

# Analytical Assessment of the Sliding Friction Coefficient Influence on Durability, Wear and Contact Pressure in Spur Gears

**M. Chernets**

Professor  
National Aviation University  
Aerospace faculty  
Ukraine

**A. Kornienko**

Associate Professor  
National Aviation University  
Aerospace faculty  
Ukraine

**Yu. Chernets**

Ph.D. student  
National Aviation University  
Aerospace faculty  
Ukraine

**S. Fedorchuk**

Senior lecturer  
National Aviation University  
Aerospace faculty  
Ukraine

*The research of influence of sliding friction coefficient on durability is carried out for gear train containing steel wheels and metal-polymer gear trains containing polyamide gears reinforced with carbon and glass dispersion fibers. The teeth engagement conditions (two teeth pairs – single tooth pair – two teeth pairs) and the change of teeth tribocontact interaction conditions due to wear are also taken into account. The gear train containing the carbon-filled composite gear has the highest durability in comparison with other types in all ranges of change of sliding friction coefficient. The highest maximum contact pressures will be at the point of entry into the one-pair engagement. The change of initial maximum contact pressures in gear train due to tooth wear was also investigated and its regularities were established. The kinetics of the tooth profile wear was studied. It is established that the maximum wear will be at the point of entry into the one-pair engagement. Close to it will be the wear at the entrance into the two-pair engagement. The course of wear at different points of engagement for the studied gear trains containing steel toothed wheels and gear trains containing steel and composite gears is almost the same except for the point of the teeth exit from the engagement.*

**Keywords:** involute spur gears, sliding friction coefficient, durability, contact pressure, wear

## 1. INTRODUCTION

In order to effectively reduce the friction coefficient in tribomechanical systems of various kinds, lubricants are widely used, including the use of antifriction and anti-wear additives, resulting in reduced wear and increased durability. In particular, this applies to gears. One of the fundamental factors determining the conditions of friction in the tribological system is the sliding friction coefficient  $f$ . Therefore, the study of the influence of this parameter on the gearing service life at the boundary, mixed and dry friction is of great practical importance. In addition, in metal-polymer gears without lubrication in the case of dry friction, the modification of polymers with dispersed particles of micron size or short fibers is used to adjust the value of the friction coefficient. In the literature of the subject there are no studies of the friction coefficient influence on the change of durability of gears made of metal (MM) and non-metallic materials using known computational methods [1-10]. Only according to the author's calculation method in [11] the influence of the coefficient of sliding friction on the durability of spur MM gears of an electric locomotive under changing contact conditions in the corrected gearing due to teeth wear was inves-

tigated. Regarding metal-polymer (MP) gears, there are no calculation methods for their study, except for the author's [12,13].

In the literature on the subject there are only some quantitative results of experimental studies of wear of polymer composite materials for MP gears [14 - 16], which can be used to predict their wear using existing calculation methods. In [14] the experimental and numerical study a loaded cylindrical PA66 gear was given. In [15], the results of experimental studies of the influence of the sliding speed on the friction force for NaPA6/steel S355J2 pairs with twin-disc setup are presented. In [16], quite extensive results of experimental studies in air and abrasive mediums of the coefficient of friction and relative volumetric wear of model MP gears for several types of polyamide composites (PA6-Mg, PA6-Na, PA66 + 30GF, POM-C) and steel S355 are presented. The results of these further studies are given in [17].

Using the method [18-22] of the estimated assessment of gearing wear and durability, the influence of the friction coefficient on the durability of metal and metal-polymer gearings under the same operating conditions will be investigated. In contrast to the above methods of gearing calculation based on the Archard law of abrasive wear, which is virtually absent in gearings, the author's method, based on the known phenomenological calculation model of friction-fatigue wear at sliding friction [23], takes into account the influence of teeth wear on durability and conditions of their engagement.

Received: January 2021, Accepted: March 2021

Correspondence to: Dr Myron Chernets  
National Aviation University, Aerospace faculty,  
1, Liubomyra Huzara ave., Kyiv, Ukraine, 03058  
E-mail: myron.czerniec@gmail.com

doi:10.5937/fme2102472C

© Faculty of Mechanical Engineering, Belgrade. All rights reserved

FME Transactions (2021) 49, 472-479 472

## 2. METHODS

### 2.1 Method of calculating linear wear

To determine the linear wear of the teeth  $h_{kijn}$  at any point  $j$  of the working surface for the cycle of interaction, the ratio [19-21] is used:

$$h'_{kijn} = \frac{v_j t'_{jh} (f p_{jh \max})^{m_k}}{C_k \tau_s^{m_k}} \quad (1)$$

where  $t_{jh} = 2b_{jh} / v_0 = \text{var}$  is the time of tribocontact interaction (wear) of the teeth during the movement of the  $j$ -th point of their contact along the contour of the tooth to the contact area variable width due to wear  $2b_{jh}$ ;  $j = 1, 2, 3, \dots$  are points of interaction on the teeth profiles from the entrance to the out of contact;  $v_0 = \omega_1 r_1 \sin \alpha$  is the velocity of movement of the contact point along the tooth profile;  $\omega_1$  is the pinion angular velocity;  $r_1$  is the pinion pitch circle radius;  $v_j$  is the sliding velocity;  $f$  is the sliding friction coefficient;  $p_{jh \max}$  is the maximum tribocontact pressure at the  $j$ -th point of interaction arising at the teeth wear;  $C_k, m_k$  are the wear resistance characteristics toothed wheels materials for selected conditions [18];  $\tau_s = 0.35R_m$  is the shear strength of metallic materials of toothed wheels;  $R_m$  is the tensile strength of materials;  $\tau_s = 0.5R_m$  is the shear strength of polymer composite materials of toothed wheels.

Due to teeth wear there is an increase in the radii of curvature of their working profiles. Accordingly, there will be a decrease in the initial maximum contact pressures  $p_{j \max}$  and an increase in the width of the contact areas  $2b_j$  at each  $j$ -th point of the teeth contact. Therefore, taking into account the teeth wear, the current values are calculated by modified Hertz formulas

$$p_{jh \max} = 0.564 \sqrt{N' \theta / \rho_{jh}}, \quad 2b_{jh} = 2.256 \sqrt{\theta N' \rho_{jh}}, \quad (2)$$

where  $N' = N/bw$ ;  $N = 9550P/r_1 n_1 \cos \alpha$  is the force acting in engagement;  $P$  is the power on the driving shaft;  $b$  is the width of the pinion;  $w$  is the number of pairs of meshing teeth;  $\theta = (1 - \nu_1^2) / E_1 + (1 - \nu_2^2) / E_2$ ;  $E, \nu$  are the Young's modulus and Poisson's ratio of toothed wheel materials;  $n_1$  is the pinion rotational speed;  $\alpha = 20^\circ$  is the pressure angle;  $\rho_{jh} = \frac{\rho_{1jh} \rho_{2jh}}{\rho_{1jh} + \rho_{2jh}}$  is

the variable at wear the reduced radius of curvature of teeth profiles in normal section;  $\rho_{1jh}, \rho_{2jh}$  are respectively, the variable radii of curvature of the teeth profiles of pinion and gear.

In the process of gearing operation, due to teeth wear, the initial radii of curvatures  $\rho_{1j}, \rho_{2j}$  of their working profiles and, accordingly, the reduced radius of curvature  $\rho_j$  will increase.

Accordingly, they are calculated as follows [18]:

$$\rho_j = \frac{\rho_{1j} \rho_{2j}}{\rho_{1j} + \rho_{2j}}, \quad \rho_{1j} = r_{b1} \tan \alpha_{1j} \quad (3)$$

$$\rho_{2j} = r_2 \sqrt{(r_{2j} / r_2)^2 - \cos^2 \alpha}$$

$$r_{b1} = r_1 \cos \alpha, \quad \alpha_{1j} = a \tan(\tan \alpha_{10} + j \Delta \varphi),$$

$$\tan \alpha_{10} = (1 + u) \tan \alpha - \frac{u}{\cos \alpha} \sqrt{(r_{20} / r_2)^2 - \cos^2 \alpha},$$

$$r_2 = m z_2, \quad r_{20} = r_{a2} - r, \quad r_{a2} = r_2 + m, \quad r_{a1} = r_1 + m,$$

$$r = 0, 2m, \quad \tan \alpha_{1s} = \sqrt{(r_{1s} / r_1)^2 - \cos^2 \alpha},$$

$$r_{1s} = r_{a1} - r = r_{a1} - 0, 2m, \quad r_{b2} = r_2 \cos \alpha, \quad r_{a1} = r_1 + m,$$

$$r_{2j} = \sqrt{a^2 + r_{1j}^2 - 2a r_{1j} \cos(\alpha - \alpha_{1j})}, \quad a = (z_1 + z_2) m / 2,$$

$$r_{1j} = r_1 \cos \alpha / \cos \alpha_{1j}, \quad \cos \alpha_{20} = \frac{r_2}{r_{20}} \cos \alpha,$$

$$\tan \alpha_{2s} = \left(1 + \frac{1}{u}\right) \tan \alpha - \frac{1}{u \cos \alpha} \sqrt{\left(\frac{r_{1s}}{r_1}\right)^2 - \cos^2 \alpha},$$

where  $r_1, r_2$  are the pinion and gear pitch circle radii;  $r_{b1}, r_{b2}$  are the pinion and gear base circle radii;  $r_{a1}, r_{a2}$  are the pinion and gear addendum circle radii;  $r$  is rounding radius of addendums;  $u$  is the gear ratio;  $a$  is the centre distance;  $\Delta \varphi$  is the angle of rotation of the pinion tooth from the point of initial contact (p.0) to point 1, etc.;  $\alpha_{10}$  is the angle corresponding to the 1-st contact point of pinion tooth on the line of engagement;  $\alpha_{1s}$  is the angle determining the position of the last contact point of pinion tooth on the line of engagement;  $\alpha_{20}, \alpha_{2s}$  are the angles of the 1-st and last contact point of gear tooth on the line of engagement.

### 2.2 Method of calculating the change in the teeth radii of curvature

The effect of wear on the change of the initial radii of tooth curvature was studied in [18,19]. Accordingly, the change in the radii of curvature is taken into account as follows:

$$\rho_{kjh} = \rho_{kj} + D_{jk} \sum_{k=1}^n K_{kj}^{-1}, \quad k=1; 2, \quad (4)$$

where  $n = n_k = 1, 2, 3, \dots$  are numbers of toothed wheel revolution;  $k$  is the toothed wheel numbering (1 - pinion, 2 - gear);  $D_{jk} = K_{kj}^2$  are the dimensionless constants at each contact point  $j$  depending on the teeth wear in the general case.

Change in the curvature of tooth profiles due to wear during each interaction

$$K_{kj} = 8h'_{kj} / l_{kj}^2 \quad (5)$$

To reduce the duration of calculations, the calculation block diagram has been developed. Here, the change of tooth profile radii of curvature, their reduced radius of curvature, maximum contact pressures, width of the contact area are not considered after each revolution (interaction cycle), but after a certain number of revolutions (interaction block). In the block, the calculation is carried out by the linear method of accumulation, i.e. under constant initial conditions. In the next block of calculations the accumulated changes are taken into account by Equations (6), (7) and according to the new current data the calculations of the above parameters continue. The calculations time

decreases in proportion to the size of the block. In this case the teeth radii of curvature are calculated by the formula [19]

$$\rho_{kjh} = \rho_{kj} + E_k \sum_{B_1}^{B_{\max}} D_{kjB} K_{kjB}^{-1}, \quad (6)$$

where  $B$  is the number of toothed wheel revolutions (the value of the interaction block) with constant contact conditions; the value of the block can be  $B = 1$  revolution at the exact solution and  $B = n_1$  revolutions for 1, 10, 100, ... hours;  $B_1$  and  $B_{\max}$  are respectively, the first and last blocks of calculations;  $E_k$  are the dimensionless constants selected depending on the teeth allowable wear  $h_{k*}$ ;  $D_{kjB} = K_{kjB}^2$  are dimensionless constants unchanged in the block, and which may remain unchanged in subsequent blocks or change in each subsequent block.

The change in the tooth profiles curvature due to wear during each individual block of tooth interactions will be

$$K_{kjB} = 8 \sum_{j=1}^B h'_{kjn} / l_{kj}^2, \quad (7)$$

Because the teeth wear during gearing operation causes a change in the initial radii of curvature, the values  $h_{\square kjn}$  are calculated at each subsequent revolution for time  $t_{\square jh} = 2b_{jh}/v_0$ , and the variable width of the contact area  $2b_{jh}$  at  $(n_k - 1)$ -th revolution or at  $(B - 1)$ -th block is calculated according to Equation (2).

The chord length of the circle replacing the involute between points  $j - 1, j + 1$ , is calculated as follows:

$$l_{kj} = 2\rho_{kjh} \sin \varepsilon_{kjh} = \text{const}, \quad (8)$$

where  $\varepsilon_{kjh} = S_{kj} / \rho_{kjh}$  is the angle between the points  $j$

and  $j + 1$ ;  $S_{kj} = \left| \frac{mz_k}{4} \left( \frac{1}{\cos^2 \alpha_{kj}} - \frac{1}{\cos^2 \alpha_{k,j+1}} \right) \cos \alpha \right|$  is the

involute length between the points  $j$  and  $j + 1$ ;  $\alpha_j, \alpha_{j+1}$  are the pressure angles for selected involute points  $j, j + 1$  (see above);  $\alpha_{2j} = \arccos \left[ (r_2 / r_{2j}) \cos \alpha \right]$ ,

$\alpha_{2,j+1} = \arccos \left[ (r_2 / r_{2,j+1}) \cos \alpha \right]$ ;  $m$  is the module;  $z_1, z_2$  are the numbers of teeth of the pinion and gear.

As a result of teeth wear, after each interaction or block of interactions, all calculated parameters will change, in particular  $h_{1j}, h_{2j}, \rho_{1jh}, \rho_{2jh}, \rho_{jh}, p_{jh\max}, 2b_{jh}, t_{\square jh}$ .

The ratio is used to calculate the sliding speed

Units should be typewritten vertically, as for example:

$$v_j = \omega_1 r_{b1} (\tan \alpha_{1j} - \tan \alpha_{2j}), \quad (9)$$

where  $r_{b1} = r_1 \cos \alpha$ .

### 2.3 Calculation of total wear

For the selected arbitrary number of revolutions of the pinion and gear  $n_{1s}$  and the corresponding number of

blocks, the total wear  $h_{1jn}$  and  $h_{2jn}$  of teeth at  $j$ -th point of contact is calculated as follows:

$$h_{1jn} = \sum_1^{n_{1s}} h_{1jB}, \quad h_{2jn} = \sum_1^{n_{2s}} h_{2jB}, \quad (10)$$

where  $n_{2s} = n_{1s}/u$ ;  $h_{kjB} = \sum h'_{kj}$  is the wear of teeth in each block;  $u$  is the gear ratio.

### 2.4 Calculation of durability

The durability of gearing for a given number of revolutions of toothed wheels  $n_{1s}$  or  $n_{2s}$  is calculated as follows [21,22]:

$$t_B = n_{1s} / 60n_1 = n_{2s} / 60n_2, \quad (11)$$

Upon reaching the accepted allowable wear of the teeth  $h_{k*}$  in one of the points of any toothed wheel is automatically calculated the corresponding maximum number of revolutions  $n_{\max 1s}$  and  $n_{\max 2s}$ , which allows determining the ultimate minimum gearing life according to Equation (10).

Provided that the maximum initial contact pressures  $p_{j \max}$  remain unchanged, the gearing life can be calculated according to the simplified method [13,18] as

$$t_{\min} = h_{k*} / \bar{h}_{kj}, \quad (12)$$

where  $\bar{h}_{kj} = 60n_k h'_{kj}$  is the linear wear of teeth for one hour;  $h_{k*}$  is the accepted allowable wear of teeth.

Then the dependence in Equation (1) for  $h_{\square kj}$  at  $p_{j \max} = \text{const}$  will take the form [13,18-20]

$$h'_{kj} = \frac{v_j t_j' (f p_{j \max})^{m_k}}{C_k \tau_S^{m_k}}, \quad (13)$$

where  $t_j' = 2b_j / v_0 = \text{const}$  is the time of teeth tribocontact interaction.

Hertz's formulas in Equation (2) also acquire a classical form, because  $\rho_{jh} \equiv \rho_j = \text{const}$ .

### 2.5 Method of determining the pressure angles

In straight spur gears, two-one-two-pair engagement of teeth is realized. The angles of transition from two-pair ( $\Delta\varphi_{1F2}$ ) to one-pair and again to two-pair ( $\Delta\varphi_{1F1}$ ) engagement and the teeth exit angle from the engagement  $\Delta\varphi_{1E}$  are calculated as follows [22]:

$$\Delta\varphi_{1F2} = \varphi_{10} - \varphi_{1F2}, \quad \Delta\varphi_{1F1} = \varphi_{10} + \varphi_{1F1}, \quad (14)$$

$$\Delta\varphi_{1E} = \varphi_{10} + \varphi_{1E},$$

where  $\varphi_{1F2} = \tan \alpha_{F2} - \tan \alpha$ ,  $\varphi_{1F1} = \tan \alpha_{F1} - \tan \alpha$ ,

$$\varphi_{10} = \tan \alpha_{10} - \tan \alpha, \quad \tan \alpha_{F2} = \frac{r_1 \sin \alpha - (p_b - e_1)}{r_1 \cos \alpha},$$

$$\tan \alpha_{F1} = \frac{r_1 \sin \alpha - (p_b - e_2)}{r_1 \cos \alpha}, \quad p_b = \pi m \cos \alpha,$$

$$e_1 = \sqrt{r_{1s}^2 - r_{b1}^2} - r_1 \sin \alpha, \quad e_2 = \sqrt{r_{20}^2 - r_{b2}^2} - r_2 \sin \alpha,$$

$$\varphi_{1E} = \tan \alpha_E - \tan \alpha, \quad \alpha_E = \arccos(r_{b1} / r_{1s}).$$

## 2.6 Evaluation of the influence of tooth wear on bending stress

The decrease in the tooth thickness near the root due to wear is determined as follows:

$$S_h = S_0 - h_s, \quad (15)$$

where  $h_s$  is the linear teeth wear near the root.

The tooth thickness near the root is calculated according to [24] by the following formula

$$S_0 = [(S / r_2) - 2(\text{inv}\alpha_x - \text{inv}\alpha)]r_0, \quad (16)$$

where  $S = \pi m / 2$  is the tooth thickness measured along the pitch diameter,  $\cos \alpha_x = r_2 / r_0$ ,  $r_0 = r_2 - m$  is the radius of the tooth root where the rounding begins.

## 3. NUMERICAL SOLUTION, RESULTS AND DISCUSSION

Data for calculations are taken as follows:  $T_{nom} = 4000$  Nmm;  $P = 0,42$  kW;  $K_g = 1.2$  is the coefficient of dynamism;  $z_1 = 20$ ;  $m = 4$  mm;  $u = 3$ ;  $n_1 = 1000$  rpm;  $\Delta\varphi = 4^\circ$ ;  $b = 20$  mm;  $a = 160$  mm;  $h^* = 0.5$  mm;  $f = 0.05, 0.1, 0.2, 0.3, 0.4$ ;  $B = 6 \cdot 10^6$  revolutions (100 hours of operation).

Materials of metal toothed wheels: a pinion – alloy steel 38KhMJuA, nitrided to a depth of 0.4 ... 0.5 mm, hardness 600 HB; ultimate strength  $\sigma = 1040$  MPa,  $\tau_{S1} = 365$  MPa,  $C_1 = 3.9 \cdot 10^6$ ,  $m_1 = 2$ ; a gear – alloy steel 40Kh, full hardening, hardness 341 HB; ultimate strength  $\sigma = 981$  MPa,  $\tau_2 = 345$  MPa,  $C_2 = 0.17 \cdot 10^6$ ,  $m_1 = 2.5$ ;  $E = 2.1 \cdot 10^5$  MPa,  $\nu = 0.3$ .

Materials of metalpolymer gears:

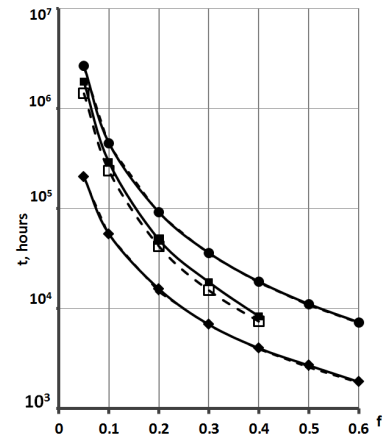
- composite 1: polyamide reinforced with 30% dispersive carbon fibers - PA6+30CF,  $\tau_{SCF} = 48$  MPa,  $E_{CF} = 5200$  MPa,  $\nu_{CF} = 0.42$ ,  $C_{CF} = 4.7 \cdot 10^6$ ,  $m_F = 2.3$  [12,13];

- composite 2: polyamide reinforced with 30% dispersive glass fibers - PA6+30GF,  $\tau_{SCF} = 52$  MPa,  $E_{GF} = 3900$  MPa,  $\nu_{GF} = 0.42$ ,  $C_{GF} = 1.2 \cdot 10^6$ ,  $m_{GF} = 1.9$  [12,13].

The results of calculations are presented in Fig. 1-7. In particular, Fig. 1 shows the minimum durability  $t_{B \min}$  of gears calculated by the block method (solid curves), when allowable wear is achieved at one of the points of teeth. Also here are the results of calculating the durability  $t_{\min}$  of gears by the simplified method (dashed curves), when the conditions of teeth contact interaction are assumed to be constant during the operation until the allowable wear is achieved, i.e. without taking into account the effect of tooth wear on the change of initial maximum contact pressures.

According to the simplified method of calculations, the durability of the metal (MM) gear train will be 1.13... 1.3 times less than according to the specified method, which takes into account the real conditions of tribocontact interaction. However, in the metal-polymer (MP) gear train the durabilities  $t_{B \min}$ ,  $t_{\min}$  will be almost the same.

Fig. 1 shows that the gear train containing a gear made of carbon-filled composite has the highest durability in the entire range of changes in the sliding friction coefficient. The gear train containing steel toothed wheels has a slightly lower durability (1.46... 1.88 times lower at  $f = 0.05 \dots 0.2$ ). In contrast, the metal-polymer gear train containing glass-composite gear has significantly lower durability than the other two types of gear trains: 8.9... 3.12 times lower at  $f = 0.05 \dots 0.2$  relative to the gear train containing steel toothed wheels; 12.9... 3.12 times lower at  $f = 0.05 \dots 0.6$  relative to the metal-polymer gear train containing carbon composite gear.



**Figure 1. The effect of friction coefficient on the minimum durability of gear trains: —●—  $t_{B \min}$  (Steel-Steel); —□—  $t_{\min}$  (Steel-Steel); —●—  $t_{B \min}$  (Steel-PA6+30CF); —○—  $t_{\min}$  (Steel-PA6+30CF); —◆—  $t_{B \min}$  (Steel-PA6+30GF); —◇—  $t_{\min}$  (Steel-PA6+30GF)**

Fig. 2 shows the relative durability  $\tilde{t}$  of MM and MP gear trains at change of the friction coefficient.

In closed gear trains with metal wheels lubricated with oils the boundary friction is realized  $f = 0.05 \dots 0.1$ . However, in case of insufficient lubrication or oil destruction during the operation, the friction coefficient will increase. In some cases, the gear train can operate in the mode of semi-dry or even dry friction, in which the friction coefficient will reach 0.2... 0.4. Metal-polymer gear trains made of the studied filled polyamide composites work quite reliably without lubrication at dry friction conditions with the friction coefficient  $f = 0.3 \dots 0.6$ . However, by changing the percentage and composition of the fillers or by applying lubrication, adding antifriction additives to oils, the friction coefficient can be reduced even to 0.05 and consequently it increases the durability of the gear train.

Analysis of the fig. 1 shows that the gear train with a carbon composite wheel at  $f = 0.05 \dots 0.6$  will have the greatest durability. At the boundary friction ( $f = 0.05 \dots 0.1$ ) MM gear train will have slightly less durability (about 1.5 times), and at higher values of the friction coefficient this difference will increase to 2.2 times ( $f = 0.4$ ).

It should be noted the significant effect of the friction coefficient on reducing the gear train durability:

- 220 times for Steel-Steel gear train at  $f = 0.05 - 0.4$ ;
- 144 times for Steel-PA6 + 30CF gear train at  $f = 0.05 - 0.4$ , and 367 times at  $f = 0.05 - 0.6$ ;
- 52 times for Steel-PA6 + 30GF gear train at  $f = 0.05 - 0.4$ , and 112 times at  $f = 0.05 - 0.6$ .

Regarding the relative durability  $\tilde{t}$  of MM and MP gear trains, shown in Fig.2, the increase in the friction coefficient  $f$  leads to its decrease in the case of gear trains (Steel-PA6 + 30CF)/(Steel-Steel). Therefore, the carbon-composite metal-polymer gear train will be significantly more durable in a wide range of changes in the sliding friction coefficient of the studied types of materials.

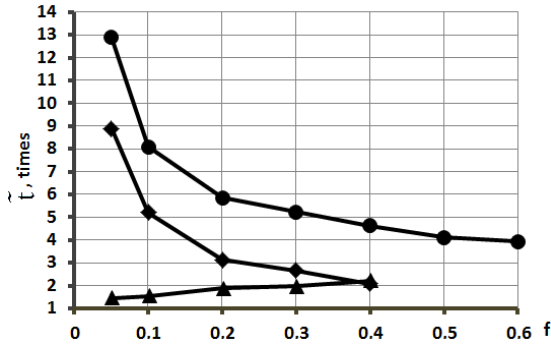


Figure 2. The influence of the friction coefficient on the relative durability of gear trains:  $\blacklozenge$  (Steel-Steel)/(Steel-PA6+30GF),  $\bullet$  (Steel-PA6+30CF)/(Steel-PA6+30GF),  $\blacktriangle$  (Steel-PA6+30CF)/(Steel-Steel),  $\blacksquare$  (Steel-Steel)/(Steel-Steel).

An important parameter for gear trains is the load carrying capacity, which is characterized by the level of maximum contact pressures  $p_{j \max}$  in the engagement. Fig. 3 for MM gear train shows the calculated values of both the initial contact pressures  $p_{j \max}$  (solid line) and their change  $p_{jh \max}$  (dashed lines) as a result of the teeth wear to a value of  $h_s=0.5$  mm when changing the friction coefficient. The left and right areas of the diagram correspond to the two-pairs engagement, and the central area corresponds to the single-pair engagement.

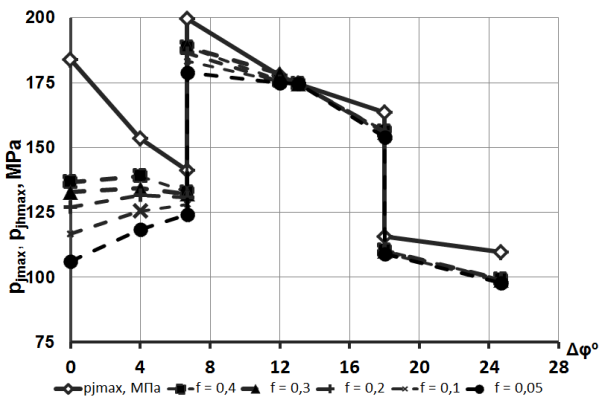


Figure 3. The change of the maximum contact pressures in the process of two teeth pairs – single tooth pair – two teeth pairs engagement and wear in MM gear trains

The friction coefficient has a significant effect on the rate of  $p_{jh \max}$  in the zone of two-pairs engagement and has a much smaller effect in the zone of single-pair engagement.

Accordingly, Fig. 4 shows the change in the initial pressures  $p_{j \max}$  in the MP gear train during the cycle of two-one-two-pairs engagement.

The nature of the change of the maximum initial pressures  $p_{j \max}$  in MP gear trains is the same as in MM gear trains. However, with the same transmitted power and geometric parameters of the toothed wheels, there is a very significant difference between the value  $p_{\max}$  in

MM gear train and MP gear trains. In particular, in the metal-polymer carbon-composite gear train it is 5.07 times smaller, and in the metal-polymer glass-composite gear train it is 5.85 times smaller.

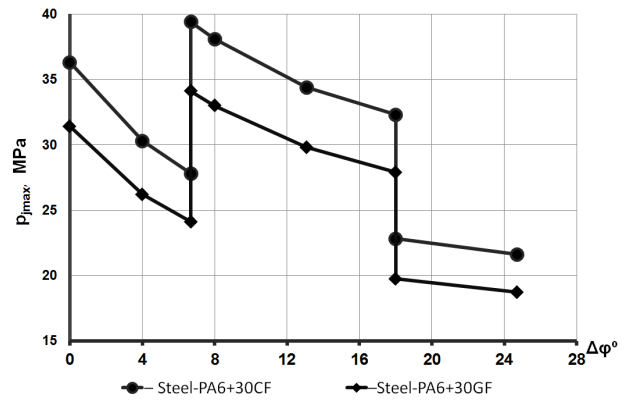


Figure 4. The change of the maximum initial pressures in the process of teeth engagement in MP gear trains.

The dependences of the change  $p_{j \max}$  in the engagement cycle and  $p_{jh \max}$  after reaching the allowable wear at the point of entry of the teeth into the single-pair engagement in MP gear trains Steel - PA6 + 30CF and Steel - PA6 + 30GF are shown in Fig. 5.

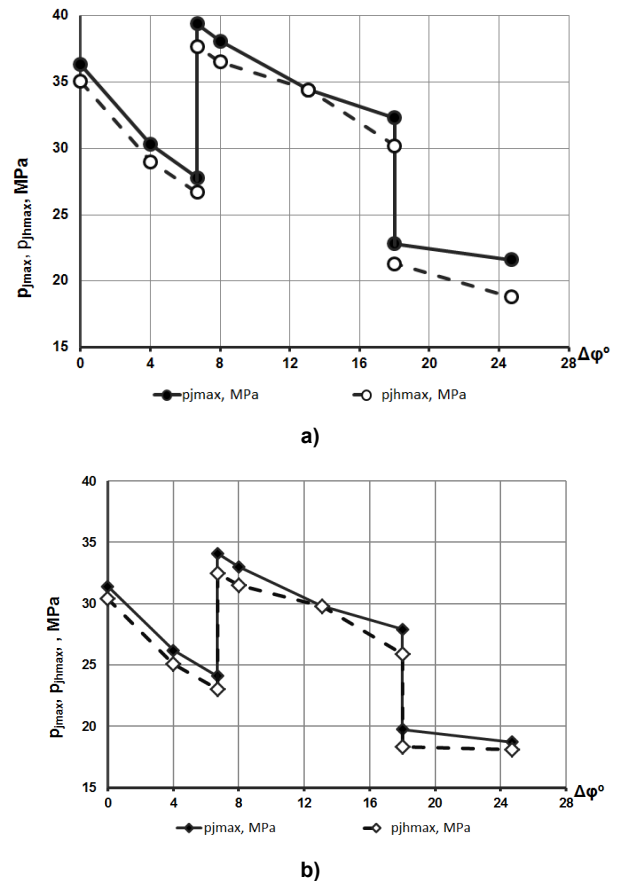


Figure 5. The change of the maximum contact pressures in the process of teeth engagement and wear in MP gear trains: a) Steel - PA6 + 30CF, b) Steel - PA6 + 30GF

The analysis of the received data testifies almost identical change of tribocontact pressures in the engagement. However, in the MP gear train Steel - PA6 + 30CF the maximum pressures are slightly higher than in the MP gear train Steel - PA6 + 30GF. As a result of

calculations it is established that the friction coefficient does not show influence on  $p_{jhmax}$  in the specified MP gear trains. Obviously, this is due to the fact that the steel teeth of the pinion are practically not worn, because it is three orders of magnitude less than the wear of the teeth of composite wheels (compared to PA6 + 30CF 1340 times, and compared to PA6 + 30GF 1610 times).

Fig. 6 shows the linear wear of the working teeth profile at different points of engagement.

The maximum (allowable) wear of the teeth will occur at the point of entry into the single-pair engagement. Wear at the entrance to the two-pair engagement will also be close to it. It is established that the values of linear wear of the teeth profile at individual points in the studied MM gear train and both types of MP gear trains will be approximately the same in magnitude and nature of change.

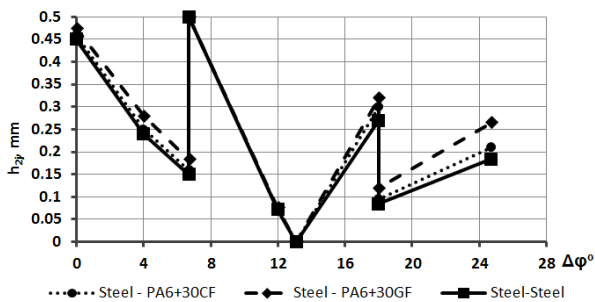


Figure 6. Wear of the tooth profile during the engagement.

The change in sliding speed during engagement is shown in Fig.7

At the entrance of the teeth into the engagement and at the exit from it, the sliding speed will have the opposite direction and approximately the same value for all types of gear trains.

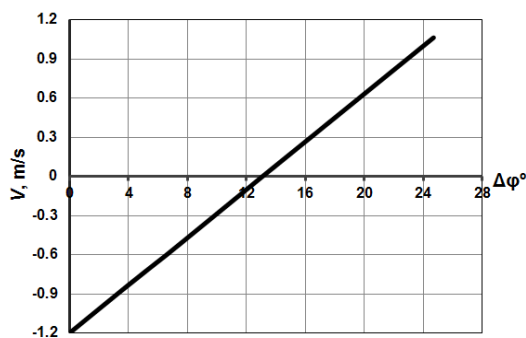


Figure 7. Sliding speed in the engagement.

According to the calculated data of the gears, the reduction of the initial thickness  $S_0$  near the root of the composite teeth is calculated. Accordingly, the initial thickness  $S_0$  is 7.39 mm. According to fig. 6 the teeth wear of the gears is: PA6 + 30CF -  $h_s \approx 0.21$  mm, PA6 + 30GF -  $h_s \approx 0.26$  mm. Due to wear, the teeth thickness  $S_h$  is: PA6 + 30CF - 7.18 mm (<2.84%), PA6 + 30GF - 7.13 mm (<3.52%). According to the results of experimental studies of gears at different loads [25], it was found that when wearing up to 10%, the bending stresses of a worn and unworn tooth in the zone of tension and compression will be almost the same. Tooth wear by 20% will already increase these stresses in the

zone of tension by 6-8%, and in the zone of compression - up to 15%. That is, for the studied MP gears with the accepted calculation data, in particular  $h_s = 0.5$  mm, the teeth wear will not affect the bending stress. This effect will be negligible even when the allowable wear is twice as high.

It should also be noted that the teeth of composite gears are more deformable and undergo greater bending than the teeth of steel gears, which leads to an increase in the engagement period, i.e. to an increase in the overlap coefficient. As a result, it is known that the performance efficiency of the gear train will be improved, because there will be some reduction of the maximum contact pressures at the entrance of the teeth into the engagement, which will increase the durability of the MP gears.

#### 4. CONCLUSION

1. The author's computational method of gear trains, in particular metal-polymer, provides at the design stage the possibility of a predictive assessment of their durability, load capacity and wear of the composite gear.
2. Irrespective of materials of toothed wheels the assessment of tribotechnical and contact parameters can be realized by the developed computational method.
3. The results of the estimated assessment of the durability of gear trains (Fig. 1) indicate that in the case of calculation of MM gear trains by the specified method, the durability is higher 1.13 ... 1.3 times than calculated by the simplified method. On the other hand, in both types of MP gear trains this difference will be insignificant (up to 3%).
4. As established, the sliding friction coefficient significantly affects the change of the initial contact pressures in the MM gear train (Fig. 3). This effect is not observed in MP gear trains (Fig. 5).
5. The developed computational method also provides an opportunity to assess the teeth wear at selected points of two-one-two-pair engagement in spur gears. In helical gears, which are not studied here, such an assessment by this method will also not be difficult.
6. As a result of the conducted researches it is established that the trend of teeth wear in MM and MP gear trains is practically the same (fig. 6).
7. The influence of tooth wear on the maximum bending stresses was also studied.
8. The presented research method, previously developed for the calculation of gear trains with metal wheels, and subsequently transformed for the calculation of metal-polymer gear trains, provides at the design stage effective assessment of durability and optimization of gear trains by choosing materials of toothed wheel, fillers of polymer composites and possible use of the appropriate type of lubricants.

#### REFERENCES

- [1] Brauer, J., Andersson, S.: Simulation of wear in gears with flank interference - a mixed FE and analytical approach, - Wear, Vol. 254, pp. 1216-1232, 2003.

- [2] Flodin, A., Andersson, S.: Wear simulation of spur gears, -Tribotest, Vol. 3, No.5, pp. 225-250, 1999.
- [3] Flodin, A., Andersson, S.: A simplified model for wear prediction in helical gears, – Wear, Vol. 249, No.3-4, 285-292, 2001.
- [4] Grib, V.: *Solution of tribotechnical tasks with numerous methods*, Science, Moscow, 1982. (in Russian).
- [5] Kahraman, A., Bajpai, P. and Anderson, N.E.: Influence of tooth profile deviations on helical gear wear, - Journal of Mechanical Design, Vol. 127, No.4, pp. 656-663, 2005.
- [6] Kindrachuk, M., Volchenko, A., Volchenko, D., Volchenko, N., Poliakov, P., Tisov, O. and Kornienko, A.: Polymeres with enhanced energy capacity modified by semiconductor materials, – Functional Materials, Vol. 26, No.3, pp. 629-634, 2019.
- [7] Kolivand, M., Kahraman, A.: An ease-off based method for loaded tooth contact analysis of hypoid gears having local and global surface deviations, – Journal of Mechanical Design, Vol. 132, No. 7, pp. 0710041-0710048, 2010.
- [8] Pasta, A, Mariotti Virzi, G.: Finite element method analysis of a spur gear with a corrected profile, - Journal of Strain Analysis for Engineering Design, Vol. 42, pp. 281-292, 2007.
- [9] Shil'ko, S., Starzhinskii, V.: Prediction of Wear Resistance of Gearing with Wheels Made of Reinforced Composites, - Journal of Friction and Wear, Vol. 14, No. 3, pp. 7-13, 1993.
- [10] Shil'ko, S., Starzhinsky, V., Petrokovets, E. and Chernous, D.: Two-Level Calculation Method for Tribojoints Made of Disperse-Reinforced Composites: Part 1, - Journal of Friction and Wear, Vol. 34, No. 1, pp. 65-69, 2013.
- [11] Chernets, M., Shil'ko S. and Starzhinsky V.: Estimation of bearing capacity and wear of spur gear meshing taking into account tooth profile modification and sliding friction coefficient in meshing, in: Goldfarb, V., Trubachev, E., Barmina, N. (Eds.): *New Approaches to Gear Design and Production*, Springer, 261-272, 2020.
- [12] Chernets, M., Shil'ko, S., Pashechko, M. and Barshch, M.: Wear resistance of glass- and carbon-filled polyamide composites for metal-polymer gears, - Journal of Friction and Wear, Vol. 39, No. 5, pp. 361-364, 2018.
- [13] Chernets M.: Method of calculation of tribotechnical characteristics of the metal-polymer gear, reinforced with glass fiber, taking into account the correction of tooth, - Eksploatacja i Niezawodnosc – Maintenance and Reliability, Vol. 21, No. 4, pp. 546-552, 2019.
- [14] Cathelin, J., Letzelter, E., Guingand, M., De Vaujany, J.P. and Chazeau, L.: Experimental and numerical study a loaded cylindrical PA66 gear, - Journal of Mechanical Design, Vol. 135, pp. 89-98, 2013.
- [15] Sukumaran, J., Ando, M., De Baets, P., Rodriguez, V., Szabadi, L., Kalacska, G. and Paeppegem, V.: Modelling gear contact with twin-disc setup, - Tribology International, Vol. 49, pp. 1-7, 2012.
- [16] Kalacska, G., Kozma, M., De Baets, P., Keresztes, R. and Zsidai, L.: Friction and wear of engineering polymer gears, in: *Proc. World Tribology Congress III*, September, 2005, Washington, pp. 12-16.
- [17] Keresztes, R. and Kalacska, G.: Friction of polymer/steel gear pairs, - Plastics and Rubber, 45, 236-242, 2008.
- [18] Chernets, M., Kelbinski, J. and Jarema, R.: Generalized method for the evaluation of cylindrical involute gears, - Materials Science, Vol. 1, pp. 45-51, 2011.
- [19] Chernets, M., Yarema, R. and Chernets, Ju.: A method for the evaluation of the influence of correction and wear of the teeth of a cylindrical gear on its durability and strength. Part 1, - Service life and wear. Materials Science, Vol. 3, pp. 289-300, 2012.
- [20] Chernets, M. and Chernets, Ju.: Evaluation of the strength, wear, and durability of a corrected cylindrical involute gearing, with due regard for the engagement conditions, – Journal of Friction and Wear, Vol. 37, No. 1, pp.71-77, 2016.
- [21] Chernets, M. and Chernets, Yu.: A technique for calculating tribotechnical characteristics of tractive cylindrical gear of VL – 10 locomotive, - Journal of Friction and Wear, Vol. 37, No. 6, pp. 566-572, 2017.
- [22] Chernets, M. and Chernets, Ju.: The simulation of influence of engagement conditions and technological teeth correction on contact strength, wear and durability of cylindrical spur gear of electric locomotive, Proceedings of the Institution of Mechanical Engineers, Part J: – Journal of Engineering Tribology, Vol. 231, No. 1, pp. 57-62, 2017.
- [23] Andreikiv, A.E., Chernets, M.V.: *Evaluation of the contact interaction of rubbing machine elements*, Naukova Dumka, Kiev, 1991, (in Ukrainian).
- [24] Shalobaev, E.V. and Kudinov, A.T.: *Plastic Gears in Instrument Mechanisms*, Metal – Polymer Research Institute, Gomel, 1998, (in Russian).
- [25] Stachowiak, G.W.: Numerical Characterization of wear particle morphology, in: Hutchings, I.M. (Ed.): *New Directions in Tribology*, Mechanical Engineering Publications Ltd., Bury St Edmunds, pp. 371-389, 1997.

---

**АНАЛИТИЧКА ПРОЦЕНА УТИЦАЈА  
КОЕФИЦИЈЕНТА КЛИЗНОГ ТРЕЊА НА  
КОНТАКТНИ ПРИТИСАК, ТРАЈНОСТ И  
ХАБАЊЕ ЦИЛИНДРИЧНИХ ЗУПЧАНИКА**

**М. Чернетс, А. Корниенко, Ј. Чернетс, С.  
Федорчук**

Истражује се утицај коефицијента клизног трења на трајност преносног склопа састављеног од челичних зупчаника и склопа састављеног од метално-полимерних зупчаника међу којима су полиамидни зупчаници ојачани угљеничним и стаклено дисперзијским влакнима. Такође су узети у обзир услови спрезања зубаца (двопарна-једнопарна-двопарна спрега) и промена услова триболошког контакта изазваних хабањем. Преносни склоп са зупчаницама од композита са угљеником има најдужу трајност у поређењу са другим типовима

спреге при свакој промени коефицијента клизног трења. Максимална вредност контактеног притиска је у тачки уласка у једнопарну спрегу. Истражена је и промена максималне вредности контактеног притиска изазвана хабањем зубаца и утврђена је правилност јављања ове пројаве. Проучена је кинетика хабања профила зубаца. Максимално хабање настаје у тачки уласка у једнопарну али и двопарну спрегу. Процес хабања челичних зубаца и склопова који имају челичне и композитне зупце је готово исти осим у тачки изласка зубаца из спреге.