Constructive Geometric Generating of Concave Pyramids of Fourth Sort

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The paper presents the results of the study of the new set of polihedra, the Concave pyramids of the fourth sort, the construction procedures for generating them and their possible application. Correspondingly to the method of generating the Concave cupolae of fourth sort, the Concave pyramids of fourth sort have the similar logic of origination, and their counterpart in regular faced convex pyramids. They are characterised by the polygonal base, deltahedral surface net, obtained by folding the planar net of unilateral triangles, the polar distribution of the unit space cells with common apex – the top of the Concave pyramid. Polihedral surface of the planar net of Concave pyramids is produced by polar distribution of unit cells, consisting of a spatial sexagon and spatial pentagon - six, or five, unilateral triangles grouped around the common vertex. In the deltahedral surface, the two neighbouring unit cells are joined by means of a unilateral triangle in the zone of the polygonal base and a spatial quadrangle with which they share common sides. The criterion of face regularity is respected, as well as the criterion of multiple axial symmetry. The sort of the Concave pyramids is determined by the number of equilateral triangle rows in thus obtained polyhedron's net. The parameters of the solids were determined constructively by geometric methods.

Keywords: concave pyramids, polyhedral, equilateral triangle, regular polygon.

1. INTRODUCTION

Polyhedral surfaces are widely used in architecture, engineering, and industrial design to form constructive details, façade covers, interior details and spatial struc–tures (Fig. 1). It is possible to shape functional spaces inside the polyhedra, while their grouping results in composite polyhedra, which provide a matrix for for–ming spatial structures [1]. Some design solutions re–quire that polyhedral structures be enlarged or reduced, and their application ranges from everyday objects to modern engineering structures. The paper [2] explores the structures consisting of bars of polyhedral units, it describes the design of polyhedral systems, its dyna–mics, optimal management and results of prototype examination.

The study of the principles governing the geo–met–rical generation of the segments of the cupolae con–sisting of flat elements with regular configuration, the sections of regular polyhedra is presented in [3]. The paper [4] examines generation of polyhedral spatial structures using the sections of Archimedes' and Plato's bodies that is the reciprocally positioned structural ele–ments of circular, rectangular, triangular and trapezoid intersections. The analysis of the forms reveals the main techniques for defining, constructing and implementing the forms suggested for use in designing spatial structures [5].
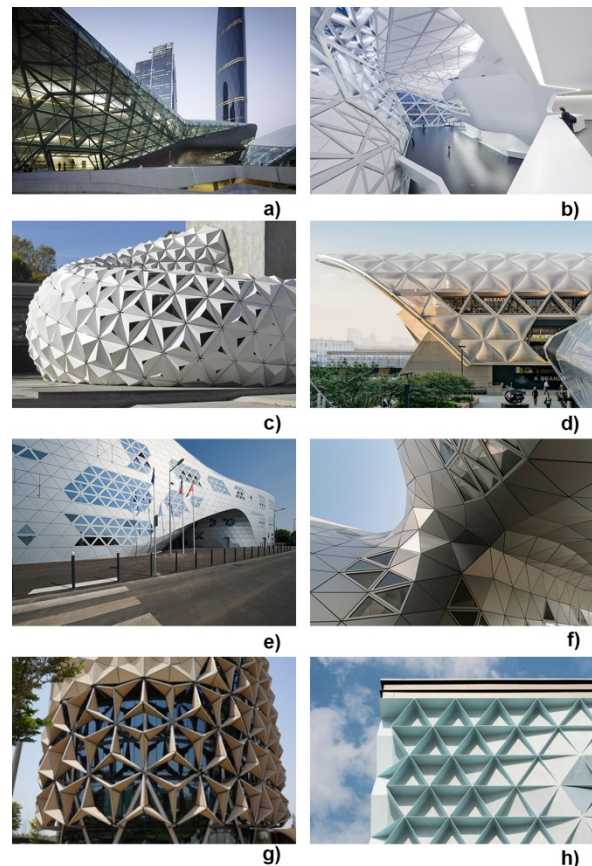

Figure 1. Application of polyhedral surfaces in architecture and engineering: a) and b) Guangzhou opera house, c) ArboSkin Pavilion, Stuttgart University, d) Canary Wharf Crossrail Station, London, e) and f) Georges-Freche School of hotel Management, Montpellier, g) Al Bahr Tower, Abu Dhabi, h) Rong Num Kaeng, Nonthaburi

Due to their characteristics, the structures generated by means of triangles stand out in the large set of polyhedral structures [6], and deltahedral surface net is the main characteristic of Concave pyramids.

Concave pyramids belong to the family of previously studied polyhedral structures comprising Concave cupolae of the second sort [7-9], Concave cupolae of the fourth (and higher) sorts [10-13], Concave antiprisms of the second sort [14] and composite polyhedra obtained by means of their mutual combination [15]. The results of the exploration of the application of these polyhedral structures in architecture and engineering were presented in [16, 17].

These polyhedra are characterized by their envelope - developmental deltahedral surface formed above the regular polygonal base. The deltahedral net is obtained by means of polar distribution of the unit cell(s) around the axis of the polyhedron that is orthogonal to the polygonal base plane and crosses the center of the in circle of the polygonal base. The unit cell consists of equilateral triangles grouped around a common vertex. The faces of the polyhedron may not protrude each other or intersect, except on the edges. Two faces of the surface net meet on each edge of the polyhedron. The edges do not intersect, except in vertices. The planes to which the sides belong may pass the inner space of the polyhedron – the surface area is a concave polyhedral area. Each face is visible from outside – there are no inner sides. There are no two neighboring coplanar faces. The type of the polyhedron is determined by the number of rows of equilateral triangles in the planar net of deltahedral surface.
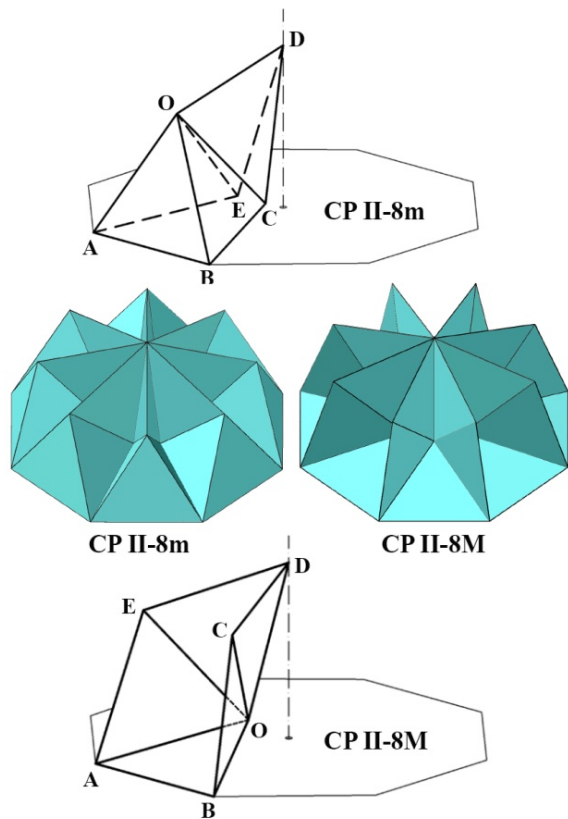

Figure 2. Method of generating the Concave pyramids of second sort by folding and creasing the plane net, obtaining two different types: CP II-8m, and CP II-8M

The previous studies have examined the Concave pyramids of the second sort (CP II) [18-20]. The unit cell consists of five equilateral triangles grouped around the common vertex. All the unit cells in thus formed deltahedral net of Concave pyramids of the second sort have a common vertex located on the rotation axis, ort–hogonal to the plane of the polygon's base. Due to the manner in which the vertex is formed and the surface area which is a developmental concave deltahedral surface, these polyhedra are termed Concave pyramids.

Geometrical generating of Concave pyramids of the second sort is based on finding the precise position of the spatial pentagon's vertex which meets the condition that the vertices A and B are located on the sides, while the vertex D lies on the axis of polygonal base (Fig. 2). Previous research has shown that there are two types of Concave pyramids of the second sort above the same polygonal base. If the common vertex O is indented, the concave pyramid of greater height is obtained (CP II-nM). Conversely, if the common vertex O protrudes, the distribution of the other vertices in the spatial pentagon is such that it generates a concave pyramid of a smaller height (CP II-nm).
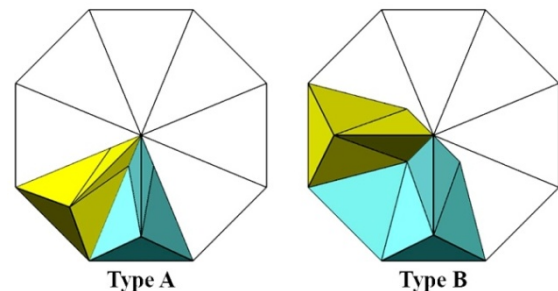

Figure 3. Methodof generating the Concave pyramids of second sort, type A and type B

Apart from this classification of the Concave pyramids of the second sort into those with smaller and larger height, resulting in differently shaped polyhedron, there also two ways in which concave pyramids can be generated. In the first manner, termed "type A" the number of unit cells in the net is equal to the number of sides (n) of the polygonal base (Fig. 3) - the unit cell is developed above every side of the polygonal base. In the second manner, or "type B", the unit cell is developed above every other side, and they are mutually connected by means of equilateral triangles. The second manner of generation can only be applied to Concave pyramids of the second sort which are formed above the polygonal base with even number of sides.

Table 1: All possible shapes of Concave pyramids of the second sort

CP II	n=6	n=7	n=8	n=9
CP II-mA	●	●	●	
CP II-MA	●	●	●	●
CP II-B	●		●	

Table 1 shows all possible shapes of Concave py–ramids of the second sort. As can be seen, they can be developed above polygonal base $6 \leq n \leq 9$. The net CP II-10B and CP II-9mA protrudes the plane of the polygon's base so that the polyhedron itself cannot be

formed, but it is possible to generate the net, which has been used [15] to form composite polyhedral structures [21].

2. CONCAVE PYRAMIDS OF THE FOURTH SORT

This study has proven the existence of the Concave pyramids of the fourth sort, type B, and it has shown that the generation of concave pyramids follows the geometrical principles for generating concave cupolae. The generation of the Concave pyramids of the fourth sort is based on predefined conditions described in the introduction, which classify those polyhedrons into a unique family of concave polyhedral structures.

The unit cell consists of the spatial hexagon ABCDEF (six equilateral triangles grouped around the common vertex O_1 and the spatial pentagon EDGHJ (five equilateral triangles grouped around the common vertex O_2), with the common edge ED (Fig. 4). Thus formed unit cell in turn forms the surface of the concave pyramid by planar distribution around the axis orthogonal to the base plane. In the deltahedral surface area CP-IV two neighboring unit cells are joined by means of an equilateral triangle (with which they share the common edge BC, or AF) and the spatial tetrahedron (four equilateral triangles grouped around a common vertex Q) with which they share the common sides CD, DG, or FE, EJ. The common vertex of all unit cells (marked as vertex H in Figure 4) lies on the poly–hedron's axis.
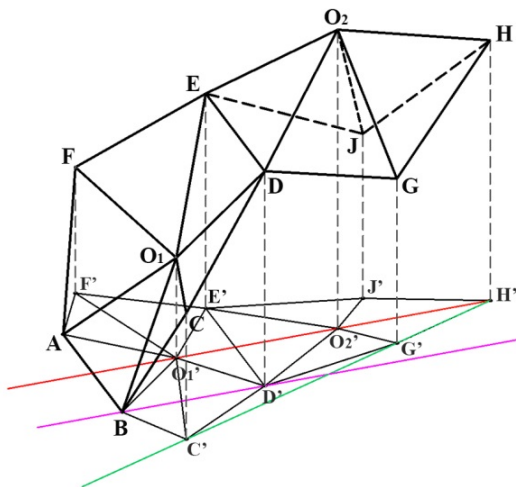

Figure 4. The unit cell of the Concave pyramid of the fourth sort and orthogonal projection onto the base plane

The geometry of the unit cell defines the possible size of the polygonal base. The condition that the radius of polygon's base incircle must be:

$$r < \frac{3a\sqrt{3}}{2} + a \qquad (1)$$

leads us to the conclusion that concave pyramids of the fourth sort can be formed above the polygonal base whose number of sides is $10 \leq n \leq 22$.

2.1 Geometric generating of Concave pyramids of fourth sort

The process of exploring the patterns for forming polyhedral surfaces is greatly facilitated by constructive and computer-aided geometry, which reveals the mechanisms behind the generation of the surface. The illustration of how computer-aided geometry helps design the form of engineering construction is given in [22], which reviews the procedures for generating spatial bar and panel structures. Constructive geometry offers a variety of geometric design methods and techniques using classic construction, which in turn allows for formation of a spatial structure.

fGeometrical generation is based on finding the unit cell vertex position which meets the following conditions:

- The vertices A and B are located on the sides, while the vertex H is on the axis of the polygonal base,
- The vertices O1, O2 and H lie on the common plane α,
- The vertices B and D lie on the common plane β,
- The vertices C, Q, G and H are on the common plane γ,
- The planes α, β and γ are orthogonal to the plane of the polygonal base of the concave pyramid.

The construction itself relies on the constructive procedure for generating Concave cupolae of the fourth sort [10]. In other words, it relies on the fact that the distance between the neighbouring vertices of the unit cell is always the same and equal to the side of the unit equilateral triangles. To illustrate, let us look at the construction of the position and height of the vertex D. The auxiliary spheres whose centers are located in the neighboring vertices of the spatial hexahedron (vertices O_1 and C) are cut by the plane β containing vertices B and D. Mutual intersection of thus obtained intersecting circles determines the position of vertex D, following the condition that vertex B must be located on the polygonal plane. By repeating the constructive proce–dure above and by determining the position for every unit cell for multiple initial positions of vertex O_1, we generate the trajectory of vertex H. When thus produced vertex H trajectory is intersected by the plane containing the polyhedron's axis, we obtain the sought position of vertex H, and the final position of all the other vertices of the unit cell.

2.2 Variations of the constructive procedure for Concave pyramids of fourth sort

The constructive procedure to generate the vertex H trajectory allows us to choose the position of vertices C, Q and O_2 with a larger or a smaller height, which results in 8 possible variations of the constructive procedure for CP IV. The same holds for the construction of Concave cupolae of the fourth sort, for which reason the same manner of denotation of the variants of the constructive procedure has been adopted:

1. **CP IV-B (CQO_2)**
   smaller height for **C**, **Q** and O_2
2. **CP IV-B ($CQ^*O_2^*$)**
   smaller height for **C**, larger height for **Q** and O_2
3. **CP IV-B (CQ^*O_2)**
   smaller height **C** and O_2, larger height for **Q**
4. **CP IV-B (CQO_2^*)**
   smaller height for **C** and **Q**, larger height for O_2

5. **CP IV-B (C*QO₂)**

   larger height for **C**, smaller height for **Q** and **O₂**

6. **CP IV-B (C*Q*O₂)**

   larger height for **C** and **Q**, smaller height for **O₂**

7. **CP IV-B (C*QO₂*)**

   larger height for **C** and **O₂**, smaller height for **Q**

8. **CP IV-B (C*Q*O₂*)**

   larger height for **C**, **Q** and **O₂**.

The change of the shape of the concave pyramids of the fourth sort depending on the choice of the cons–tructive procedure is in this study illustrated by the case of the concave pyramid above the dodecagonal base.

Concave pyramids of the fourth sort above the dodecagonal polygonal base can be formed by using the following variations of the constructive procedure: CPIV-B(CQ*O₂), CPIV-B(C*QO₂) and CPIV-B(C*QO₂*) (Figure 5-7).
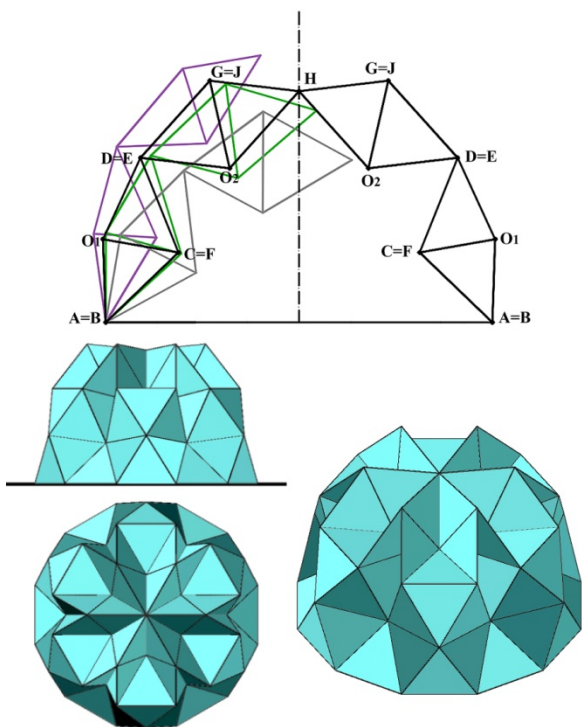

Figure 5. CP IV-12B(CQ*O₂) - vertical section of three posi–tions of a unit cell, orthogonal projections and spatial model

Figure 5 shows the orthogonal projections and spatial model of the Concave cupolae of the fourth sort generated by the constructive procedure which requires the choice of the larger height of the apex Q, and lower height for apexes C and O₂. The orthogonal section of the Concave pyramid of the fourth sort shows three positions of the unit cell for three different initial positions of the apex O₁. The top of the unit cell, the apex H, moves through space generating the spatial curve which intersects with the vertical plane of the orthogonal axes and identifies the position of the apex H as the common apex of all unit cells. In that manner, the polyhedron obtains a common vertex, corresponding to the vertex at the top of classic pyramids. The pyramid above the dodecagonal polygonal base has three vertical planes of symmetry, and three identical sections, one of which is presented in Figure 5. All unit cells of Concave

pyramid of the fourth sort share the common vertex – the apex H, and such formed polyhedral structure encloses the space within the polyhedral surface.
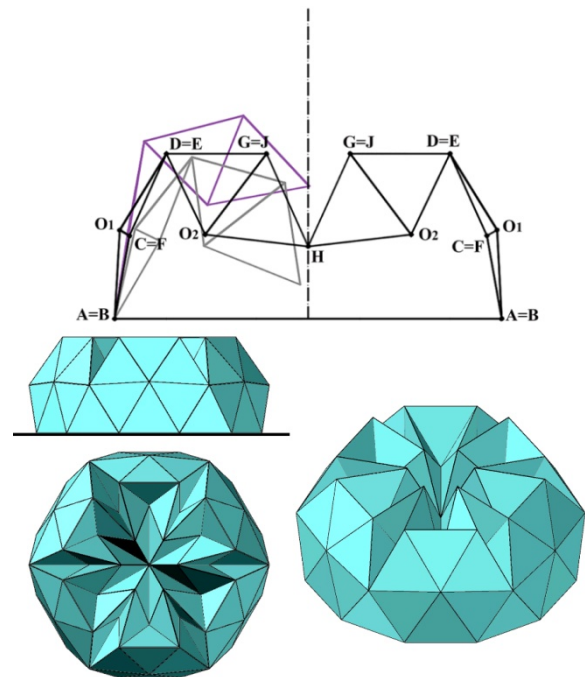

Figure 6. CP IV-12B(C*QO₂) - vertical section of three positions of a unit cell, orthogonal projections and spatial model
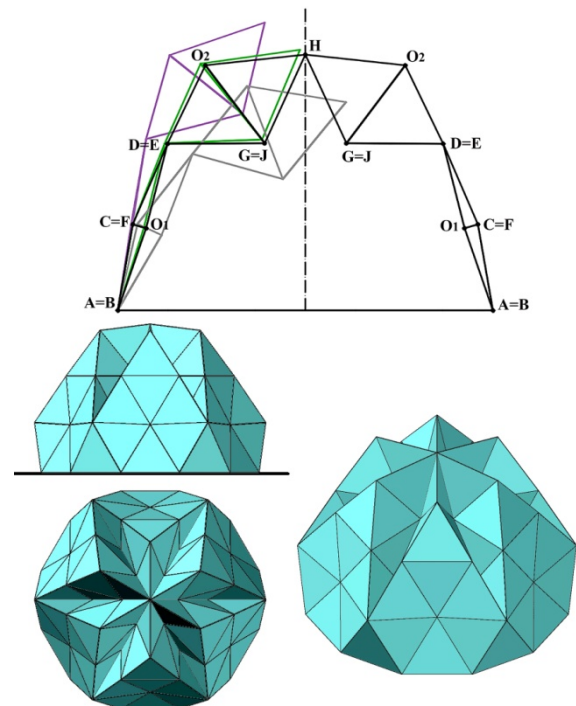

Figure 7. The CP IV-12B(C*QO₂*) - vertical section of three positions of a unit cell, orthogonal projections and spatial model

The exploration of generation of Concave pyramids of the fourth sort has shown that the common vertex H for all the unit cells within the polyhedron can have the height larger than that of any apex of polyhedral (Fig. 7), and the formed polyhedral surface is in shape most similar to that of classic architectural dome. This cha–racteristic determines future architectural and engi–neering use of the Concave pyramids of the fourth sort

generated by constructive procedure applying the larger height of the apex C and Q_2. On the other hand, if we follow the generating procedure opting for the const–ruction that uses the larger height of apex C, and smaller height for apexes Q and O_2, we obtain a polyhedral structure in which the vertex H has the smallest height, as shown in Figure 6.
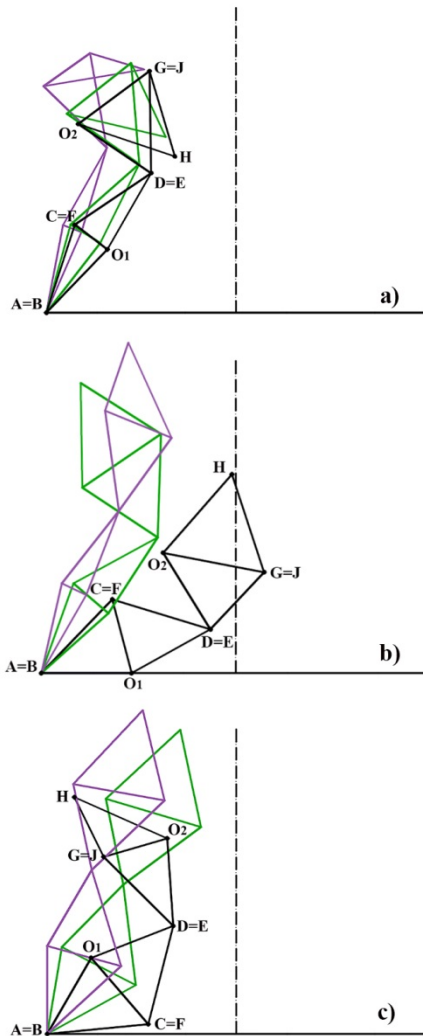

Figure 8. a) CP IV-12B($C^*Q^*O_2$), b) CP IV-12B($C^*Q^*O_2^*$), c) CP IV-12B($CQ^*O_2^*$) - vertical section of three positions of a unit cell

In other cases the vertex H trajectory does not intersect the plane to which the polyhedron's axis belongs or the faces of the polyhedron protrude each other or intersect - constructive procedure variations CP IV-B (CQO_2) and CP IV-B (CQO_2^*), so that it is not possible to generate a concave pyramid which meets the predefined starting conditions.

The vertex H trajectory passes by the plane – con–structive procedure variations CP IV-12B ($C^*Q^*O_2$) and CP IV-12B ($C^*Q^*O_2^*$), or distances itself from the plane of the polyhedron's central axis when the position of vertex O_1 is altered (constructive procedure variations: CP IV-12B ($CQ^*O_2^*$) is shown in Figure 8.

The planar net CPIV-12B is shown in Figure 9. Concave pyramids are obtained by folding the illus–trated net, so that polyhedral generated in such manner can be regarded as folding surfaces. The shape of the net does not depend on the size of the polygonal base,

although the number of unit cells changes, but remains the half of the number of sides of the polygonal base.
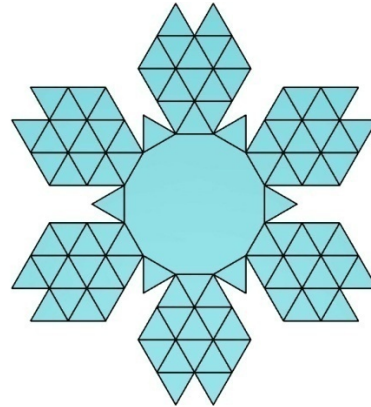

Figure 9. The planar net CP IV-12B

Heights of all vertices of Concave pyramids of the fourth sort above the dodecagonal polygonal base are shown in Table 2.

Table 2. Heights of all vertices of CP IV-12B

$a=100$	CP IV-12B (CQ^*O_2)	CP IV-12B (C^*QO_2)	CP IV-12B ($C^*QO_2^*$)
A=B	0	0	0
O_1	80,6309	85,2150	81,8419
O_2	149,0974	80,9576	243,8864
C=F	67,7033	79,7117	85,8551
Q	139,7877	73,0726	80,5445
D=F	159,2014	159,1148	165,9209
G=J	233,6810	158,2282	166,4273
H	223,4533	69,7217	254,7003

Bearing in mind that the above described generation procedure for concave pyramids of the second and fourth sort, we maintain that for any concave pyramid above an n-sided polygonal base the following applies:
• The number of vertices is calculated by means of the formula:

$$v = \frac{9n}{2} + 1 \tag{2}$$

• The number of edges is calculated by means of the formula:

$$e = \frac{25n}{2} \tag{3}$$

• The number of faces is calculated by means of the formula:

$$f = 8n + 1 \tag{4}$$

The study has shown that above every polygonal base $10 \leq n < 22$ it is possible to generate CP-IV by applying different variations of the constructive pro–cedure. However, this research has not uncovered the rule which defines the possibilities for forming CP IV for the given polygonal base and the given variation of the construction procedure. For that reason, we examined all the possible cases, and the results are shown in Table 3.

CP IV-B	N						
	10	12	14	16	18	20	22
CQO_2				•			
$CQ^*O_2^*$							
CQ^*O_2	•	•	•				
CQO_2^*						•	
C^*QO_2		•					
$C^*Q^*O_2$						•	
$C^*QO_2^*$	•	•	•	•	•		
$C^*Q^*O_2^*$							

3. APPLICATION POSSIBILITIES FOR CONCAVE PYRAMIDS OF THE FOURTH SORT

Many existing constructions have the shape that resembles the geometry of natural crystals, and can be derived from the geometry of regular polyhedral, be–cause they sometimes resemble each other, and have a characteristic distribution of sides and building angles [23]. It is important to explore the new possibilities for generation of polyhedral forms, as well as means for their visualisation and implementation, thus offering a choice of polyhedral structures which can be used in civil engineering. However, in this process, we must bear in mind that the complex spatial structure must be built out of numerous components, which have to be designed, manufactured, packed and transported.

The advantage of spatial structures with the geo–metry of concave pyramids of the fourth sort lies in the fact that they offer solutions for easy pre-fabrication, installation and transport, thanks to their geometrical characteristics, primarily to their symmetry and modu–larity. Concave pyramids consist of unit cells with polar distribution around the axis of the polygonal base (Fi–gure 4), so that the potential pre-fabrication is based on solving a production of a single building cell, and its multiplication depending on the chosen base. The structure can be regarded as a spatial grid – a set of bars of uniform length equal to that of the size of the side of a unilateral triangle which forms the surface net. On the other hand, the spatial structure can be produced by joining the building segments with the geometry of unilateral triangles. The appearance of the unilateral tri–angle segments and their connection is determined by the function of the structure which is being built, and it requires future study. The same stands for the appe–arance of joints in Concave pyramids of the fourth sort when observed as spatial grid.

The presented solution for generation of Concave pyramids of the fourth sort is in line with current trends in researching new tools to help the designer make the final decision regarding the choice of the shape of the spatial structure, especially when this aspect is com–bined with other design aims, such as efficiency, eco–nomy, usefulness and beauty of the design of the structure which covers large areas [24]. The presented study offers a solution based on deltahedral surface, which is a novel perspective on how to cover large areas, as opposed to the existing, frequently treated

solutions which utilize the geometry of geodesic cupolae [25, 26], or use the building blocks with irregular polygonal structure, or with regular polygons of unequal dimensions [27, 28].

The concave pyramids of the fourth sort offer an economic constructive solution to cover both small and large surfaces, without indirect support. The fact that their planar net is a folding surface, which can be easily formed on any terrain, accounts for the potential of the structures we explored in this paper. They can be used as shelters for people and goods in freak weather, military conflict or current migrant crisis. The fact that concave pyramids are generated by polar distribution of the unit cell and that their net is a developmental surface, allows us to regard and apply these polyhedrons as folding constructions [29].

Above the same base, simply by altering the height of the given vertices, it is possible to form Concave pyramids of different shapes. As in [30], it is possible to alter the geometry of the entire system without changing the dimensions of the constructive elements. Concave pyramids can be regarded as structures which can transform from an open compact configuration into a predetermined spatial form, and remain statically stable. They have the properties of structural mechanisms, the properties of both mechanisms and bearing construc–tions. Consequently, once formed network of concave pyramids can be used to form a transformable structure adaptable to the given terrain and function.

We have seen that geodesic cupolas have helped researchers develop new ideas in different fields of science outside architecture and engineering [31]. This is why future studies could focus on how to apply the whole family of concave polyhedrons (Concave pyra–mids, Concave cupolas and Concave antiprisms) in fields such as biology, agriculture, physics, mathematics and design.

The generation of spatial structure of Concave pyramids of the fourth sort was achieved in this paper by means of construction geometry. As in [32], we have shown that construction geometry enables exploration of metric characteristics, as well as generation and visualisation of spatial stuctures, and represents a foundation for their further study in other disciplines.

4. CONCLUSION

By observing predefined conditions for generating the family of concave polyhedral structures which include the previously studied Concave cupolae of the second and higher sort, Concave antiprisms of the second sort and Concave pyramids of the second sort it is possible to constructively generate type B Concave pyramids of the fourth sort. There are 12 polyhedrons of this type whose generation relies on the constructive procedure for forming the Concave cupolae of the fourth sort. It has been proven that for the same polygonal base it is possible to generate more than one CP IV-nB.

Concave pyramids of the fourth sort are a new set of polyhedra belonging to the family of concave poly–hedra. They are characterised by a polygonal base, a deltahedral outer surface, obtained by folding the planar net of unilateral triangle, and by polar distribution of

spatial unit cells with common vertex, the top of the Concave pyramid. The generation of Concave pyramids above the polygonal base with even number of sides, with the range $10 \leq n < 22$, has been examined using the procedures of constructive geometry.

The presentation of thus formed new polyhedrons and the confirmation of accuracy of the selected cons–tructions was performed by means of the software pac–kage AutoCAD 3D models. Future research may focus on the confirmation of the results presented in this paper, as well as on analytical methods, application of appropriate iterative numerical procedures and the exp–loration of the applicative potential of these polyhedrons in architectural geometry.

Note: The results of the examination of generation of Concave pyramids of the fourth sort by means of constructive geometry were presented at the international scientific conference "moNGeometrija 2020", organized by the Serbian Society for Geometry and Graphics and the Faculty of Mechanical Engineering of the University in Belgrade.

REFERENCES

[1] Markov I.J., Gabriel J.F.: Spatial and structural aspects of polyhedra, Engineering Structures, Volume 23, Issue 1, pp 4-11, 2001.

[2] Agrawal K.S., Kumar S., Yim M.: Polyhedral Single Degree-of-freedom Expanding Structures: Design and Prototypes, Journal of Mechanical Design, Volume 124, Issue 3, pp 473-478, 2002.

[3] Zubkov V.V.: Shape Formation of Stereometric Dome Shells, International Journal of Space Structures, Volume 7, Issue 1, pp. 39-60, 1992.

[4] Rizzuto J.P., Saidani M., Chilton J.C.: Polyhedric Space Structures Using Reciprocally Supported Elements of Various Cross-Sections, Journal of the Internacional Association for Shell and Spatial Structures, Vol. 42, No. 3, pp. 149-159, 2001.

[5] Golov G.M.: Techniques of Compositional Geometrical Shape Forming for Crystalline Dome Shells, International Journal of Space Structures, Volume 7, Issue 1, pp. 25-38, 1992.

[6] Skuratov S.V., Shchutsky S.V.: The Polyhedral Wooden Dome Triangular Panel Plywood Covering Static Tests, Materials Science Forum, Vol. 974, pp. 693-697, 2019.

[7] Obradović, M.: Geometric Redesign of the Subdivided Surface of CC II: Application In Architecture, Journal of Industrial Design and Engineering Graphics (JIDEG), Vol. 14, No. 1, pp. 79-84, 2019.

[8] Obradović, M.: Tiling the Lateral Surface of the Concave Cupolae of the Second Sort, Nexus Network Journal, Vol. 21, No. 1, pp. 59-77, 2019.

[9] Obradović, M., Mišić, S.: Concave Regular Faced Cupolae of Second Sort, *Proceedings of 13th ICGG,* Dresden, Germany*, El Book, pp. 1-10. 2008.

[10] Mišić, S.: *Generation of the cupolae with concave polyhedral surfaces (Konstruktivno-geometrijsko generisanje kupola sa konkavnim poliedarskim površima)*, PhD thesis, University of Belgrade, Faculty of Architecture, Belgrade, 2013.

[11] Mišić, S., Obradović, M.: Forming the CupolaeWith Concave Polyhedral Surfaces by Corrugating a Fourfold Strip of Equilateral Triangles, *Proceedings of International Conference for Geometry and Graphics moNGeometrija 2010*, Belgrade, pp. 363-374, 2010

[12] Mišić, S., Obradović, M., Lazović, G., Popkonstantinović B.: Generating a Type of Concave Cupolae of Fourth Sort. *Proceedings ofInternational Conference on Engineering Graphics and Design - ICEGD 2013*, Timişoara, pp. 79-82, 2013.

[13] Mišić, S., Obradović, M., Popkonstantinović, B.: The Structural Transformation of Concave Cupolae of Fourth Sort Using Different Variants of Constructive Procedure, *Proceedings of the 4th International Scientific Conference moNGemetrija 2014*, Vlasina, pp. 147-156, 2014.

[14] Obradović, M., Popkonstantinović B., Mišić S.: On the Properties of the Concave Antiprisms of Second Sort, FME Transactions, Vol. 41, No. 3, pp 256-263, 2013.

[15] Mišić, S., Obradović, M., Đukanović, G.: Composite Concave Cupolae as Geometric and Architectural Forms, Journal for Geometry and Graphics, Vol. 19, No. 1, pp. 67-79, 2015.

[16] Obradović, M., Mišić, S., Popkonstantinović, B., Petrović, M.: Possibilities of Deltahedral Concave Cupola Form Application in Architecture. Proceedings of ICEGD Conference, Iasi, pp. 123-140, 2011.

[17] Obradović, M., Mišić, S., Popkonstantinović, B., Petrović, M., Malešević B., Obradović, R.: Investigation of Concave Cupolae Based Polyhedral Structures and Their Potential Application in Architecture, Technics Technologies Education Management, Vol 8, No. 3, pp. 1198–1214, 2013.

[18] Obradović, M., Mišić, S., Popkonstantinović, B.: Concave Pyramids of Second Sort - the Occurrence, Types, Variations, *Proceedings of the 4th International Scientific Conference moNGemetrija 2014*, Vlasina, pp. 157-168, 2014.

[19] Obradović, M., Mišić S., Popkonstantinović B.: Variations of Concave Pyramids of Second Sort With an Even Number of Base Sides, Journal of Industrial Design and Engineering Graphics (JIDEG), Vol. 10, No. 1, Special Issue, pp. 45-51, 2015.

[20] Obradović, M., Stavrić, M., Wiltsche, A.: Polyhedral forms obtained by combinig lateral sheet of CP II-10 and truncated dodecahedron, FME Transactions, Vol. 45, No. 2, pp. 256-261, 2017.

[21] Emmerich D.G.: Composite polyhedral, Topologie Strucutrale, Vol. 13, pp. 5-32, 1986.

[22] Díaz J.A., Togores R., Otero C.: Geometry Applied to Designing Spatial Structures: Joining Two Worlds. In: Laganá A., Gavrilova M.L., Kumar V., Mun Y., Tan C.J.K., Gervasi O. (eds) Computational Science and Its Applications – ICCSA 2004. ICCSA 2004. Lecture Notes in Computer Science, vol 3045. Springer, Berlin, Heidelberg, pp 158-167, 2004.

[23] Huybers P.: Comuter-aided design of polyhedral building structures, Design Studies, Volume 14, Issue 1, pp 19-39, 1993.

[24] Pilarska D.: Two subdivision methods based on the regular octahedron for single- and duble-layer spherical geodesic domes, International Journal of Space Structures, Volume 35, Issue 4, pp 160-173, 2020.

[25] Haghnazar R., Nooshin H., Golabchi M.: Improving the Regularity of Geodesic Domes Using the Concept of Stepping Projection, International Journal of Space Structures, Volume 29, Issue 2, pp 81-95, 2014.

[26] Kaveh A., Talatahari S.: Geometry and topology optimization of geodesic domes using charged system search, Structural and Multidisciplinary Optimization 43, pp 215-229, 2011.

[27] Jiang C., Tang C., Vaxman A., Wonka P., Potman H.: Polyhedral patterns, ACM Transactions on Graphics, Volume 34, Issue 6, pp 1-12, 2015.

[28] Glymph J., Shelden D.R., Ceccato C., Mussel J., Schober H.: A parametric strategy for free-form glass structures using quadrilateral planar facets, Automation in Construction, 13, pp 187-202, 2002.

[29] Tucyek F.: Folding a dome, Proceedings of IASS Annual Symposia, IASS 2019 Barcelona Symposium: Origami and Folded Systems, pp 1-9, 2019.

[30] Akgün Y., Gantes J.C., Sobek W., Korkmaz K., Kalochairetis K.: A novel adaptive spatial scissor-hinge structural mechanism for convertible roofs, Engineering Structures, Volume 33, pp 1365-1376, 2011.

[31] Tarnai T.: Geodesic Domes: Natural and Man-Made, International Journal of Space Structures, Volume 26, Issue 3, pp 215-227, 2011.

[32] Diaz-Severiano, J.A., Gomez-Jauregui, V., Manchado, C., Otero, C.: Symmetry in Regular Polyhedra Seen as 2D Möbius Transformations: Geodesic and Panel Domes Arising from 2D Diagrams, Symmetry 10, 356, 2018.

КОНСТРУКТИВНО ГЕОМЕТРИЈСКО ГЕНЕРИСАЊЕ КОНКАВНИХ ПИРАМИДА ЧЕТВРТЕ ВРСТЕ

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У раду су презентовани резултати истраживања нове групе полиедара Конкавних пирамида четврте врсте, конструктивни поступци њиховог генерисања и могућности примене. У складу са методом генерисања Конкавних купола четврте врсте, Кон–кавне пирамиде четврте врсте имају сличну логику настанка и њихов пандан у конвексним пирамидама са правилном полигоналном основом. Карактерише их полигонална основа, делтаедарски омотач који настаје пресавијањем раванске мреже једнако–стра–ничних троуглова, поларни распоред јединичних просторних ћелија које све имају заједничко теме – врх Конкавне пирамиде.

Полиедарска површ омотача Конкавних пира–мида четврте врсте настаје поларним распо–редом јединичних ћелија које се састоје од прос–торног шестоугла и просторног петоугла - шест, од–носно пет, једнакостраничних троуглова групи–саних око заједничког темена. У делтаедарској површи омотача две суседне јединичне ћелије спојене су једнакостраничним троуглом у зони полигоналне основе и просторним четвороуглом са којим деле заједничке странице. Поштује се критеријум правилности површи, као и критеријум вишеструке аксијалне симетрије. Расподела једна–костраничних троуглова заснива се на строго утвр–ђеним и дефинисаним параметрима, што омогућава генери–сање структура на начин који их квалификује као аутономну групу полиедара. Врста конкавне пира–миде одређена је бројем редова једнакостра–ничних троуглова у тако добијеној прежи полиедра. Параметри Конкавних пирамида четврте врсте одређени су методама конструктивне геометрије.