

**Leila I. Aliieva**

Professor,  
Department of Computerized Design and  
Modeling of Processes and Machines,  
Donbass State Engineering Academy,  
Ukraine

**Oleg E. Markov**

Professor,  
Department of Computerized Design and  
Modeling of Processes and Machines,  
Donbass State Engineering Academy,  
Ukraine

**Igramotdin S. Aliiev**

Professor,  
Department of Metal Forming Department,  
Donbass State Engineering Academy,  
Ukraine

**Natalia S. Hrudkina**

Docent,  
Department of Computerized Design and  
Modeling of Processes and Machines,  
Donbass State Engineering Academy,  
Ukraine

**Vladymyr N. Levchenko**

Docent,  
Department of Metal Forming Department,  
Donbass State Engineering Academy,  
Ukraine

**Khrystyna V. Malii**

PhD,  
Department of Metal Forming Department,  
Donbass State Engineering Academy,  
Ukraine

## Analysis of Power Parameters of Combined Three-Direction Deformation of Parts with Flange

*The upper bound power method is used to simulate the process of combined radial-backward-forward extrusion of hollow parts of the «cup with flange and branch pieces» type from a continuous workpiece. The calculation scheme with autonomous deformation zones is used, which contains modules with an inclined boundary and the condition for the equality of powers acting on both sides of the intermediate hard zone is accepted. This made it possible to more accurately determine the power regime and the prevailing direction of the metal flow, which is necessary to assess the character of the forming of the part. The comparison of theoretical and experimental values of the deformation pressures and the flow velocities with each other, as well as with the results obtained by the finite element method shows the feasibility of using the obtained functions for technological calculations of power parameters and evaluating of part forming.*

**Keywords:** extrusion; power mode; hollow stem part with flange; upper bound method; velocity field; kinematic module; finite element method

### 1. INTRODUCTION

Hollow parts such as sleeves and cups with flange made of light alloys are very common in the aviation, instrument making and machine building industries. The effective resource-saving method of manufacturing them instead of cutting is the technology of die forging and, in particular, cold extrusion. Extrusion processes demonstrate the steady tendency to increase the production volumes, to expand the range of parts and materials, to create new ways of forming, types of specialized technological equipment and tooling [1, 2]. The addition of the traditional schemes of longitudinal (forward and backward) extrusion with new methods of radial and side extrusion provides great opportunities for manufacturing more complex configurations in sectional dies [3, 4]. The combination of these extrusion methods in the manufacturing of hollow parts with flange increases the efficiency of die forging processes due to the decrease in technological operation number, by lowering energy consumption and increasing production efficiency.

Combined processes involve various simple deformation methods. In combined extrusion the combination

of simple extrusion schemes can be a sequential or incorporated one – with one common center or adjacent deformation centers [4]. Combined methods are characterized by the presence of several degrees of freedom, i.e. possible directions of the outflow of workpiece metal.

Three-direction combined radial-backward-forward extrusion provides for the simultaneous flow of metal in radial, forward and backward directions. Therefore, the method can be recommended for hollow parts with flanges and branch pieces of various shapes, which are widely used in industry [5].

### 2. LITERATURE REVIEW

Using new schemes of cross side (radial and side) extrusion in the combination promotes the qualitative development of technological capabilities of stamping technologies (Table 1).

Depending on the nature of the combination of simple schemes of cross side and longitudinal extrusion in one combined process, the methods can also be divided into sequential (in time or along deformation path) and incorporated ones. Among the methods of incorporated combined cross side-longitudinal extrusion, we single out the scheme of three-direction radial-backward-forward extrusion as the most universal one (scheme 1) to obtain hollow part with flange and branch pieces.

Using the combined method of radial-backward extrusion, it is possible to obtain both hollow parts such

Received: November 2020, Accepted: December 2020

Correspondence to: Prof. Oleg E. Markov  
Donbass State Engineering Academy, Akademichna  
(Shkadinova) Str. 72, 84313 Kramatorsk, Ukraine  
E-mail: oleg.markov.ond@gmail.com

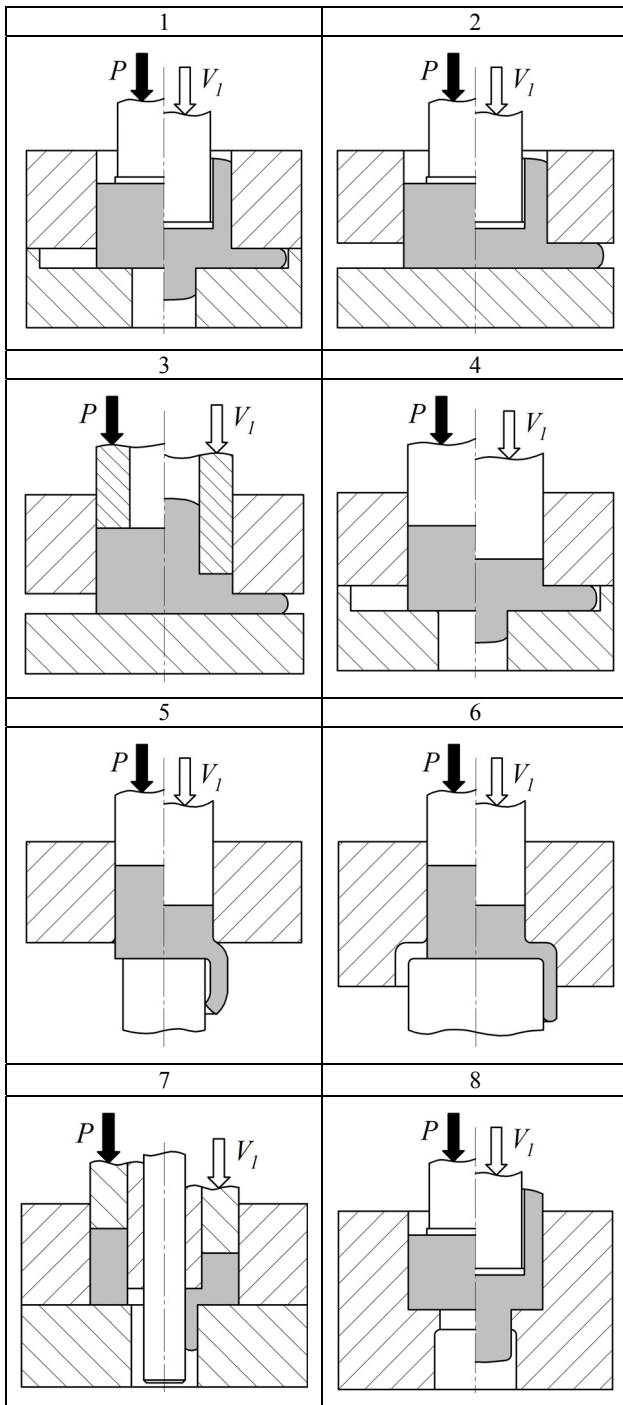
doi: 10.5937/fme2102344A

© Faculty of Mechanical Engineering, Belgrade. All rights reserved

FME Transactions (2021) 49, 344-355 344

as the cup with flange at the bottom (scheme 2) and parts like stem with flange (scheme 3), and using radial-forward extrusion one can produce parts with flange of continuous and tubular workpieces (scheme 4) [4].

**Table 1 – Schemes of combined radial-longitudinal extrusion**



Methods of sequential radial-longitudinal extrusion are designed to obtain hollow parts. There are two variants of this method: with expansion (scheme 5 and 6) [5] and with reduction of metal during its radial flow (scheme 7) [4, 6].

To obtain hollow products such as cups with branch piece, the incorporated combined extrusion method is used that combines forward and backward extrusion schemes (scheme 8) [7]. With such combined longitudinal extrusion the extrusion force of hollow

cups with an external stem (branch pieces) at the bottom decreases almost twice [4, 8].

In recent years, many researchers have shown interest the processes of radial-longitudinal extrusion. In the work of Y.S. Lee [9] the power characteristics of the process of sequential radial-forward extrusion are presented and the influence of design parameters is investigated, such as the diameter of the mandrel, the radius of the die and the friction coefficient. To verify the reliability of the simulation results performed by the FEM under the same conditions as in the experiment, experimental data concerning the force of deformation are used.

In the work [10], the influence of some important design geometric parameters of the sequential radial-forward extrusion process, such as the flange thickness, value of the circular clearance and the radius of mandrel on development and fluctuations of load, is investigated. The reliability of the results of FE simulation obtained in the study under consideration is confirmed by using experimental data presented in the reference. The conclusion that the radius of the counter-punch and the radii of curvature of the transitional zone of a tool have negligible impact on the formation of load on the punch and counter-punch is doubtful.

In the works [11–13], a finite-element analysis of two-direction radial-backward extrusion of hollow parts with the flange is carried out for various variable factors, such as the size of clearances, the radius of curvature of the die and friction conditions at the contact. The authors argue that friction conditions have negligible impact on power parameters and deformation of the sample. Under certain conditions and geometric parameters of the die, the height of the part may be smaller than that of the original workpiece, i.e. the flow in the opposite direction can be reduced to zero or to a negative value.

A number of works are devoted to the evaluation of the deformed state, the conditions for the appearance of defects in parts and the metal deformability of workpieces at cold combined extrusion. The work [14] is devoted to the estimation of the limit form changing of workpieces of various materials in the processes of cold volumetric deformation. In that work, the plasticity resource is calculated according to various criteria for the process of combined radial-forward extrusion. The obtained ways of material particle deformation make it possible to evaluate the plasticity resource and technological capabilities of the combined radial-forward extrusion process for various materials for which plasticity diagrams are known. Based on the studies, the evaluation of the plasticity exhaustion resource in the flange zone for various materials is made.

To prevent the appearance of defects in parts during the combined reverse-forward cold extrusion of a piston, the extrusion process in a movable die was developed [15].

The analysis of defect formation in the processes of forward, radial and radial-backward extrusion is the subject of the work [16]. That work also considered the possibilities of predicting the process of defect occurrence in the form of dimple at the bottom of the cup with the flange using the upper bound power method

(UBPM). The proposed calculation scheme for the plane-deformed process of combined radial-backward extrusion makes it possible to determine the critical value of the thickness of the cup bottom from the point of view of dimple occurrence.

The calculation functions for the power regime of combined extrusion processes were obtained mainly for flat problems by the upper bound method. The study of the combined radial-backward extrusion process of the "cup with flange" type parts was carried out in the works for the plane-deformed state [17]. The developed models allow the power deformation regime to be approximately determined.

The works [18, 16] devoted to the study of combined backward-forward and radial-backward extrusion processes with the divided deformation center are of particular interest. These results were obtained by the upper bound method using L. G. Stepansky assumption that with the combined flow along with the power balance the condition for equal powers acting on both sides of the plane separating two independent deformation zones must be fulfilled [19].

For the axisymmetric process of radial-forward extrusion, R. Ebrahimi proposed the analytical approach based on the determination of simple-velocity-field components of parallel flow using the upper bound power method [20].

The analysis of the three-direction forward-backward-radial extrusion process of a hollow part of a sleeve with the jumper type using the finite element method in the ABAQUS software package was carried out by A. Farhoumand [21]. The influence of geometric parameters, such as the radius of die curvature and the height of the clearance, as well as friction conditions, was studied. A comparative analysis of the results of finite element analysis with experimental data for loads and flow of the material in various directions, as well as on the distribution of deformations, was carried out.

The hollow parts of the cup-with-flange and axial-branch-piece type are widespread in industry and obtaining the calculation relationships for the design of the processes for their extrusion is an urgent task. Despite the high efficiency of combined extrusion processes, examples of their practical development are limited, which makes their in-depth study necessary. This is due to the fact that, along with the above mentioned advantages of the combined processes, there are a number of problem tasks associated with issues of ambiguous nature of the form changing and the complex stress-strain state of a workpiece under conditions of increased degree of freedom of metal flow [3, 4]. Moreover, the main attention should be paid to the development of refined mathematical models for calculating technological regimes, and first of all, power parameters, as well as parameters of gradual and limit form changing, allowing us to predict the forming of parts in deformation processes with several directions of the metal flow.

The analysis of literature data shows that the main problem that restrains the use of combined extrusion processes in industry is the lack of recommendations for technology development and tool design.

The purpose of the work is to study power parameters of the technological modes of combined radial-reverse-forward extrusion.

To achieve the said purpose, the following tasks were set:

- to develop the calculation process scheme of combined radial-backward-forward extrusion using modules with a non-parallel flow;

- to study the effect of the geometry of chamfers of the split die on the power mode of the process;

- to calculate the unit pressure depending on the relative geometrical parameters of the part (thickness and radius of the flange, the radius of the branch piece) and the radius of the punch at different values of the contact friction coefficient, to research the stress-strain state of the workpiece and compare the calculated power parameters of the three-direction extrusion process with experimental data and data obtained by finite element simulating.

- on the basis of experimental studies and finite element simulating the stress-strain state, limiting and gradual form changing, power parameters to identify the features of the combined radial-backward-forward extrusion process.

### **3. EXPERIMENTAL RESEARCH AND FINITE ELEMENT SIMULATION OF THE THREE DIRECTION EXTRUSION PROCESS**

The aim of the work is increasing of the mechanical properties for parts with responsible destination by means of welding of the internal defects, and improvement of ingot's structure by upsetting the four-beam workpieces. There are the following tasks in this research:

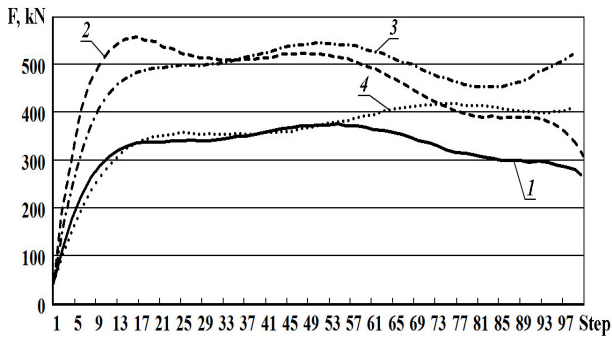
- a shape influence of the four-beam workpieces to strain and stress distribution, and welding of the internal defects after upsetting need to determine;

- a new scientifically-based method of the workpieces upsetting with concave faces, which increases a density of the parts structure need to develop;

- an approbation of improved upsetting process of four-beam workpieces need to introduce.

To obtain parts of the cup with the flange type is possible in various ways of combined extrusion (with two and three degrees of freedom of metal flow). Moreover, the parts obtained by three-direction extrusion are characterized by stability of dimensions, since the deformation occurs in the same die, without reinstalling, which is necessary according to modern technologies to form the elements of a complex part. The comparison of diagrams «deformation path – extrusion force» (Fig. 1) obtained by simulating various options for combining extrusion schemes confirmed the significant decrease in power parameters during three-direction extrusion. Although at the initial stage of the process the increasing curves for extrusion forces during two-direction (radial-forward) and three-direction processes almost coincide [22].

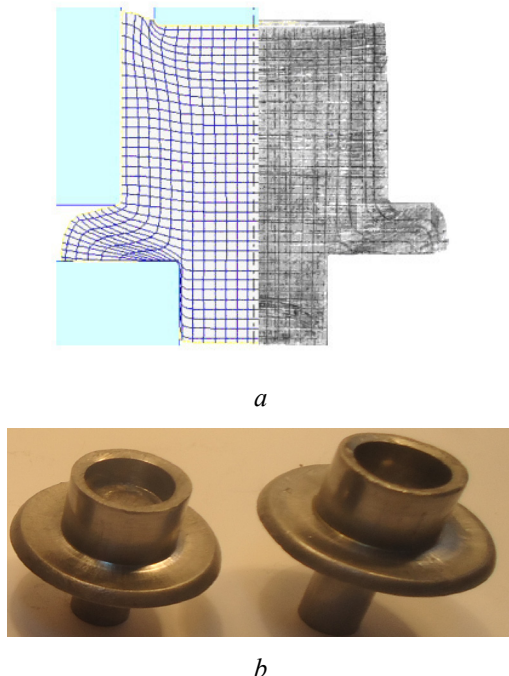
Thus, one can state the feasibility of using three-direction combined extrusion and the advantage of this process over the processes with two degrees of material flow.



1 – backward-forward-radial, 2 – backward-forward, 3 – backward-radial, 4 – radial-forward

**Figure 1. The comparison of deformation force diagrams for various combined extrusion schemes (aluminium alloy Al99)**

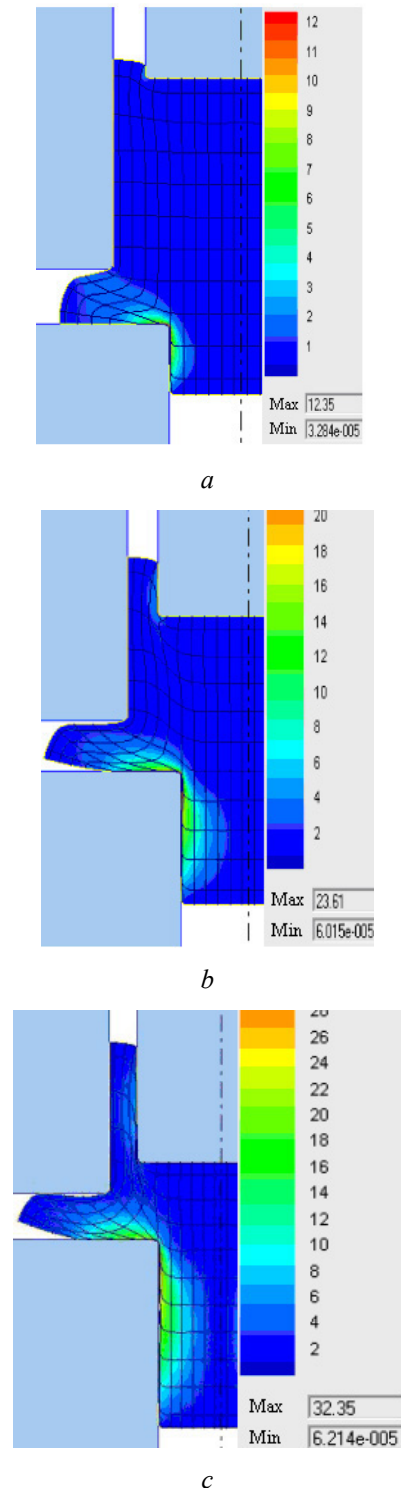
The experimental studies and finite element simulating of the stress-strain state, limit and gradual form changing and power parameters allow the features of the combined extrusion process to be identified. This contributes to the construction of adequate mathematical models of the process under study and the following verification of theoretically obtained calculations. To study the stress-strain state during three-direction radial-backward-forward extrusion, composite lead workpieces with coated mesh with a 2.1 mm step are used (Fig. 2, a). Based on the picture of the coordinate mesh deformation (see Fig. 2, a), one can assume about the presence of intermediate rigid zone, in which the material practically does not deform during the process.



**Figure 2. Distortion of mesh, received experimentally and by simulating in QFORM2D/3D (a) and the obtained parts (b)**

The outflow of a metal into the clearance between the punch and the upper die and into the flange and branch piece zone is carried out from two autonomous deformation zones separated by a rigid one. Studies of the degree of deformation during simulating in

QFORM2D/3D made it possible to identify characteristic zones at the initial stage of deformation as well (Fig. 3).



**Figure 3. Gradual changing in the deformed state during combined three-direction extrusion obtained by simulating in QFORM2D/3D for stroke 5 (a), 15 (b) and 25 mm (c)**

When studying deformed meshes and the distribution of deformations, it is seen that the maximum shear deformations are concentrated on the line between the edges of the punch and the die of the workpiece. Also, it should be noted that there are low-deformed zones that are concentrated on the outer surface of the cup wall, in the internal axial zones of the branch piece

and in the peripheral zone of the flange. In addition, the picture of gradual change in the deformed state during the combined three-direction extrusion confirms the presence of the rigid zone, the height of which decreases during the deformation [22]. At the same time, the heights of these deformation zones practically do not change during the process, which is achieved due to the expenditure of metal in the intermediate hard zone (Fig. 3 a, b). With further deformation, there comes the moment of complete degenerating the intermediate zone and integrating the upper and lower deformation centers (Fig. 3, c).

The analysis of the picture of mesh distortion made it possible to clearly determine the location and shape of deformation zones: one of which is located between the punch and the upper die in the formed wall of the cup and the second, which is formed when material flows in radial-forward direction. Mesh elements adjacent to the die wall undergo compression in the radial direction, stretching upward. Mesh distortion practically does not occur in the intermediate zone (the height of this zone decreases during the process).

#### 4. MATHEMATICAL MODEL CREATION OF THE COMBINED THREE-DIRECTION EXTRUSION PROCESS

The power method is the most widely used one in solving the problems of die forging. It allows us to estimate the upper bound of the deformation force using directly receiving solutions, bypassing the integration of differential equilibrium equations [23]. At present, two modifications of the power method are distinguished: the method of balance powers or works and the upper bound method. The first is based on simulating the metal flow in the deformation zone using continuously deformable modules, and the second consists in using rigid (non-deformable), as a rule, triangular units.

When calculating axisymmetric extrusion processes, the deformable volume of the workpiece is divided into kinematic elements (modules), which are considered in cylindrical coordinate system taking into account axial symmetry and the circumferential velocity component is equal to zero  $v_\theta = 0$ . The flow of material inside each module is described using pair of functions that explicitly determine the kinematically possible velocity field (KPVF). The choice of set of kinematic modules, their configuration, and KPVF is carried out on the basis of tentatively fulfilled experimental studies and the analysis of metal flow features [16, 23–25]. The simplest scheme for receiving KPVF for axisymmetric kinematic elements with rectangular frontal section is based on the assumption of parallel metal flow. If the velocity components along each coordinate direction are independent of the coordinates in other directions, i.e.  $v_z = v_z(0)$ ,  $v_r = v_r(r)$ , then the velocity components in a general form can be determined by the formulas [16] (1):

$$v_z = C_1 \cdot z + C_2; \quad v_r = -0,5 \cdot C_1 \cdot r + \frac{C_3}{r}; \quad (1)$$

where  $C_1, C_2, C_3$  are arbitrary constants.

The constants  $C_1, C_2, C_3$  are determined based on the kinematic boundary conditions at velocities and the continuity conditions of the normal component of the velocity on the surface of the velocity discontinuity.

The relative linear and shear strain rates for the modules considered in cylindrical coordinate system are calculated using formulas (2):

$$\dot{\epsilon}_z = \frac{\partial v_z}{\partial z}; \quad \dot{\epsilon}_r = \frac{\partial v_r}{\partial r}; \quad \dot{\epsilon}_\theta = \frac{v_r}{r}; \quad \dot{\gamma}_{rz} = \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \quad (2)$$

After choosing the appropriate set of KPVF for the deformation process, the main equation of the energy balance in powers is made that connects the power of external active forces  $N_a = p \cdot F \cdot v_0$  with the power of internal

$$p \cdot F \cdot v_0 = \sum_i N_{di} + \sum_k N_{ci-j} + \sum_j N_{tj-n}, \quad (3)$$

where  $p$  is deforming pressure;  $v_0$  is velocity of the punch,  $F$  is cross sectional area of the punch,  $N_{di}$  is the power of plastic deformation forces of the module  $i$ ;  $N_{ci-j}$  is the power of shear forces between neighboring modules  $i$  and  $j$ ;  $N_{tj-n}$  is the power of friction forces developed on the contact surface of the module  $j$  with the tool  $n$ .

Dividing both parts of the resulting equality by factor  $F \cdot v_0$ , the unit pressure  $\bar{p} = p/\sigma_s$  is calculated by the formula

$$\bar{p} = \frac{\sum_i N_{di} + \sum_k N_{ci-j} + \sum_j N_{tj-n}}{F \cdot v_0 \cdot \sigma_s} \quad (4)$$

where  $\sigma_s$  is yield stress.

Note that the unit pressure  $\bar{p}$  is a criterial value, which allows further finding the deformation force  $P$  for a certain deformation scheme regardless of the material brand.

All schemes of incorporated combined extrusion (with several degrees of flow freedom) are classified depending on the features of the deformation center (DC) formation. There occurs a division into schemes with the attached, united, disconnected and combined DC. The greatest labor consumption among them is represented by extrusion schemes with disconnected deformation centers and an intermediate rigid zone. The optimal value of the rigid zone movement speed is determined from the condition of equality of powers acting on both sides of the plane separating two independent deformation centers.

When simulating the process of combined radial-backward-forward extrusion based on experimental data, autonomous (disconnected) deformation centers were identified (Fig. 4): the upper one is the backward extrusion unit and the lower one is the combined radial-forward extrusion unit with the attached deformation center (Fig. 4, 5). The complication of the energy analysis problem consists in the use of modules with non-parallel flow to describe the metal flow at the transition edges of the tool.

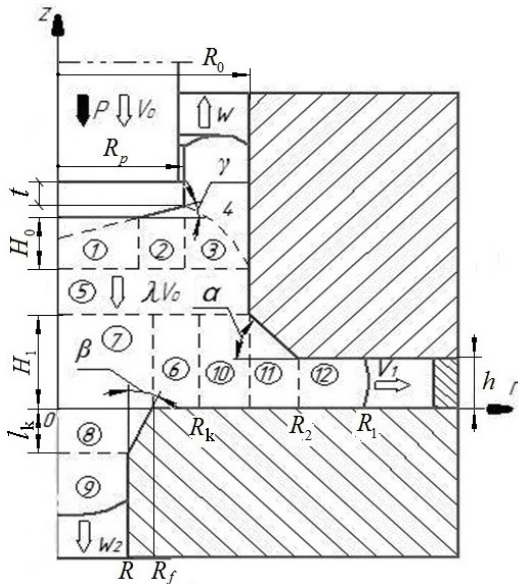


Figure 4. The calculation scheme of combined radial-backward-forward extrusion)

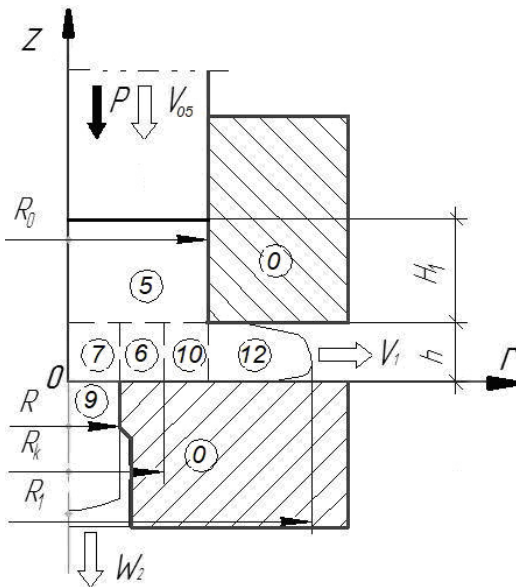


Figure 5. The process scheme of radial-forward extrusion of a stem part with the flange)

Considering the combined radial-forward extrusion unit with the attached deformation center, one can note that the kinematic modules 5, 6, 7, 9, 10, and 12 are elementary and do not cause difficulties in subsequent calculations. The presented kinematic rectangular modules have the inversion property, which ensures the ease of their use (of obtained values of unit pressures) and integration into more complex calculation schemes [26]. However, the considered KPVPs for elementary kinematic elements of this shape do not allow considering the flow of material in kinematic elements with curved inclined boundaries that repeat the shape of the tool contact surfaces. For axisymmetric processes, various KPVPs that meet these requirements are considered in the works [22–24]. For the lower deformation center, one can use the results of the analysis of the radial-forward extrusion block with rectangular modules with the addition of the unit deformation pressures in two new kinematic modules 8

and 11, which are presented below and are intended to describe the flow through the transition edges of dies.

Thus, the calculation scheme with modules with non-parallel flow includes two new modules 8 and 11 (see Fig. 4). The KPVP of the module 8 can be considered as a special case for the axial trapezoidal module, considered in the article [24]. Taking into account the absence of metal flow velocity through an inclined rectilinear boundary, the KPVP of the module 8 takes the form:

$$\begin{cases} v_{z8} = -\frac{v_{7-8} \cdot R_f^2}{T^2(z)}, \\ v_{r8} = -\frac{v_{7-8} \cdot R_f^2 \cdot r}{T^3(z)} \cdot k, \end{cases} \quad (5)$$

where  $v_{7-8} = -\frac{v_0 \cdot R_k^2}{R_f^2}$  is the velocity at the boundaries

of the kinematic modules 7 and 8;  $T(z) = k \cdot z + R_f$  is the equation of the inclined boundary;

$k = \text{tg}\beta = \frac{R_f - R}{l_k}$ ;  $R$  is the radius of the axial branch

piece;  $R_k$  is the radius of the boundary of the deformation centers forming the axial branch piece and flange;  $\beta$  is the chamfer angle of the lower die;  $l_k$ ,  $R_f$  is the parameters of the lower die defining the chamfer of an axial branch piece (see Fig. 4).

The power of forces and the unit deformation pressure for the module 8 were calculated by the formulas [26–28]:

$$N_{d8} = \frac{2\pi \cdot \sigma_s \cdot v_{7-8}}{9 \cdot \text{tg}^2 \beta} \cdot \left( (4 + 3 \cdot \text{tg}\beta)^{3/2} - 8 \right) \cdot R_f^2 \cdot \ln \frac{R_f}{R} \quad (6)$$

$$\Delta \bar{p}_8 = \frac{2}{9 \cdot \text{tg}^2 \beta} \cdot \left( (4 + 3 \cdot \text{tg}\beta)^{3/2} - 8 \right) \cdot \ln \frac{\bar{R}_{f1}}{\bar{R}} + \frac{4}{3 \cdot \sqrt{3}} \cdot \text{tg}\beta + \frac{4}{\sqrt{3}} \cdot \mu \cdot (1 + \text{tg}^2 \beta) \cdot \ln \frac{\bar{R}_{f1}}{\bar{R}} \quad (7)$$

where  $\mu$  is a friction coefficient,  $\bar{R}_{f1} = \frac{R_f}{R_k}$ ,  $\bar{R} = \frac{R}{R_k}$ .

Note that the value  $\Delta \bar{p}_8$  is obtained in an analytical form and takes into account the power of deformation forces, shear and friction for the kinematic module 8.

Similarly the KPVP for the kinematic trapezoidal module 11 with an inclined rectilinear boundary in the absence of flow velocity through an inclined boundary [24] has the form:

$$\begin{cases} v_{z11} = -\frac{v_{10-11} \cdot R_0 \cdot H_1 \cdot \text{tg}\alpha}{r \cdot z^2(r)} \cdot z, \\ v_{r11} = \frac{v_{10-11} \cdot R_0 \cdot H_1}{r \cdot z(r)}, \end{cases} \quad (8)$$

where  $v_{10-11} = \frac{v_0 \cdot (R_0^2 - R_k^2)}{2 \cdot H_1 \cdot R_0}$  is the velocity at the

boundaries of the kinematic modules 10 and 11;

$z(r) = -tg\alpha \cdot (r - R_0) + H_1$  is the equation of the inclined boundary;  $tg\alpha = \frac{H_1 - h}{R_2 - R_0}$ ;  $R_0$  is the radius of the upper die;  $\alpha$  is the chamfer angle of the upper die;  $H_1$  is the height of the lower deformation center.

The calculation of the deformation force power for a given KPVF causes difficulties in integrating the intensity of the strain rates and the impossibility of expressing a given value through elementary functions. To obtain the value of the unit pressure  $\Delta\bar{p}_{11}$  in an analytical form the upper bound of the power of deformation forces is performed in the form:

$$N_{d11} \leq \sigma_s \cdot \sqrt{V_{11} \cdot \iiint_{V_{11}} \dot{\varepsilon}_{i11}^2 dV_{11}} \quad (9)$$

where

$$V_{11} = 2\pi \cdot \int_{R_0}^{R_2} r dr \int_0^{z(r)} dz = 2\pi \cdot \left( \begin{array}{l} (H_1 + tg\alpha \cdot R_0) \cdot \frac{R_2^2 - R_0^2}{2} \\ -tg\alpha \cdot \frac{R_2^3 - R_0^3}{3} \end{array} \right)$$

is the volume of zone 11;  $\dot{\varepsilon}_{i11}$  is the intensity of the strain rate in the module 11;  $R_2$  is the geometric parameter of the lower die (see Fig. 4).

Given (9), the unit deformation pressure for module 11 takes the form:

$$\Delta\bar{p}_{11} = \frac{2 \cdot \sqrt{2}}{3} \cdot \sqrt{\left( N \cdot \frac{R_2^2 - R_0^2}{2} - tg\alpha \cdot \frac{R_2^3 - R_0^3}{3} \right) \cdot (Q_1 + Q_2 + Q_3) + \frac{tg\alpha}{2 \cdot \sqrt{3}} + \frac{2 \cdot \mu \cdot (2 + tg^2\alpha)}{\sqrt{3} \cdot tg\alpha} \cdot \ln \left| \frac{H_1}{h} \right|} \quad (10)$$

$$\text{where } Q_1 = \frac{6 + 0,5 \cdot tg^2\alpha}{2 \cdot N} \cdot \left( \frac{1}{R_2^2} - \frac{1}{R_0^2} \right);$$

$$Q_2 = -\frac{tg\alpha \cdot (2,5 \cdot tg^2\alpha + 12)}{N^2} \cdot \left( \frac{1}{R_2} - \frac{1}{R_0} \right) +$$

$$+ \frac{tg^2\alpha \cdot (1 + tg^2\alpha)}{N} \cdot \left( \frac{1}{A^2} - \frac{1}{B^2} \right);$$

$$Q_3 = \frac{4 \cdot tg^2\alpha \cdot (2 + tg^2\alpha)}{N^2} \cdot \left( \frac{1}{A} - \frac{1}{B} \right) +$$

$$+ \frac{tg^2\alpha \cdot (20 + 6,5 \cdot tg^2\alpha)}{N^3} \cdot \ln \left| \frac{B \cdot R_2}{A \cdot R_0} \right|;$$

$A = N - tg\alpha \cdot R_2$ ;  $B = N - tg\alpha \cdot R_0$ ;  $N = H_1 + tg\alpha \cdot R_0$ ;  $h$  – flange thickness.

The general equation of the power energy balance for the lower block of combined radial-forward extrusion has the form:

$$\bar{p}_{kp} = \frac{\bar{p}_{6-9} \cdot R_k^2 + \bar{p}_{10-12} \cdot (R_0^2 - R_k^2)}{R_0^2}, \quad (11)$$

where  $\bar{p}_{6-9}$  is the total unit pressure for kinematic modules 6-9 (forward extrusion);  $\bar{p}_{10-12}$  is the total unit pressure for kinematic modules 10-12 (radial extrusion).

The upper block in the deformation center is the zone of the backward extrusion of the cup using the punch with a conical end face. The optimization of the punch geometry for backward extrusion is a quite investigated problem [23]. We use the simplified formula of P. D. Chudakov [25] for the punch extrusion unit pressure with the chamfer  $\gamma = 15$ , radius of flat end face of the punch  $R_p$  and the width of the calibrating punch belt  $t = 0.1 \cdot R_p$ , which in the notation of this scheme takes the form:

$$\begin{aligned} \bar{p}_1 = \frac{1}{\sqrt{3}} \cdot \left[ \frac{\mu \cdot R_p}{12 \cdot H_0} + \frac{H_0}{R_p} + \frac{R_0 - R_p}{a} + 3 \cdot \mu + \right. \\ \left. + 0.226 + \frac{b}{R_p^2} \cdot \left( b \cdot \ln \frac{a}{H_0} - 0.5 \cdot R_p \right) \right] \cdot (8 \cdot \mu - 1.57) + (12) \\ \left. + \frac{2}{R_0^2 - R_p^2} \cdot \left[ R_0^2 \cdot \ln \frac{R_0}{R_p} + 2 \cdot \mu \cdot t \cdot (R_0 + R_p) + \right. \right. \\ \left. \left. + 2 \cdot a \cdot \left( \mu \cdot R_0 + \frac{1}{3} \cdot \frac{R_0^3 - R_p^3}{R_0^2 - R_p^2} \right) \right] \right] \end{aligned}$$

where  $a = H_0 + 0.134 \cdot R_p$ ;  $b = 3.73 \cdot H_0 - 0.5 \cdot R_p$ ;  $H_0$  is the height of the upper deformation center.

After finding the pressures for the incoming blocks and modules, one can find the pressure value for the process of three-direction extrusion of a hollow part with a stem and flange. As noted above, the optimal value of the movement velocity of the rigid zone 5 is determined from the condition of equality of powers acting on both sides of the plane separating two independent centers of deformation. Taking into account the relation  $\bar{q} = \bar{p}_1 \cdot \frac{R_0^2 - R_p^2}{R_0^2}$ , for the coefficient  $\lambda$  characterizing the movement velocity of the rigid zone 5, one can write:

$$\lambda = \frac{\bar{q} \cdot \varepsilon^2}{\bar{q} + \bar{p}_{kp} + G/(1 - \varepsilon^2)} \quad (13)$$

where  $\varepsilon = R_p/R_0$ ;  $G = \mu \cdot \frac{(R_k + R_0) \cdot t}{R_0^2}$ .

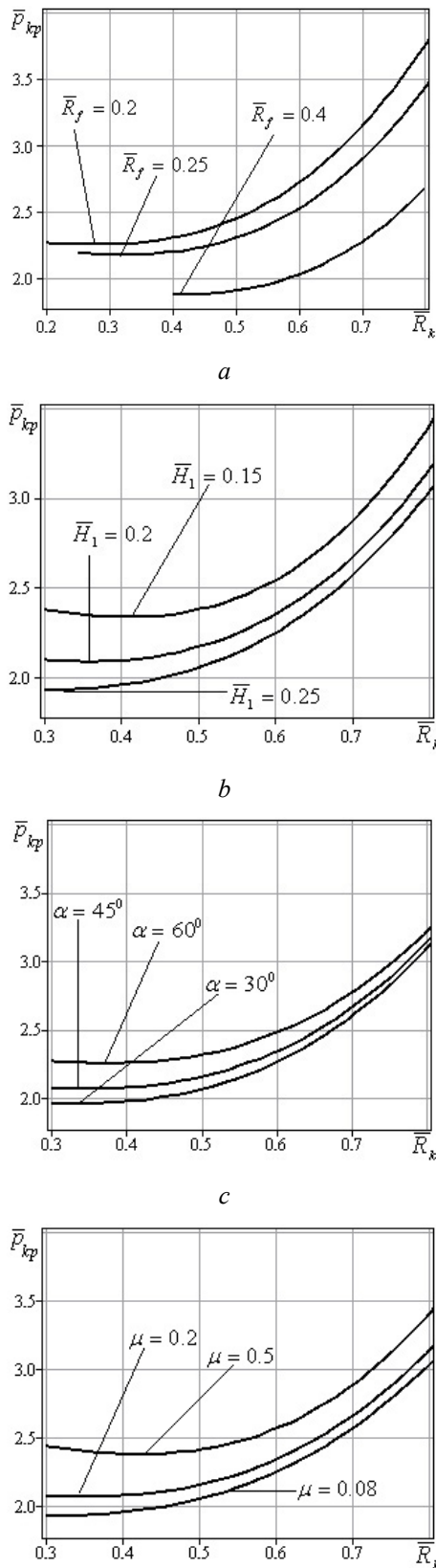
For the total unit pressure of the combined extrusion, one can write:

$$\bar{p}_k = \bar{p}_1 \cdot (1 - \lambda) + \bar{p}_{kp} \cdot \frac{R_0^2}{R_p^2} \cdot \lambda - G \cdot \frac{R_0^2}{R_p^2} \cdot \lambda \quad (14)$$

## 5. RESULTS AND DISCUSSION

Based on the developed calculation scheme with the presence of chamfers in the zone of formation of the axial branch piece and flange, the relationships between the unit pressure value in the lower autonomous defor-

mation center  $\bar{p}_{kp}$  according to (11) and the technological process parameters were analyzed (Fig. 6).



**Figure 6.** Changing in the value of the unit pressure  $\bar{p}_{kp}$  versus the critical value  $\bar{R}_k$  for various parameters of the radial-forward extrusion process:

**a** – at  $\bar{H}_1 = 0.2, h = 0.15, \alpha = \beta = 45^\circ, \mu = 0.08$  ;

**b** – at  $\bar{R}_f = 0.3, \bar{R}_2 = 1.05, \alpha = \beta = 45^\circ, \mu = 0.08$  ;

**c** – at  $\bar{H}_1 = 0.25, \bar{R}_f = 0.3, \bar{R}_2 = 1.1, \beta = 45^\circ, \mu = 0.2$  ;

**d** – at  $\bar{H}_1 = 0.25, \bar{R}_f = 0.3, \bar{R} = 0.15, \bar{R}_2 = 1.15, \alpha = \beta = 45^\circ$ .

The obtained calculation formula of the value  $\bar{p}_{kp}$  is the function of fixed geometric parameters of the process, friction conditions and the variable parameter  $R_k$  that is responsible for separation boundary of the flow of the forward (forming the axial branch piece) and radial (forming the flange) extrusion.

Within the framework of this calculation scheme the variation of the parameter  $\bar{R}_k$  is possible in the value range from  $\bar{R}_f = R_f / R_0$  to 1. The study of the optimal relative critical radius of the material flow distribution  $\bar{R}_k = R_k / R_0$  showed that the decrease in the parameter  $\bar{R}_f$  while maintaining the rest parameters of the deformation process led to the increase in the unit pressure  $\bar{p}_{kp}$  (Fig. 6, a).

In this case, the optimal value  $\bar{R}_k = 0.4$  for  $\bar{R}_f = 0.4$  takes boundary one and corresponds to the degeneration of the module 6. For the rest (smaller) values  $\bar{R}_f$  slight shift occurs  $\bar{R}_k$  towards increasing with regard to  $\bar{R}_f$  that indicates the involvement of portion of module 6 metal in the formation of the axial branch piece. Decreasing in the height of lower deformation center  $\bar{H}_1 = H_1 / R_0$  while maintaining the rest parameters of the deformation process also leads the increase in the value of the unit pressure  $\bar{p}_{kp}$  (Fig. 6, b).

In this case for  $\bar{H}_1 = 0.25$ , the degeneration of the module 6 occurs, that corresponds to favorable conditions (larger volume of metal) for the formation of the flange zone. At lower values of  $\bar{H}_1$  one can see the presence of minimum point (the presence of module 6, that additionally forms the axial branch piece). The influence of technological factors as chamfer angles and friction conditions on the picture of changing separation boundary of the flow  $\bar{R}_k$  (Fig. 6, c, d) is also of interest. Decreasing the chamfer angle  $\alpha$  while maintaining the rest process parameters is the favorable factor from the point of view of flange forming and it leads to decreasing the value of the unit pressure. Moreover, at the highest value  $\alpha = 30^\circ$ , the degeneration of the module 6 occurs, i.e. forming the branch piece takes place only due to the axial zone directly located above the module 8 (Fig. 6, c). Friction conditions also have the significant effect on the optimum value  $\bar{R}_k$ . The maximum friction in the bottom of the workpiece shifts the optimal value  $\bar{R}_k$  to 0.42 with a relative value  $\bar{R}_f = 0.3$  and helps to reduce the zone 10, which is responsible for the forming of the flange at  $\bar{R}_2 = R_2 / R_0$  (Fig. 6, d).



Moreover, with  $\mu = 0.08$  there is a decrease in the value of the unit pressure and degeneration of the module 6, that is a favorable factor for forming the flange zone.

The nature of the studied parameter effect on changing the optimal value  $\bar{R}_k$  does not change when an autonomous deformation center of backward extrusion is connected to the general scheme of combined three-direction extrusion.

The analysis of changing the value  $\lambda$  that is responsible for the speed of movement of the intermediate rigid zone 5 depending on  $\bar{R}_p$  ( $\bar{R}_p = R_p / R_0$  is the relative radius of the punch) is of interest as well. The study of changing the relative velocity of the rigid zone movement shows that with increasing the radius of the punch  $\bar{R}_p$ , this value increases (Fig. 7). And at  $\bar{R}_p \rightarrow 0$ ,  $\lambda \rightarrow 0$ . With increasing  $\bar{R}_p$  (and therefore with decreasing in the thickness of the cup wall) relative velocity of the rigid zone movement increases, and at  $\bar{R}_p = 1$  (there is no wall),  $\lambda = 1$  and radial direct extrusion is implemented. This trend of changing  $\lambda$  persists for different values of the height of the lower deformation center  $\bar{H}_1$  and the radius of  $\bar{R}$ , but the parameter  $\lambda$  values, ceteris paribus, are higher for larger values  $\bar{H}_1$  (Fig. 7, a) and  $\bar{R}$  (Fig. 7, b) at the same  $\bar{R}_p$  value. The latter is particularly evident for the range of  $\bar{R}_p$  values from 0.3 to 0.9, which corresponds to most sizes of hollow parts obtained by cold volume extrusion. This is due to the fact that at large relative values of flange thickness for radial extrusion and the radius of branch piece under direct extrusion the resistance of deformation decreases, the total power of forces for the lower deformation center becomes smaller and  $\lambda$  increases. But nevertheless, the distribution of the flow in the backward and radial-forward directions and the  $\lambda$  value are most significantly influenced by the value of the relative radius of the punch  $\bar{R}_p$ .

Using the analysis of the obtained function (14) for combined three-direction extrusion, the nature of the process parameters effect on the unit deformation pressure  $\bar{p}_k$  is found. With increasing the relative radius of the punch  $\bar{R}_p$  to 0.8...0.85 the value  $\bar{p}_k$  decreases, and then it grows significantly (Fig. 8, a). This is explained by the fact that with decreasing in thickness of the cup wall, the influence of the lower deformation center increases and grows in the total energy balance up to certain point, i.e. the scheme of three-direction radial-backward-forward extrusion is implemented, and then the unit pressure increases due to degenerating of the upper deformation center and transiting from three-direction extrusion to radial-forward one.

Increasing the values of the relative radius of the branch piece  $\bar{R}$  leads to decreasing in the value of  $\bar{p}_k$  (Fig. 8, b), which is explained by decreasing the degree of deformation caused by forward extrusion during the

formation of the axial branch piece. This nature of the change in unit pressure versus parameters  $\bar{R}_p$  and  $\bar{R}$  is maintained under all the given friction conditions.

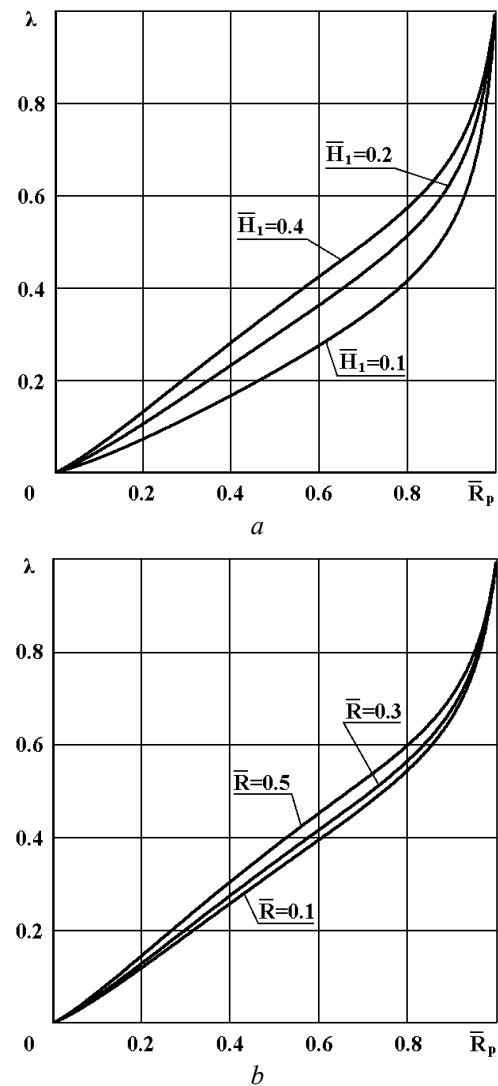
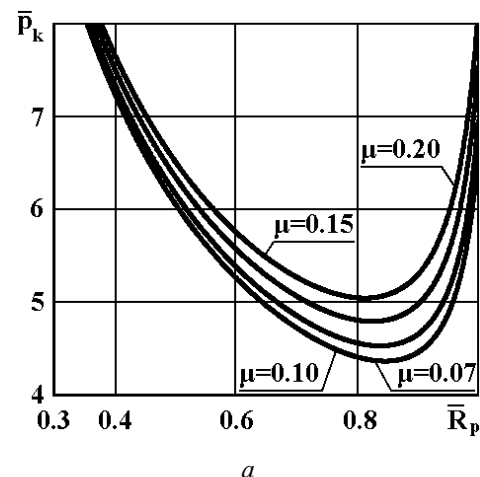


Figure 7. Graphs of  $\lambda$  versus relative radius of the punch  $\bar{R}_p$  for different flange height  $\bar{H}_1$  (at  $\bar{R} = 0.2$ ) (a) and different radius of the branch piece  $\bar{R}$  (at  $\bar{H}_1 = 0.3$ ) (b) at  $\mu = 0.1, \bar{l}_k = 0.1, \alpha = \beta = 45^\circ$ ,

$$\bar{H}_0 = \frac{2}{\sqrt{6}} \cdot \bar{R}_p, \bar{t} = 0.1 \cdot \bar{R}_p, \bar{R}_1 = 1.2$$



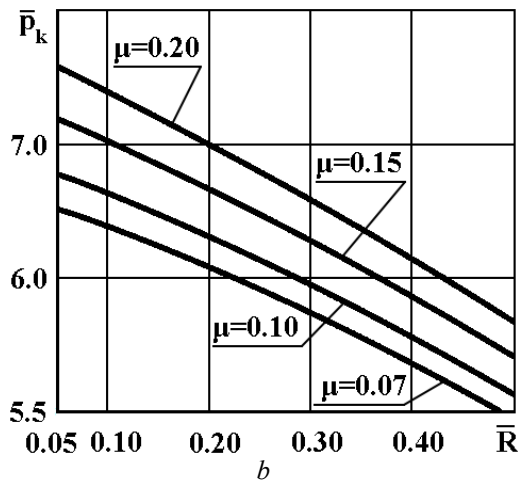


Figure 8. The graphs of the unit pressure versus relative parameters: punch radius  $\bar{R}_p$  (at  $\bar{R} = 0.2$ ) (a) and radius of

the branch piece  $\bar{R}$  (at  $\bar{R}_p = 0.5$ ) (b) for various values of the contact friction coefficient  $\mu$  at

$\bar{l}_k = 0.1, \alpha = \beta = 45^\circ, \bar{L}_k = 0.1, \bar{H}_1 = 0.3,$

$$\bar{H}_0 = \frac{2}{\sqrt{6}} \cdot \bar{R}_p, \bar{t} = 0.1 \cdot \bar{R}_p, \bar{R}_1 = 1.2$$

and  $H^* = 0.5$  (height of the rigid zone 5)

The experimental studies of the effect of the branch piece diameter shows that with the increase in the diameter of the branch piece from 14.2 mm to 21.2 mm, the extrusion force decreased by average of 5%. Increasing the punch diameter from 21.2 mm to 36 mm (correspondingly, decreasing the wall thickness of the cup) led to increasing the extrusion force by 17% [22]. When comparing the data obtained by the power method and experimental studies, it is found that theoretical calculations give results 18% higher than the experimental ones; the FEM showed the results 12% lower than the experiments (Fig. 9).

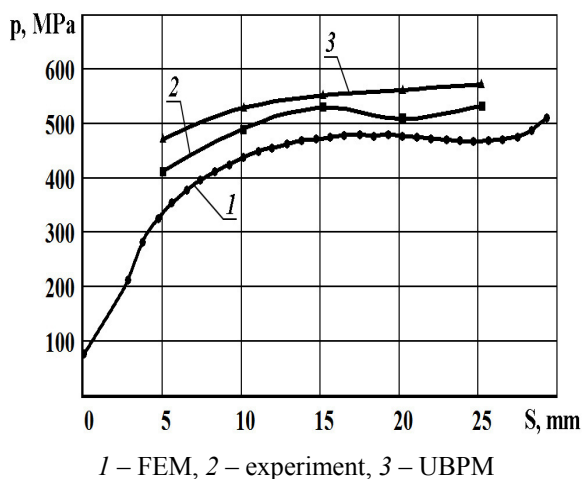


Figure 9. Deformation forces in the function of the punch stroke for a hollow part with a flange and branch piece (aluminum alloy Al99) at  $R_p = 14$  mm,  $R = 10.5$  mm,

$$R_0 = 22.5$$
 mm,  $h = 7$  mm

Thus, the validity of the created calculation scheme for simulating the process of combined radial-backward-forward extrusion is confirmed.

## 6. CONCLUSIONS

Based on experimental studies and finite element simulating in QFORM2D/3D, the regularities of forming and developing the deformed state of the workpiece in the process of combined three-direction extrusion are evaluated. It is found that during combined extrusion with metal outflow in the radial, forward and backward directions, the centers of intensive plastic deformation are concentrated in the zones of the outlet openings on the transition edges of the deforming tool. The picture of gradual changing the deformed state at the initial stage confirms the fact of the presence of undeformed rigid zone separating the autonomous centers of backward and radial-forward extrusion. The height of these autonomous centers remains the same throughout the process, and the height of the rigid intermediate zone decreases during deformation. Based on the identified features, the power method of the upper bound is used to model the process of combined radial-backward-forward extrusion of hollow parts of the «cup with flange and branch piece» type of a continuous workpiece. For the ratios of the geometric parameters of the process with the diameter of the axial process less than the diameter of the punch, a design scheme with a disconnected deformation zone and with the condition equality of capacities acting on both sides of the intermediate hard zone. The proposed scheme contains modules with an inclined boundary, which made it possible to more accurately evaluate the formation of the semi-finished product with a predominant radial-forward metal flow. The relations of the value of the unit pressure in the lower autonomous deformation center  $\bar{p}_{kp}$  and the technological parameters of the process are analyzed. The criterion value (variable parameter) is the relative value  $\bar{R}_k$ , namely the boundary of centers forming the axial branch piece and flange. The influence of geometric parameters (radius of the axial branch piece, flange thickness, chamfer geometry) and friction conditions on the optimal value of  $\bar{R}_k$  and the value of the unit pressure is analyzed. The conditions under which the module 6 additionally forming the axial branch piece degenerates are mentioned. The changing in the speed of the rigid zone movement from the initial height of the workpiece and the effect of the geometric parameters of the process on its change is analyzed. The influence of the radius of the punch and the radius of the axial branch piece for various values of the contact friction coefficient  $\mu$  on the total unit pressure of the process is also analyzed. The theoretical and experimental data, the simulation results in QFORM2D/3D are compared according to the deformation force. It was found that Theoretical calculations are found to give the results 18% higher than the experimental ones, and the FEM shows results 12% lower than the experiments.

The purpose of further research is to expand the capabilities of the power method in the development of calculation schemes for combined extrusion processes with various designs of parts and accordingly types of deformation centers (divided, incorporated), which allow the formation of sizes in different directions to be efficiently predicted.

## REFERENCES

- [1] Aliev, I.S.: Radial Extrusion Processes, Soviet Forging and Sheet Metal Stamping Technology, (English Translation of Kuznechno-Shtampovnoe Proizvodstvo), No 3, pp. 54-61, 1988.
- [2] Balendra, R., Qin, Y.: Injection Forging: Engineering and Research, Journal of Material Processing, Vol. 145, pp.189-206, 2004.
- [3] Aliev, I.S., Lobanov, A.I., Borisov, R.S. and Savchinskij, I. G.: Investigation of Die Blocks with Split Dies for the Processes of Cross Extrusion, Forging and Stamping Production. Materials Working by Pressure, No. 8, pp. 21-26, 2004.
- [4] Cho, H.Y., Min, G.S., Jo, C.Y. and Kim, M.H.: Process Design of the Cold Forging of a Billet by Forward and Backward Extrusion, J. Mater Process Technol., Vol. 135, pp. 375-381, 2003.
- [5] Jayasekara, V.R., Noh, J.H. and Hwang, B.B.: Tool Wear in Forward Rod and Backward Tube Forming Processes, in: *Proceedings of the Int. Conf. «Metal Forming 2012»*, 16-19.09.2012, AGH, Krakow, pp. 522-526, 2012.
- [6] Hrudkina, N., Aliieva, L. Markov, O., Malii, K., Sukhovirska, L. and Kuznetsov, M.: Predicting the shape formation of parts with a flange and an axial protrusion in the process of combined aligned radial direct extrusion, Eastern-European Journal of Enterprise Technologies, Vol. 5, No. 1, (107), pp. 110-117, 2020. <https://doi.org/10.15587/1729-4061.2020.212018>
- [7] Kalyuzhnyi, V.L., Aliieva, L.I., Kartamyshev, D.A. and Savchinskii, I.G.: Simulation of Cold Extrusion of Hollow Parts, Metallurgist, Vol. 61, No. 5-6, pp. 359-365, 2017. <https://doi.org/10.1007/s11015-017-0501-1>.
- [8] Hrudkina, N., Aliieva, L., Markov, O., Marchenko, I., Shapoval, A., Abhari, P. and Kordenko, M.: Predicting the shape formation of hollow parts with a flange in the process of combined radial-reverse extrusion, Eastern-European Journal of Enterprise Technologies, Vol. 106, No. 4/1, pp. 55-62, 2020. <https://doi.org/10.15587/1729-4061.2020.203988>
- [9] Lee Y.S., Hwang S.K., Chang Y.S. and Hwang B.B.: The Forming Characteristics of Radial-Forward Extrusion, Journal of Materials Processing Technology, Vol. 113, pp. 136-140, 2001.
- [10] Jafarzadeh H., Zadshakoyan M. and Abdi Sobbouhi E.: Numerical Studies of Some Important Design Factors in Radial-forward Extrusion Process, Materials and Manufacturing Processes, No. 25, pp. 857-863, 2010.
- [11] Lee H.Y., Hwang B.B. and Lee S.H.: Forming Load and Deformation Energy in Combined Radial Backward Extrusion Process, in: *Proceedings of the Int. Conf. «Metal Forming 2012»* 16-19.09.2012, AGH, Krakow, pp. 487-490, 2012.
- [12] Yang D.Y. and Kim K.J.: Design of Processes and Products through Simulation of Three-dimensional Extrusion, J. Mater Process Technol., Vol. 191, pp. 2-6, 2007.
- [13] Choi H.J. and Hwang B.B.: The Forming Characteristics of Radial-backward Extrusion, J. Mater Process Technology, Vol. 113, pp. 141-147, 2001.
- [14] Markov O.E., Kukhar V.V., Zlygoriev V.N., Shapoval A.A., Khvashchynskiy A.S., Zhytnikov R.U.: Improvement of Upsetting Process of Four-Beam Workpieces Based on Computerized and Physical Modeling, FME Transactions 48, pp. 946-953, 2020.
- [15] Lee D.J., Kim D.J., Kim B.M.: New Processes to Prevent a Flow Defect in the Combined Forward-backward Cold Extrusion of a Piston-pin, Journal of Materials Processing Technology, Vol. 139, pp. 422-427, 2003.
- [16] Aliiev I., Aliieva L., Grudkina N. and Zhbankov I.: Prediction of the Variation of the Form in the Processes of Extrusion, Metallurgical and Mining Industry, Vol. 7, No. 3, pp. 17-22, 2011.
- [17] Hwang B.C., Lee H.I. and Bae W.B.: A UBET Analysis of the Non-axisymmetric Combined Extrusion Process, Journal of Materials Processing Technology, Vol. 139, pp. 547-552, 2003.
- [18] Evdokimov A.K., Antonkina K.A.: Combined Extrusion with Backpressure. Forging and Stamping Production, Materials Working by Pressure, No. 12, pp. 6-12, 2012.
- [19] Stepanskij L.G.: *Calculations of Working of Metal by Pressure Processes*, Machine Building, Moscow, 217 p., 1982.
- [20] Ebrahimi R., Reihanian M., Moshksar M. M.: An Analytical Approach for Radial-forward Extrusion process, Materials and Design, Vol. 29, pp. 1694-1700, 2008.
- [21] Farhoumand A., Ebrahimi R.: Analysis of Forward-backward-radial Extrusion Process, Materials and Design, Vol. 30, pp. 2152-2157, 2009.
- [22] N. Hrudkina et al.: Modeling the process of radial-direct extrusion with expansion using a triangular kinematic module, Eastern-European Journal of Enterprise Technologies, Vol. 105, No. 3/1, pp. 17-22, 2020. <https://doi.org/10.15587/1729-4061.2020.203989>
- [23] Unksov E.P., Jonson U., Kolmogorov V.L. and Ogorodnikov V. A.: *Theory of Forging and Stamping*, Machine Building, Moscow, 720 p., 1992. ISBN 5-217-00846-6.
- [24] Hrudkina, N., Aliieva, L., Abhari, P., Markov, O. and Sukhovirska, L.: Investigating the process of shrinkage depression formation at the combined radial-backward extrusion of parts with a flange, Eastern-European Journal of Enterprise Technologies, Vol. 101, No. 5/1, pp. 49-57, 2019. <https://doi.org/10.15587/1729-4061.2019.179232>.
- [25] Chudakov P.D., Kalenichenko I.S.: Researching of Two-Direction Cold Extrusion, Forging and Stamping Production, No. 5, pp. 4-6, 1975.
- [26] Hrudkina, N.S. and Aliieva, L.I.: Modeling of cold extrusion processes using kinematic trapezoidal

modules, FME Transactions, Vol.48. No. 2, pp. 357-363, 2020, doi:10.5937/fme2002357H.

[27] Aliieva L., Hrudkina N., Aliiev I., Zhbakov I. and Markov O.: Effect of the tool geometry on the force mode of the combined radial-direct extrusion with compression, Eastern-European Journal of Enterprise Technologies. Vol. 2, N 7 (98), pp. 49-57, 2020.

[28] Hrudkina, N.S.: Process modeling of sequential radial-direct extrusion using curved triangular kinematic module, FME Transactions, Vol.49. No. 1, pp. 56-63, 2021, doi:10.5937/fme2101056H

#### NOMENCLATURE

|                        |  |
|------------------------|--|
| $R$                    | Radius of part   |
| $v_z, v_r$             | velocity components along each coordinate direction                    |
| $\dot{\epsilon}_{ij}$  | components of strain rates, [1/s]                                      |
| $N_{di}$               | power of plastic deformation forces, [N]                               |
| $p$                    | deforming pressure, [MPa]  |
| $F$                    | cross sectional area, [mm <sup>2</sup> ]                               |
| $v_0$                  | velocity of the punch, [mm/s]  |
| $\bar{p}$              | unit pressure, []  |
| $\sigma_s$             | yield stress, [MPa]  |
| $\beta$                | chamfer angle of the lower die, [°]                                    |
| $\alpha$               | chamfer angle of the upper die, [°]                                    |
| $\mu$                  | friction coefficient, []   |
| $H_1$                  | height of the lower deformation center, [mm]                           |
| $h$                    | flange thickness, [mm]   |
| $\lambda$              | coefficient characterizing the movement velocity of the rigid zone, [] |
| $\bar{\epsilon}$       | strain, []   |
| $\dot{\bar{\epsilon}}$ | strain rate, [1/s]   |

#### Abbreviations

|      |                                       |
|------|---------------------------------------|
| FE   | Finite Element                        |
| UBPM | upper bound power method              |
| KPVF | kinematically possible velocity field |
| DC   | deformation center                    |
| FEM  | Finite Element Method                 |

### АНАЛИЗА ПАРАМЕТАРА СНАГЕ ПРИ КОМБИНОВАНОЈ ТРОСМЕРНОЈ ДЕФОРМАЦИЈИ КОД ДЕЛОВА СА ПРИРУБНИЦОМ

Л.И. Алиева, О.Е. Марков, И.С. Алиев, Н.С. Хрудкина, В.Н. Левченко, К.В. Мали

Метод горње границе снаге је коришћен за симулацију процеса комбинованог радијалног-напред-назад истискивања код шупљих делова „шоље са прирубницом и цевним огранцима“ при континуираном варирању брзина обратка. Примењена је шема срачунавања са независном зоном деформације која има модуле са границом под нагибом.

Прихваћени су услови једнаке снаге која дејствује на обе стране средишне тврде зоне, чиме је омогућено прецизно одређивање режима снаге и преовлађујући проток метала, а што је потребно за процену начина формирања машинског елемента. Упоредивање теоријских и експерименталних вредности напрезања на притисак и брзине протока, као и резултата примене ФЕМ методе показало је да се добијене функције могу користити за технолошке прорачуне параметара снаге и процену формирања машинског елемента.