(\left|s_i\right|^{\frac{3}{2}} sign(s_i)\xi_b^T(i)+\frac{1}{\gamma_b(i)}\dot{\theta}_b^T(i)\right)\tilde{\theta}_b(i)+ \sum_{i=1}^{p} s_i\left(u_a^*(i)+u_b^*(i)-\varphi_i\right)\end{aligned} \tag{28}$$

By considering equations (24) and (27), one gets:

$$\begin{aligned}\dot{v} &= \sum_{i=1}^{p} s_i\left(u_a^*(i)+u_b^*(i)-\varphi_i\right)\\ &= \sum_{i=1}^{p} s_i\left(-a_i^*\int_0^{t_i^r} sign(s_i)dt-b^*(i)\left|s_i\right|^{0.5} sign(s_i)-\varphi_i\right)\\ &\le 0\end{aligned} \tag{29}$$

Thus, the proof 2 is completed.

5. SIMULATION RESULTS AND DISCUSSIONS

5.1 Dynamical Model of a Two Link Robot Manipulator

We present in this section the simulation results of a two link robot manipulator in order to confirm the effectiveness of the developed method of tracking control. A schematic representation of a two link robot manipulator is given in Figure 1.

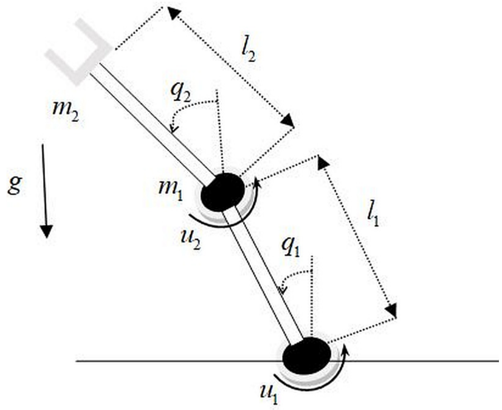

Figure 1. A schematic representation of a two link robot manipulator

where l_1 and l_2 are the links lengths; $q=\left[q_1 \quad q_2\right]^T$ is the vector of joint angles; m_1 and m_2 are the masses at the end of each joint axe; and g is the gravity acceleration.

The dynamical model of a two link robot manipulator can be expressed as:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G_r(q)=u+d \tag{30}$$

where $M(q)=\begin{bmatrix}(m_1+m_2)\ l_1^2 & M' \\ M' & m_2\ l_2^2\end{bmatrix}$ is the inertia matrix, such that: $M'=m_2 l_1 l_2\left(\sin(q_1)\sin(q_2)+(\cos(q_1)\cos(q_2)\right)$; $q,\dot{q},\ddot{q}\in\mathbb{R}^2$ are the vectors of joint positions, velocities, and accelerations, respectively;

$C=C'\begin{bmatrix}0 & -\dot{q}_2 \\ -\dot{q}_1 & 0\end{bmatrix}$ is the centripetal Coriolis matrix, such that : $C'=m_2 l_1 l_2\left(\cos(q_1)\sin(q_2)-\sin(q_1)\cos(q_2)\right)$; $G_r=\left[-(m_1+m_1)l_1 g\sin(q_1) \quad -m_2 l_2 g\sin(q_2)\right]^T$ is the gravity vector; $u=\left[u_1 \quad u_2\right]^T$ is the control torque vector; $d=\left[d_1 \quad d_2\right]^T$ denotes the unknown disturbances.

Let $x=\left[x_1 \quad x_2\right]^T=\left[q_1 \quad q_2\right]^T$. Then, by introducing the state vector $\underline{x}=\left[x^T \quad \dot{x}^T\right]^T=\left[q^T \quad \dot{q}^T\right]^T$ in equation (30), one gets:

$$\ddot{x}=f(\underline{x})+g(x)u+\underline{d} \tag{31}$$

where $f=-M^{-1}(C\dot{q}+G_r)$, $g=M^{-1}$ and $\underline{d}=M^{-1}d$

We assume that the masses m_1 and m_2 are unknown, and that they are subject to the time variation Δm_1 and Δm_2, respectively. Hence, the system (31) can be reformulated as:

$$\begin{aligned}\ddot{x} &= \left(f(\underline{x})+\Delta f(\underline{x})\right)+\left(g(x)+\Delta g(x)\right)u+\Delta \\ &= F(\underline{x})+G(x)u+\Delta\end{aligned} \quad (32)$$

where $\Delta f(\underline{x})$ and $\Delta g(x)$ represent the time varying disturbances that influence the system dynamics caused by the mass variation vector $\Delta m=\begin{bmatrix}\Delta m_1 & \Delta m_2\end{bmatrix}^T$; and $\Delta=(M+\Delta M)^{-1}d$, such that ΔM represents the parametric variation of the inertia matrix M caused by the variation Δm.

The dynamics $f(\underline{x})$ and $g(x)$ of the system (32) are assumed to be unknown and suffer respectively from the unknown time varying disturbances $\Delta f(\underline{x})$ and $\Delta g(x)$. Therefore, we can apply the developed control law expressed by equation (23) to achieve the desired objective of control.

5.2 Simulation results

The physical parameters of the robot manipulator used for simulation are as follows:
$l_1=l_2=0.5m$; $m_1=2kg$ and $m_2=1kg$;
and $g=9.8m/s^2$.

The mass variation vector is given as:

$$\Delta m(kg)=\begin{bmatrix}0.5\sin(t) & 0.4\sin(t)\end{bmatrix}^T.$$

The unknown disturbances vector is represented as:

$$\Delta=(M+\Delta M)^{-1}\begin{bmatrix}0.6\sin(2t)+0.4\sin(\dot{q}_1)+0.2q_1 \\ 0.5\sin(2t)+0.4\sin(\dot{q}_2)+0.2q_2\end{bmatrix}$$

The initial condition of the vector of joint angles is given as $q(rad)=[1.2 \quad 0.4]^T$.

The sliding surface is expressed as:

$$s=\begin{bmatrix}s_1 & s_2\end{bmatrix}^T=\dot{e}+\lambda e$$

where $e=\begin{bmatrix}e_1 & e_2\end{bmatrix}^T=\begin{bmatrix}q_{1d}-q_1 & q_{2d}-q_2\end{bmatrix}^T$ is the tracking error vector, with q_{1d} and q_{2d} denote the desired joint angles; and $\lambda=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}$, such that λ_1 and λ_2 denote respectively the slopes of s_1 and s_2.

We assume that q_1 and q_2 belong to $\left[-\frac{\pi}{2} \quad \frac{\pi}{2}\right]$ (rad), and $\dot{q}_1$ and $\dot{q}_2$ belong to $\left[-\frac{\pi}{2} \quad \frac{\pi}{2}\right]$ (rad/s).

The main objective of control is to force the state $q=\begin{bmatrix}q_1 & q_2\end{bmatrix}^T$ to track the desired reference $q_d=\begin{bmatrix}q_{1d} & q_{2d}\end{bmatrix}^T=\begin{bmatrix}\sin(t) & \cos(t)\end{bmatrix}^T$.

The designed IT2-AFSTC law defined by equation (23) can be expressed for the two-link robot manipulator system (32) as:

$$\begin{aligned}u &= \begin{bmatrix}u_1 & u_2\end{bmatrix}^T \\ &= \hat{G}^{-1}\left(\ddot{q}_d-\hat{F}+\lambda\dot{e}-\hat{u}_{ST}\right)\end{aligned} \quad (33)$$

The IT2-AFS $\hat{F}$ uses four inputs $q_1, q_2, \dot{q}_1$ and $\dot{q}_2$ to estimate F, whereas the IT2-AFS $\hat{G}$ uses two inputs q_1 and q_2 to estimate G.

Three MFs are associated to each input of the IT2-AFS $\hat{F}$, as shown in Figure 2. Likewise, three MFs are associated to each input of the IT2-AFS $\hat{G}$, as depicted in Figure 3.

The AF-RSTCL $\hat{u}_{ST}$ has the sliding surface as input vector, and uses three MFs for each of its inputs, as presented in Figure 4.

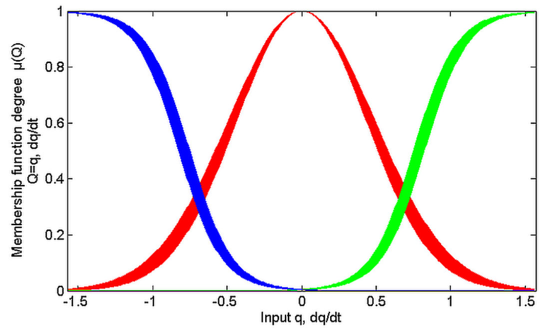

Figure 2. Membership functions used by the IT2-AFS $\hat{F}$

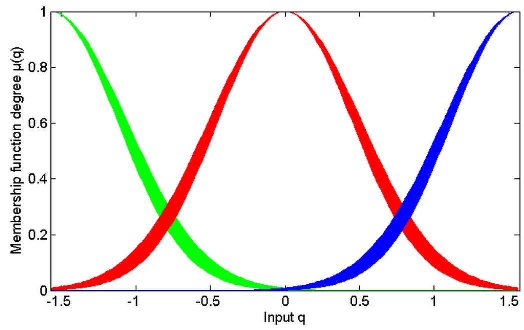

Figure 3. Membership functions used by the IT2-AFS $\hat{G}$

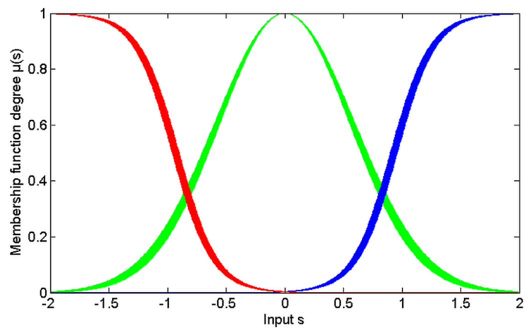

Figure 4. Membership functions used by the AF-RSTCL $\hat{u}_{ST}$

In order to confirm the effectiveness of the designed tracking control method, a comparison was carried out with two approaches of tracking control, namely 1) fuzzy SMC (FSMC) approach, and 2) fuzzy STC

(FSTC) approach. The first approach uses two type-1 adaptive fuzzy systems (T1-AFSs) to approximate F and G, and a conventional reaching SMC law to deal with the unknown disturbances and approximation errors; and the second approach uses two T1-AFSs to approximate F and G, and a STC law to reject the undesired effects caused by unknown disturbances and approximation errors.

The control laws used by the FSMC and FSTC approaches are given respectively by equations (34) and (35):

$$V = [V_1 \quad V_2]^T = \bar{G}^{-1}\left(\ddot{q}_d - \bar{F} + \lambda\dot{e} - v_{AFSMC}\right) \tag{34}$$

$$U = [U_1 \quad U_2]^T = \underline{G}^{-1}\left(\ddot{q}_d - \underline{F} + \lambda\dot{e} - u_{AFSTC}\right) \tag{35}$$

ehere ($\bar{F}$, $\bar{G}$) and ($\underline{F}$, $\underline{G}$) are the T1-AFSs respectively used by the FSMC and FSTC algorithms to approximate F and G; $v_{AFSMC} = -\begin{bmatrix}\mu_1 & 0\\ 0 & \mu_2\end{bmatrix}\begin{pmatrix}sign(s_1)\\ sign(s_2)\end{pmatrix}$ is the reaching SMC law, μ_1 and μ_2 are positive constants;

$$u_{AFSTC} = -\begin{bmatrix}\alpha_1 & 0\\ 0 & \alpha_2\end{bmatrix}\begin{bmatrix}\int_0^t sign(s_1)dt\\ \int_0^t sign(s_2)dt\end{bmatrix} - \begin{bmatrix}\beta_1 & 0\\ 0 & \beta_2\end{bmatrix}\begin{bmatrix}|s_1|^{0.5}\\ |s_2|^{0.5}\end{bmatrix}$$

,with α_1, α_2, β_1 and β_2 are the gains of the fuzzy reaching STC law u_{AFSTC} of the FSTC approach.

The MFs used by ($\bar{F}$, $\underline{F}$) and ($\bar{G}$, $\underline{G}$) are depicted respectively in Figures 5 and 6.

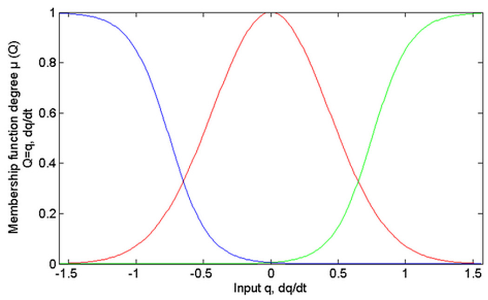

Figure 5. Membership functions used by the two T1-AFSs $\bar{F}$ and $\underline{F}$

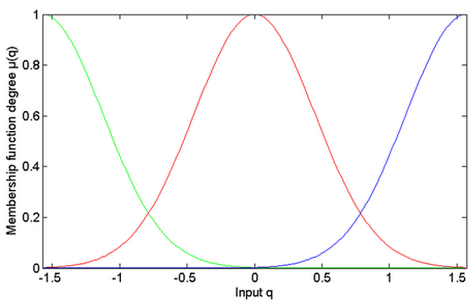

Figure 6. Membership functions used by the two T1-AFSs $\bar{G}$ and $\underline{G}$

The constant parameters used by the proposed IT2-AFSTC, FSMC and FSTC approaches, are given in Table 1 below.

Table 1. The constant parameters used by the three control approaches

Parameters	IT2-AFSTC	FSTC	FSMC
λ_1	5	5	5
λ_2	6	6	6
$\gamma_f(1)$	130	225	220
$\gamma_f(2)$	140	230	227
$\gamma_g(1,1)$	0.005	0.006	0.005
$\gamma_g(1,2)$	0.003	0.002	0.002
$\gamma_g(2,1)$	0.002	0.001	0.001
$\gamma_g(2,2)$	0.004	0.004	0.003
$\gamma_a(1)$	2800	-	-
$\gamma_a(2)$	2880	-	-
$\gamma_b(1)$	200	-	-
$\gamma_b(2)$	120	-	-
α_1	-	2	-
α_1	-	3	-
β_1	-	6	-
β_2	-	7	-
μ_1	-	-	2.5
μ_2	-	-	2.2

The simulation results obtained from the comparison that was carried out between the three approaches of control are illustrated in Figures 7-16. Figures 7 and 8 depict the tracking error trajectories; the evolution of joint angles vector $q = [q_1 \quad q_2]^T$ and its desired reference $q_d = [q_{1d} \quad q_{2d}]^T$ are represented in Figures 9 and 10; Figures 11-16 depict the different control laws of the three compared approaches of control.

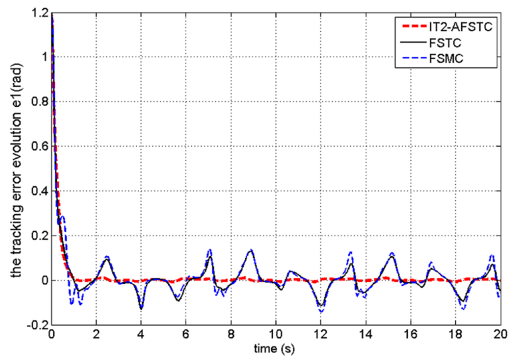

Figure 7. The tracking error $e_1(rad)$ of the three approaches of control

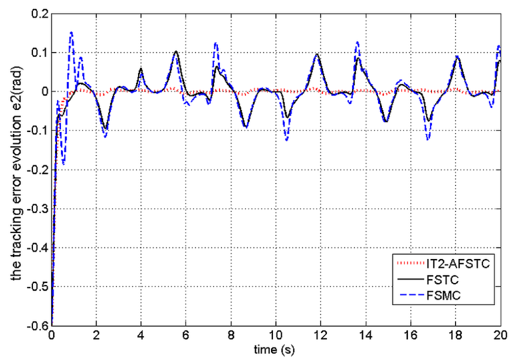

Figure 8. The tracking error $e_2(rad)$ of the three approaches of control

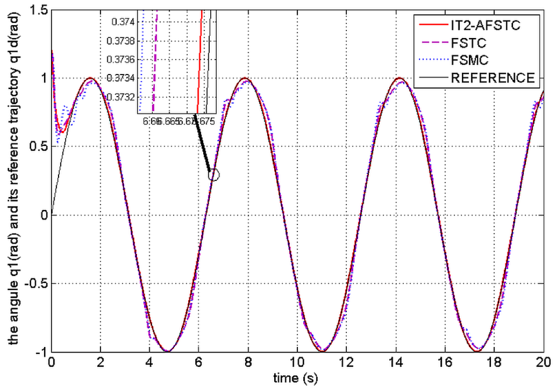

Figure 9. The angle $q_1(rad)$ and its desired reference $q_{1d}(rad)$ of the three approaches of control

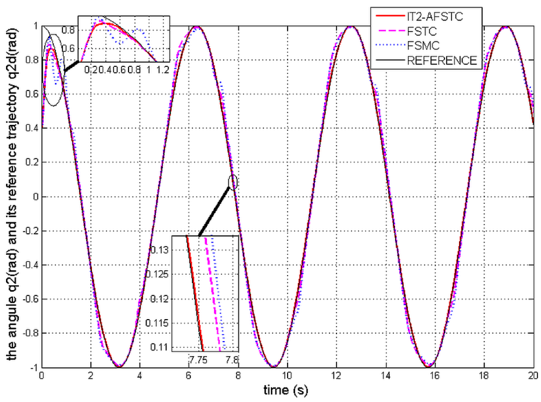

Figure 10. The The angle $q_2(rad)$ and its desired reference $q_{2d}(rad)$ of the three approaches of control

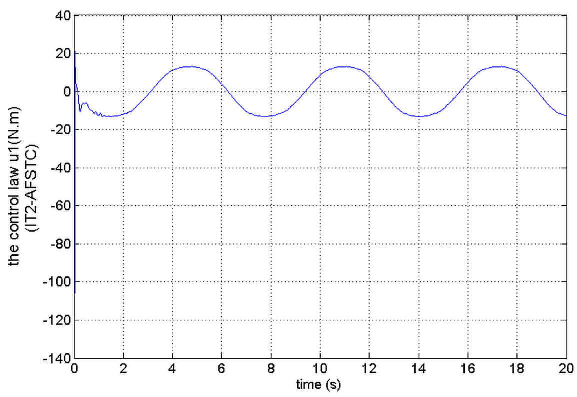

Figure 11. The control input $u_1(N.m)$ of the proposed IT2-AFSTC approach

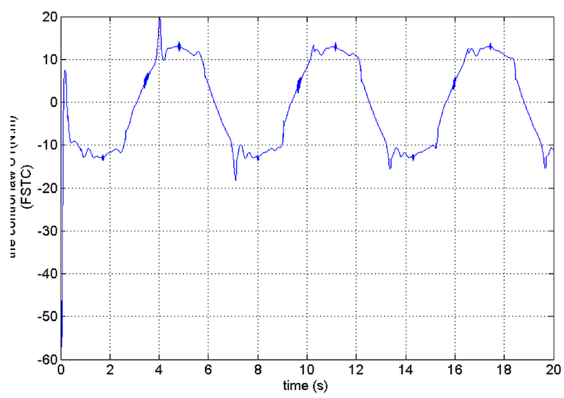

Figure 12. The control input $U_1(N.m)$ of the FSTC approach

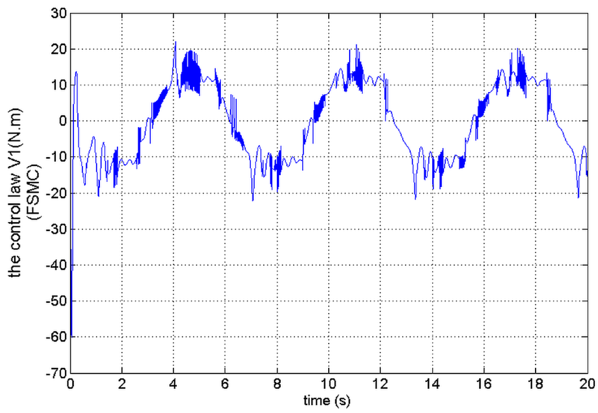

Figure 13. The control input $V_1(N.m)$ of the FSMC approach

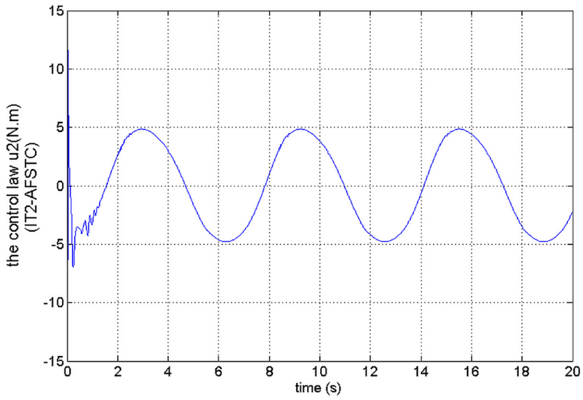

Figure 14. The control input $u_2(N.m)$ of the proposed IT2-AFSTC approach

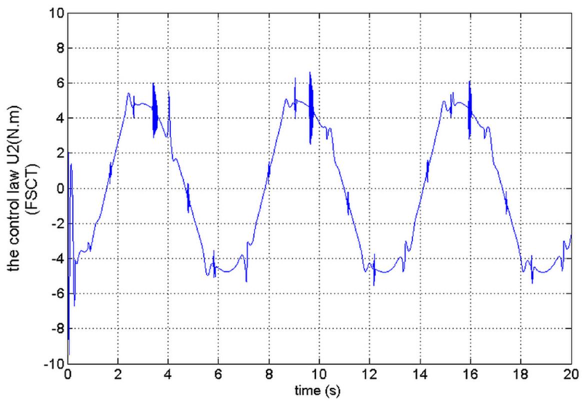

Figure 15. The control input $U_2(N.m)$ of the FSTC approach

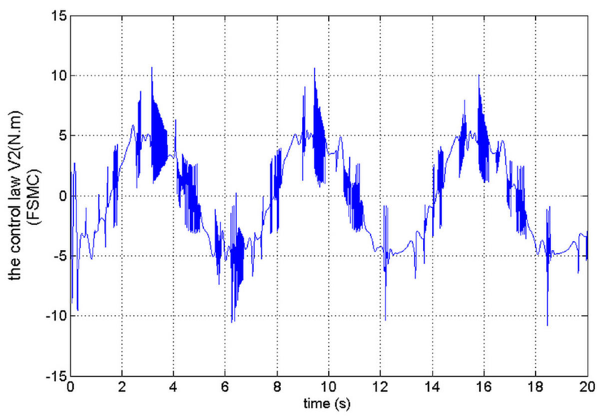

Figure 16. The control input $V_2(N.m)$ of the FSMC approach

In comparison with the FSTC and FSMC approaches, the proposed IT2-AFSTC approach shows the best tracking performance with a smooth continuous generated control input vector.

This superiority of the IT2-AFSTC approach in ensuring the desired objective of control is due to its both features, namely 1) an optimal estimation of unknown dynamics, and 2) a great efficiency in rejecting all disturbances that influence the system robustness. On the other hand, the FSTC approach presents the better tracking performance in comparison with the FSMC.

In Figures 17 and 18 below, we notice that the more the gains of u_{ASFTC} and v_{AFSMC} become large, the more the tracking performance increases. However, this improvement in tracking performance comes at the expense of the smoothness of the applied control law. Indeed, as shown in Figures 19-22, the FSTC law presents big variations, and in the FSMC law, the chattering becomes more severe, which may be harmful to the system actuators.

And yet, even with this improvement in tracking control performance of both FSTC and FSMC approaches, which generate the chattering in their control inputs, the proposed IT2-AFSTC approach still shows better tracking performance while avoiding the chattering.

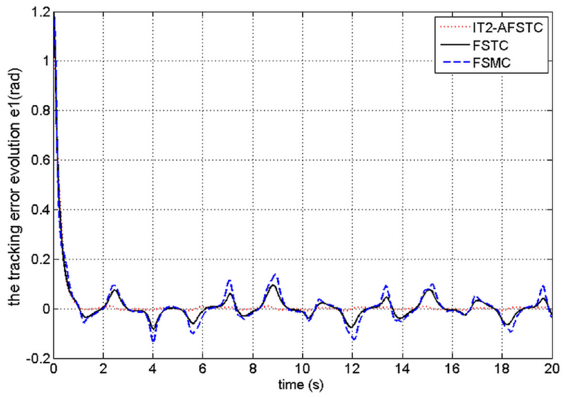

Figure 17. The tracking error $e_1(rad)$ of the three approaches of control, for $\alpha_1 = 7,\ \alpha_2 = 8,\ \beta_1 = 12,\ \beta_2 = 15$; $\mu_1 = 5,\ \mu_2 = 7$

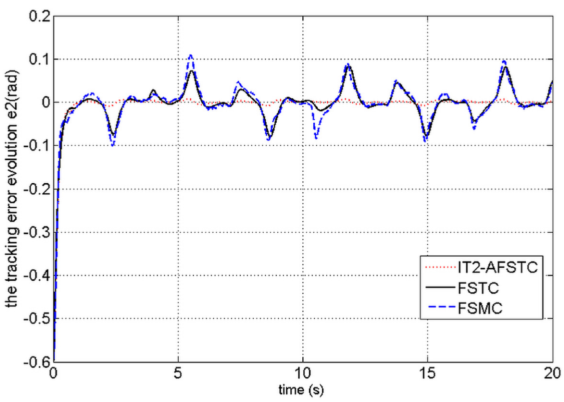

Figure 18. The tracking error $e_2(rad)$ of the three approaches of control, for $\alpha_1 = 7,\ \alpha_2 = 8,\ \beta_1 = 12,\ \beta_2 = 15$; $\mu_1 = 5,\ \mu_2 = 7$

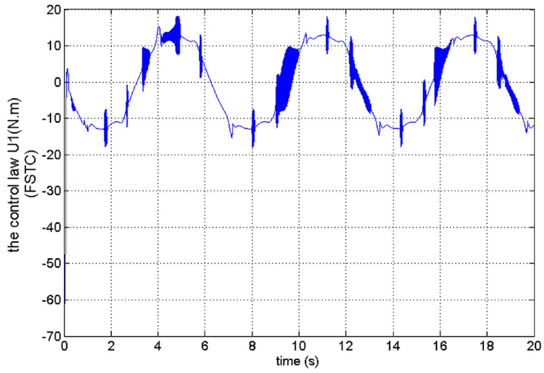

Figure 19. The control input $U_1(N.m)$ of the FSTC approach, for $\alpha_1 = 7,\ \alpha_2 = 8,\ \beta_1 = 12,\ \beta_2 = 15$

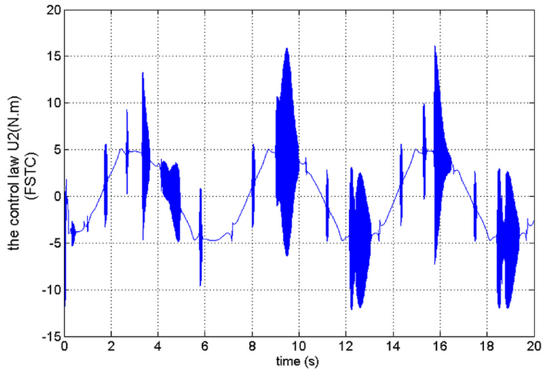

Figure 20. The control input $U_2(N.m)$ of the FSTC approach, for $\alpha_1 = 7,\ \alpha_2 = 8,\ \beta_1 = 12,\ \beta_2 = 15$

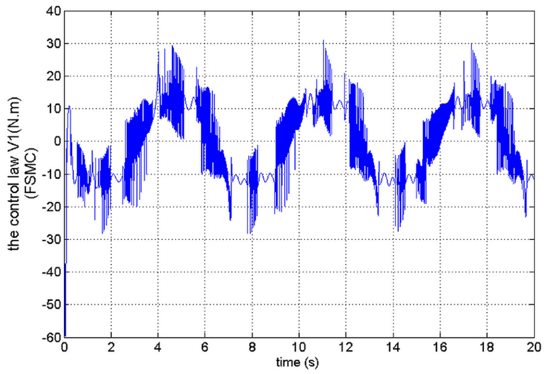

Figure 21. The control input $V_1(N.m)$ of the FSMC approach, for $\mu_1 = 5,\ \mu_2 = 7$

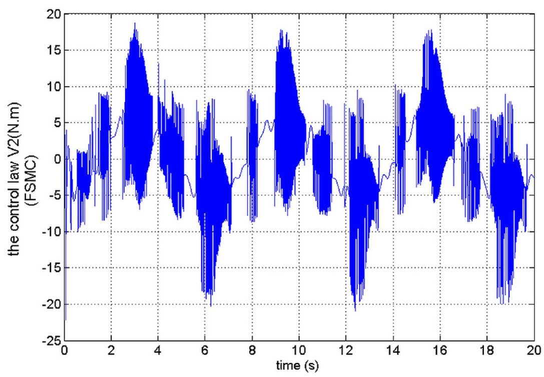

Figure 22. The control input $V_2(N.m)$ of the FSMC approach, for $\mu_1 = 5,\ \mu_2 = 7$

6. CONCLUSION

In this study, we designed a new rigorous tracking control approach for a general class of disturbed MIMO nonlinear systems with unknown dynamics. First, in order to effectively estimate the unknown dynamics, two IT2-AFSs have been constructed. Then, in the presence of approximation errors and unknown disturbances, a new robust synthesized AF-RSTCL has been introduced. Based on IT2-AFSs, the AF-RSTCL gains are optimally online estimated in order to guarantee the best tracking control performance while simultaneously avoiding the chattering. The adaptation laws are obtained using the stability analysis of Lyapunov. The simulation results confirm the mathematical proof of the proposed approach of control in ensuring the desired tracking performance with a smooth continuous control law, despite all the constraints that disturb the control system.

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ДИЗАЈН РОБУСТНОГ IT2 АДАПТИВНОГ ФАЗИ СУПЕР УПРАВЉАЊА УВИЈАЊЕМ ЗА ДАТУ КЛАСУ MIMO НЕЛИНЕАРНИХ СИСТЕМА СА ПОРЕМЕЋАЈЕМ

Н. Нафиа, А. Ел Кари, Х. Ајад, М. Мјахед

Рад се бави проблемом контроле праћења велике класе MIMO нелинеарних система са непознатом динамиком који су изложени поремећајима непознатог порекла. Прво су конструисана два IT2-AFS у циљу ефикасне процене непознате нелинеарне динамике. Потом је на основу IT2-AFS и STA алгоритма нови робустни адаптивни фази STC закон (AF-RSTCL) додат закону глобалног управљања у циљу побољшања робустности система уз апроксимацију грешака и поремећаје непознатог порекла. Да би се избегао феномен четеринга и истовремено гарантовале најбоље перформансе праћења учинка дизајнираног AF-RSTCL праћење је вршено онлајн. Адаптивни параметри глобалног закона управљања добијеног синтезом су изведени из анализе стабилности у смислу Љапунова. На једном примеру симулације потврђена је ефикасност развијене методе у постизању унапред одређених циљева контроле праћења.