Design of a Robust Controller for a GearboxConnected Two-Mass System Based on a Hybrid Model

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Backlash is a common nonlinear phenomenon in mechanical systems, producing undesired behavior such as inaccuracies and oscillations. Instability thereby may limit the speed and position in industrial robots, automotive, and other applications. In this paper, a two-mass system connected by a gearbox is modeled as a hybrid system based on a two modes approach. First, the size of the backlash gap is assumed known; thus, when the motor and load are in negative or positive-contact, the system becomes an equivalent system (rigid body) and can be modeled as one degree of freedom, which is described as a second-order system, this mode is called Contact Mode. Second, when the motor reverses its direction, the system behaves as two separated subsystems so that each subsystem can be modeled as one degree of freedom; this mode is called Backlash Mode. A sliding mode controller (SMC) has been proposed for the above two modes in this work. Hence, two sliding mode controllers are designed, one for the contact mode to achieve tracking position performance, while the other is for the backlash mode to achieve stability. Finally, the two controllers are connected by designing a switching control mode based on the gap conditions and size. The proposed control system is tested considering two different desired references. The simulation result proved the ability and robustness of the designed SMC controllers to force the load position to track the desired reference position and overcome the nonlinearities and drawbacks of SMC, such as chattering.

Keywords: *Sliding Mode Control; Backlash Mode; Contact Mode; Gearbox; Switching Control Mode*

1. INTRODUCTION

In mechanical systems, the backlash is a typical non–linear phenomenon that causes undesirable behavior such as errors and oscillations; in robotics, industrial, automotive, and many other applications, such instabilities can limit the speed and position.

Concerning the modeling of backlash, three different models of backlash were used. The backlash model is a popular nonlinear behavior in electromechanical systems; the system will behave differently if the backlash occurs. So, different mathematical models are considered by relying on conditions of the machine's operating or mechanical conditions or the accuracy level of the model, which are as follows: Physical model for backlash [1], Dead-zone model [2], Modelling with describing functions [3-5]. To achieve good performance for undesired behavior caused by backlash demands, adaptive or non-linear approaches or higher feedback gains could be unrealizable due to stability qualifications. Therefore, many authors have proposed different approaches to overcome backlash problems, as reported in reference [3].

In [6, 7], a fault detection model was proposed to identify simulated undesirable mechanical backlash. The disturbing backlash can be distinguished from the useful one inside the considered electromechanical system through the normal operation of an electromechanical system. Thus, the useful backlash is located by the initial dead zone area without perturbation, while the backlash phenomenon is generated when a dead zone area overtakes its acceptable value with perturbation. Designing high-performance controllers for automotive powertrains requires high-quality measurements of the state and size of the backlash. Hence, nonlinear estimators based on the Kalman filtering theory have been proposed in [8] for backlash size and the powertrain's current state estimation. A linear estimator is described for fast and precise estimation of the position of the wheels and engine. The adaptive controller was applied in [9], depending on different areas of the angular position error for an electromechanical system. This controller also needs information about backlash size; thus, it is estimated by the learning unit in the adaptive controller.

Effective control strategy such as the anti-windup method has been proposed in [8]. The research idea was that the advantage of the inverting concept could be employed when considering a dead-zone or backlash with saturation acting in series as an input to a system. Thus, inserting the right inverse of the dead-zone (or backlash) in series with the system will cancel its effects. However,

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in another paper, a robust feedback controller, such as a linear H_∞ controller, able to stabilize the electromechanical system with the presence of nonlinearity backlash and external disturbances was proposed in [9]. The backlash term was viewed as an external perturbation modeled by the dead-zone function; thereby, perfectly achieving the control design. In [10-12], the problem of the driving part in the DC servomotor system with wide backlash nonlinearity (for which many authors have been considered a narrow backlash angle) is the angular position. To overcome this drawback, an adaptive super twisting controller was proposed. To realize this proposed controller, necessary data about angular velocity must exist, estimated by a sliding model differentiator. This controller adequately compensates model uncertainties and un-modeled dynamics such as backlash nonlinearity, resulting from its robustness properties.

Design nonlinear H_∞ for non-smooth systems with backlash was proposed in [13]. The non-smooth servo-system is considered a non-minimum phase because of the transmission torque passing through the dead-zone area, which presents between the motor and the load. The non-smooth H_∞ synthesis is developed to asymptotically track the motion of the motor while reducing the external disturbances and uncertainties. As a result, load tracking is achieved by a suitable switched reference trajectory of the motor. Unified modeling using a multi-state dynamic model, including the motion state, motor state, the mechanical contact state, and the friction state, was introduced in [14]. A robust controller for precision transmission system with friction and backlash using perturbation separation parameters of the model was designed, where the control method is related to the dynamic modeling, which includes all different aspects such as the mechanical and electrical features of the precision transmission system. A two-stage controller design to solve the tracking problem for a two degree of freedom (2DOF) under-actuated system described by a mass-spring-damper system with non–linear backlash at the un-actuated joint was proposed in [15]. In the first stage, it is considered that all state vectors and perturbations of the system are obtainable. Then, the model is dispensed into two subsystems, actuated and un-actuated. The control input signal of the un-actuated subsystem is designed as an ideal controller to solve the tracking control problem for the un-actuated subsystem, which is the position of the actuated mass. Then, the control input for the actuated subsystem is considered as a reference signal of the un-actuated mass. Active disturbances rejection control (ADRC) is the second stage used to solve the problem of the realization of the previously designed ideal controller.

Another researcher has a different perspective where they proposed a robust control approach depending on the quadratic integral sliding mode surface [16, 17]. The nonlinear harmonic gear system model with nonlinear parts is established. The control design approach is compensated nonlinearly, comprised of time-variant nonlinear torques and parameter variations caused by frictions and backlash, making the system more robust against model uncertainties and nonlinear external disturbances. It was proved that the approach reached in a finite time, and the system was asymptotically stable. In [18-20], a sliding mode controller for the unknown backlash in the electrically driven vehicle was proposed. An observer was proposed to estimate the torque of the driveshaft and load.

load-position control for two mass systems with backlash was accomplished by applying the back-stepping technique [21]. To apply this approach, two measurable signals from the motor and load sides were assumed available. Also, a pre-control system was designed, using the feedback available signals, to realize the desired nonlinear controller goals. On the other hand, for a gear transmission system, which includes an elastic dead-zone, the back-stepping controller cannot be applied straightforward. Therefore, the system was first converted into two cascaded subsystems; then, the adaptive back-stepping control method was employed to achieve tracking control [22]. Moreover, using adaptive back-stepping control with an approximate smooth backlash model and certain well-defined functions was proposed in [23]. The backlash in the electromechanical systems divides the system into two modes: contact and backlash modes. A hybrid system approach was used to treat the control of the mechanical systems with the presence of friction and backlash nonlinearities. It was assumed that the size of the backlash gap is unknown, and the position/velocity of the load side cannot be measured. In this case, the stabilizing controller will only rely on a certain measurement on the motor side; a fixed structure linear state feedback controller was used for stabilization. Nevertheless, in [24, 25], this cont–roller's parameters were designed using the linear-quadratic (LQ) design approach, which was used instead of combining linear and nonlinear backlash and friction compensator.

The occurrence of backlash and flexibility in electric powertrain systems leads to driveline oscillations, which could cause degradation in the vehicle driveability and the mixed brake performance. This problem can be avoided using a mode-switching and active control algorithm with a hierarchical architecture, as it was applied to reduce the effect of backlash and flexibility in [26].

The flexible joint connecting the two masses in this work is the gearbox, which transfers the torque from the actuator side to the load side. The backlash phenomenon is well-known in many mechanical systems, which often occurs in gear systems [3]. In these systems, the backlash is excited by the small gaps between a pair of mating teeth of the gear. So, backlash can be defined as "the play between adjacent movable parts (as in a series of gears)" [27]. As a result of the gap, the two engaged gears are separated, Backlash mode (BM) and Contact mode (CM).

In this paper, two sliding mode controllers will be designed and connected using a switching control method. The first controller is applied for the contact mode to achieve tracking torque. The second controller is for the backlash mode to reduce the impact of motor force on the load when the contact is re-coupled again, achieving the motor's motion asymptotically, thereby avoiding the chattering phenomenon.

The structure of this paper is organized as follows. First, the problem statement is introduced in Section 2.

Then, the considered two-mass with gearbox model is discussed in Section 3. Next, the sliding mode control (SMC) strategy for controlling the load position is designed in Section 4. The numerical simulation results for the two-mass system are given in Section 5, and finally, the paper is wrapped up by the conclusions in Section 6.

2. PROBLEM STATEMENT

The backlash phenomenon or gear play is one of the most important non-linear behaviors in mechanical systems, and it is considered the main source of performance limitations in these systems. It limits their position and speeds control accuracy by causing delays and undesired oscillation, so the stability will also be affected. Additionally, uncontrolled backlash in the system produces impacts between the teeth when the direction of rotation is reversed, leading to wear in the mechanical parts and producing audible noise. Hence, it is obvious that control of the backlash is a complicated task, and it is particularly difficult when tracking angular position or constant angular speed is desired.

3. SYSTEM MODELLING

Generally, hybrid or switched systems happen when different dynamic modes exist, where continuous or discrete dynamics states can describe each mode. Many typical hybrids systems compounded with computer and real-world components, such as robotics and process control systems, are available. Such systems are difficult to describe and analyze [28-32]. The schematic diagram in Figure 1 shows the setup for a typical hybrid rotational system. This system consists of a motor, one-stage reduction gear, and load. The backlash phenomenon induces nonlinearity, which can cause instability in the gearbox [33]. In a typical hybrid system, the perturbations are comprised of motor and load frictions, external disturbance, and model uncertainties. These perturbations must be taken into account in modeling. During the system's operation and due to backlash appearance, the dynamic model of the hybrid system can be described using two separated regimes, namely, the contact mode and the backlash mode. However, these two modes are discussed in the following subsections, where Table 1defines the two-mass system parameters.

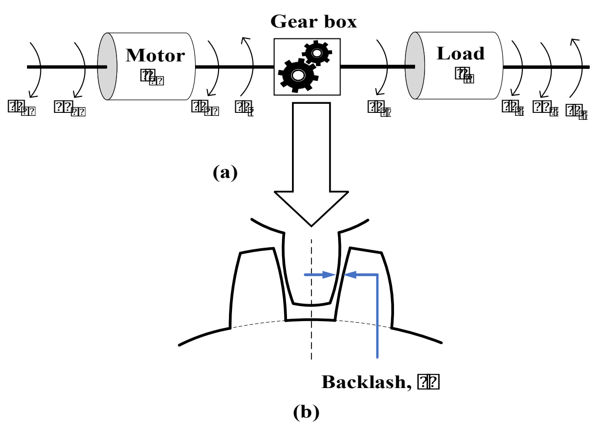

Figure 1. Two mass systems with gearbox

Table 1. The parameters of the two-mass system connected by a gearbox [34]

Parameters	*Description*
B	Backlash gap size
N	Gear ratio
J_m	Motor inertia
J_l	Load inertia
B_m	Coefficient of motor viscosity
B_l	Coefficient of load viscosity
T_m	The applied torque of the motor
T_L	The load torque disturbance

3.1 Contact mode (equivalent system model

By applying Newton's second law, the dynamics of the driving side (the motor side) can be modeled as follows[34]:

$$J_m \frac{d\omega_m}{dt} = T_m - T_i - B_m \omega_m \tag{1}$$

while the dynamics of the driven side is modeled as:

$$J_l \frac{d\omega_l}{dt} = T_g - B_l \omega_l - T_L \tag{2}$$

To write the dynamic model for the two-mass system in the contact model, the following correlations are defined:

$$\begin{aligned} T_g &= J_l \frac{d\omega_l}{dt} + B_l \omega_l + T_L \\ \omega_l &= \frac{1}{N}\omega_m \\ T_g &= N T_i \\ N &= \frac{\dot{\theta}_m}{\dot{\theta}_l} = \frac{T_g}{T_i} \end{aligned} \tag{3}$$

where N is the gear ratio, accordingly, the following is got:

$$J_m \frac{d\omega_m}{dt} = T_m - \frac{J_l}{N^2}\frac{d\omega_m}{dt} - \frac{B_l}{N^2}\omega_m - \frac{1}{N}T_L - B_m \omega_m \tag{4}$$

From Equation 4 above and for simplification purposes, let assume the effect of viscous friction and moments of inertia in the output represented as:

$$\begin{aligned} B &= B_m + \frac{B_l}{N^2} \\ J &= J_m + \frac{J_l}{N^2} \end{aligned} \tag{5}$$

Hence, by substituting Equation 5 into Equation 4, Equation 6 can be obtained:

$$J \frac{d\omega_m}{dt} = T_m - B\omega_m - \frac{1}{N}T_L \tag{6}$$

The system model in the state-space form, as it is the basis for the designed controller in the next section, is obtained after defining $x_1 = \theta_m$ and $x_2 = \dot{\theta} = \omega_m$ by:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{B}{J}x_2 - \frac{1}{JN}T_L + \frac{1}{J}T_m \tag{7}$$

Let $a_c = -\frac{B}{J}$, $d = -\frac{1}{lN}T_L, b = \frac{1}{l}, u_c = T_m$

where the controller of the contact mode is u_c; then Equation 7 can be rewritten as:

$$k_c > \max\left|\delta_c\left(t, x, u\right)\right| \tag{8}$$

The uncertainty in the above equation is given by:

$$a_c = a_{cn} + \Delta a_c$$
$$b = b_n + \Delta b$$

Eventually, the system in the state-space structure is given by:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = a_{cn}x_2 + b_n u_c + \delta_c\left(t, x, u\right) \tag{9}$$

where the perturbation term δ_c (t,x,u), as a result of model uncertainties for the contact mode, is given by Equation 10 below where the subscript c refers to contact mode:

$$\delta_c\left(t, x, u\right) = \Delta a_c x_2 + \Delta b u_c + d \tag{10}$$

3.2 Backlash mode

Concerning the model of the backlash mode, in the real world, when the backlash gap is open, the two masses are decoupled. In this case, the two-mass system can be described as a hybrid system. So, each mass is modeled separately, as in the following:

The dynamics of the first mass (the motor side) is:

$$J_l \frac{d\omega_l}{dt} = B_l \omega_l - T_L \tag{11}$$

The dynamics of the second mass (the load side) is:

$$J_l \frac{d\omega_l}{dt} = B_l \omega_l - T_L \tag{12}$$

Furthermore, for the convenience of the controller design, the following can be redefined:

$$\left[\theta_m, w_m, \theta_m, w_l\right]^T = \left[x_1, x_2, x_3, x_4\right]^T \tag{13}$$

Thus, the state space equations become as:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = a_1 x_2 + b u_B$$
$$\dot{x}_3 = x_4 \tag{14}$$
$$\dot{x}_4 = a_2 x_4 + d$$

where $a_1 = -\frac{B_m}{J_m}, b = \frac{1}{J_m}, a_2 = -\frac{B_l}{J_l}, d = -\frac{1}{h}T_L$ and $u_B = T_m$.

The uncertainty in the above equation consists of

$$a_1 = a_{1n} + \Delta a_1$$
$$b = b_n + \Delta b$$
$$a_2 = a_{2n} + \Delta a_2$$

Therefore, Equation 14 can be rewritten to be as follows:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = a_{1n}x_2 + b_n u_B + \delta_1\left(x, u\right)$$
$$\dot{x}_3 = x_4 \tag{15}$$
$$\dot{x}_4 = a_{2n}x_4 + \delta_2\left(x\right)$$

where $\delta_1(x,u)$ and $\delta_2(x)$ are the perturbation terms as a result of model uncertainties and disturbance in the backlash mode.

$$\delta_1\left(x, u\right) = \Delta a_1 x_2 + \Delta b u_B$$
$$\delta_2\left(x\right) = \Delta a_2 x_4 + d \tag{16}$$

The constructed state-space model in Equation 15 will be used as the basis for the controller design in the next section.

3.3 The condition for the contact mode

The primary condition for the contact mode is:

either $\dot{\theta}_m > 0$ & $\theta_l N - \theta_m = -\beta$

or $\dot{\theta}_m < 0$ & $\theta_l N - \theta_m = \beta$

Otherwise, the system is in backlash mode.

4. CONTROLLER DESIGN

The main parts of the systems that could produce backlash are gears or transmission elements. Therefore, position and speed control, which are important in such systems, is achieved in this section.

In this section, a nonlinear controller is designed to consider the backlash gap when it is open or near to open to avoid impacts and oscillations. Two separate controllers will be designed. The first one is designed for the system in contact mode, while the second is for the backlash mode. The principle concept in switching control is that two different controllers are designed: the contact mode controller and the backlash mode controller. The control task in the contact mode controller is to track the motor side's target position. Its task in the backlash mode controller is to re-establish the contact mode and attenuate or prevent the effect of the impact force of the motor side on the load side. The re-contact between the motor and load side is achieved asymptotically (soft landing), avoiding the chattering phenomenon. This can be realized by making the position deference (θ_m - θ_l) as small as possible. Also, the fast motion with a short time delay is desired because the time delay limits the position tracking performance. Thus, the backlash control objectives are to make the motor position (θ_m) track the reference position (θ_l), which is the load position.

4.1 Sliding mode control design for the contact mode

The contact mode can be seen as a position tracking problem. So, the sliding mode control is proposed because it can accomplish good performance, fast response, and efficiently treat non-linearities, such as discontinuous frictions, model uncertainty, disturbance load, etc. Thus, the control form of the contact mode, according to Equation 9, is:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_{cn} x_2 + b_n u_c + \delta_c(t, x, u) \end{aligned} \tag{17}$$

The first step in designing the sliding mode controller is proposing a sliding variable and its time derivative for the contact mode to be as follows:

$$\begin{aligned} s_{cm} &= \dot{e}_{cm} + \lambda_{cm} e_{cm} \\ \dot{s}_{cm} &= \ddot{e}_{cm} + \lambda_{cm} \dot{e}_{cm} \\ &= a_n x_2 + b_n u_c + \delta_c(t, x, u) - \ddot{x}_{1d} + \lambda_{cm} \dot{e}_{cm} \end{aligned} \tag{18}$$

where $\lambda_m > 0$ is a positive design parameter, while e_{cm} is the error function and $\dot{e}_{cm}, \ddot{e}_{cm}$ are its derivatives, which is defined as:

$$\begin{aligned} e_{cm} &= x_1 - x_{1d} \\ \dot{e}_{cm} &= x_2 - \dot{x}_{1d} \\ \ddot{e}_{cm} &= \dot{x}_2 - \ddot{x}_{1d} \end{aligned} \tag{19}$$

In the error function, x_{1d} is the desired position for the load side and $\dot{x}_{1d}, \ddot{x}_{1d}$ are its derivatives in the contact mode. Note that the subscript *cm* refers to contact mode. To this end, the proposed controller for the contact mode is;

$$u_c = \left(\frac{1}{b_{cn}}\right)(u_{cn} + u_{cs}) \tag{21}$$

where u_{cn} is the nominal control for the contact mode, which is determined as:

$$u_{cn} = -a_{cn} x_2 + \ddot{x}_{1d} - \lambda_m \dot{e}_{cm} \tag{22}$$

while the controller u_{cs} is given by:

$$u_{cs} = -k_c * sign(s_{cm}) \tag{23}$$

The value of k_c, which achieves the attractiveness of the sliding manifold ($s_{cm} = 0$), is determined using the non-smooth candidate Lyapunov function:

$$V_{cm} = \frac{1}{2} s_{cm}{}^2 \tag{24}$$

To ensure the attractiveness of the sliding manifold, k_c is selected such that the derivative of the Lyapunov function V_{cm} is negative definite, as can be shown in the following steps.

$$\begin{aligned} \dot{V}_{cm} &= s_{cm} * \dot{s}_{cm} \\ &= s_{cm} * \left(a_n x_2 + b_n u_c + \delta_c(t, x, u) - \ddot{x}_{1d} + \lambda_m \dot{e}_{cm}\right) \end{aligned} \tag{25}$$

After that, by substituting the proposed controller in $\dot{V}_{cm}$, the following can be obtained:

$$\begin{aligned} &\dot{V}_{cm} = s_{cm} * \left(a_{cn} x_2 + u_{cn} + u_{cs} + \delta_c(t, x, u) - \ddot{x}_{1d} + \lambda_m \dot{e}_{cm}\right) \\ &= s_{cm} * \left(-k_c * sign(s_{cm}) + \delta_c(t, x, u)\right) = \\ &= -|s_{cm}| k_c + s_{cm} \delta_c(t, x, u) \le \\ &\le -|s_{cm}|\left(k_c - |\delta_c(t, x, u)|\right) \end{aligned} \tag{26}$$

Thus, to ensure the stability and reachability of the SMC system, the gain k_c has to satisfy the following inequality.

$$k_c > \max|\delta_c(t, x, u)| \tag{27}$$

4.2 Sliding mode control design for the backlash mode

The controller's task in the backlash mode is to make the motor-load combination returns to the contact mode. The two-mass systems model in the backlash mode is given according to equation (15):

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_{1n} x_2 + b_n u_B + \delta_1(x, u) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= a_{2n} x_4 + \delta_2(x) \end{aligned} \tag{28}$$

The error term is firstly described as:

$$e_{bm} = \theta_m - N * \theta_l \tag{29}$$

where e_{bm} the error functions in the backlash mode, θ_m is the position of the motor, θ_l is the position of the load, and N is the gear reduction ratio. The proposed SMC will try to make this error as small as possible and avoid the impact and chattering effect. Next, the sliding variable and its derivative are given by:

$$\begin{aligned} s_{bm} &= \dot{e}_{bm} + \lambda_{bm} e_{bm} \\ \dot{s}_{bm} &= \ddot{e}_{bm} + \lambda_{bm} \dot{e}_{bm} \end{aligned} \tag{30}$$

where $\lambda_{bm} > 0$, while $e_{bm}, \dot{e}_{bm}$ and $\ddot{e}_{bm}$ are the error function and its derivatives in the backlash mode, which are defined as:

$$\begin{aligned} e_{bm} &= x_1 - N x_3 - \beta * sign(\dot{x}_{1d}) \\ \dot{e}_{bm} &= x_2 - N x_4 \\ \ddot{e}_{bm} &= \dot{x}_2 - N \dot{x}_4 \end{aligned} \tag{31}$$

The subscript *bm* refers to the backlash mode.To this end, the proposed controller for the backlash mode is:

$$u_B = \left(\frac{1}{b_{bn}}\right)(u_{Bn} + u_{BS}) \tag{32}$$

where u_{cn} is the nominal control, which is determined as:

$$u_{Bn} = -a_{1n} x_2 + N a_2 x_4 - \lambda_{bm} e_{bm} \tag{33}$$

While the discontinuous control u_{Bs} is given by:

$$u_{Bs} = -k_b * sign(s_{bm}) \tag{34}$$

As in the previous subsections, the value of k_b, should be selected to ensure the attractiveness of the sliding manifold ($s_{bm} = 0$). This is accomplished using the following chosen non-smooth candidate Lyapunov function:

$$V_{bm} = \frac{1}{2} s_{bm}^2 \tag{35}$$

Using the proposed controller above, the derivative of the Lyapunov function V_{cm} is negative definite, as can be seen in the following steps.

$$\dot{V}_{bm} = s_{bm} * \dot{s}_{bm} =$$
$$= s_{bm} * \left(a_{1n}x_2 + b_n u_B + \delta_1(t,x) - N a_{2n} x_4 - N\delta_2(t,x) + \lambda_{bm}\dot{e}_{bm}\right) \tag{36}$$

After that, substituting the proposed controller in V_{bm}, the following can be obtained

$$\begin{aligned}\dot{V}_{bm} &= s_{bm} * (a_{1n}x_2 + u_{Bn} + u_{Bs} + \delta_1(tx) - N a_{2n} x_4 \\ &\quad - N\delta_2(t,x) + \lambda_{hm}\dot{e}_{hm}) \\ &= s_{bm} * \left(-k_b * sign(s_{bm}) + \delta_1(t,x) - N\delta_2(t,x)\right) \\ &\le -|s_{bm}| k_b + |s_{bm}||\delta_1(t,x)| + N|s_{bm}||\delta_2(t,x)|\end{aligned} \tag{37}$$

Therefore, to ensure the stability and reachability of the SMC system, the gain k_b has to satisfy the following inequality.

$$k_b > \max\left\{|\delta_1(t,x)| + N|\delta_2(t,x)|\right\} \tag{38}$$

The discontinuous control terms $sign(s_{cm})$ and $sign(s_{bm})$ in Equation 23 and Equation 34 ensure the desired performance; they produce the chattering phenomenon. However, several approaches have been developed to address the chattering problem (see references [35-43]). The most straightforward approach is used in this work to reduce chattering where the discontinuous terms $sign(s_{cm})$ and $sign(s_{bm})$ are replaced by[44]:

$$\begin{aligned} &sign(s_{cm})\frac{2}{\pi}\tan{-1}(\gamma_{cm}s_{cm}), \gamma_{cm} > 1 \\ &sign(s_{bm})\frac{2}{\pi}\tan{-1}(\gamma_{bm}s_{bm}), \gamma_{bm} > 1 \end{aligned} \tag{39}$$

5. SIMULATION RESULTS AND DISCUSSIONS

The proposed control system in Section 4, which is based on the sliding mode control, will be simulated in this section. The simulations are performed using the numerical computer solver of MATLAB (R2016b) software, with the initial condition θ_l, w_l, θ_m, w_m = 0, 0, 0, 0, respectively. The parameters used in the simulations are shown in Table 1, while the parameters of the controllers, which are needed in Equations 18, 30, and 39, are given in Table 2.

Additionally, the source of uncertainty in this work is due to the uncertainty in the system parameters. Therefore, the uncertain terms are those coefficients in the system model, which contains J_m, J_l, B_m, and B_l. In addition, the external torque that was considered here is $T_L = 0.00035 * \sin\left(\frac{2\pi}{50} * t\right)$. Table 3 presents the nominal system model coefficients, the maximum bound on their uncertainties, and the maximum bound on the external torque. Also, the gains k_c, k_b in Equation 23 and Equation 34, which were used in the numerical simulation in this work, are given by:

$$\begin{aligned} k_c &= 0.0403 * |x_2| + 0.00412 \\ k_b &= 0.05 * |x_2| + N * \left(0.02121 * |x_4| + 0.00303\right) \end{aligned}$$

Table 2. Nominal parameters for the two-mass system with a gearbox[45]

Parameter	Description	Nominal Values	Unit
β	Backlashgap	0.1	Rad
N	Gear ratio	4	-
J_m	Motor inertia	0.02	Kg.m^2
J_l	Load inertia	0.165	Kg. m^2
B_m	Coefficient of motor viscosity	0.002	Nm/rad/s
B_l	Coefficient of load viscosity	0.007	Nm/rad/s

Table 3. Control parameters

Controller parameters	Value
$\lambda_{cm} = \lambda_{bm}$	25
$\gamma_{cm} = \gamma_{bm}$	100

Table 4. Nominal and uncertainty parameters of the two-mass system

Contact mode	Nominal	Uncertainty
a_c	0.0804123	$\lvert\Delta a_c\rvert<0.0403$
b_c	32.9896	$\lvert\Delta b_c\rvert<16.5$
d	-	$\lvert d\rvert<0.00412$
Backlash mode	Nominal	Uncertainty
a_1	0.1	$\lvert\Delta a_1\rvert<0.05$
a_2	0.04242	$\lvert\Delta a_2\rvert<0.02121$
b_b	50	$\lvert\Delta b_b\rvert<25$
d	-	$\lvert d\rvert<0.00303$

Thus, the simulation in this section will examine the proposed controllers based on testing two different desired references. Each test will show the ability and robustness of the designed SMC to force the load position to track the desired reference position and overcome the nonlinearities and drawbacks of SMC, such as chattering.

5.1 Test No. 1

The reference angle is sine wave:

$$x_{3d} = 2\sin\left(\frac{2\pi}{50}t\right)$$

Let β is the backlash's gap size; this leads to the desired angular position for the motor angle x_1 to be in the following form:

$$x_{1d} = N * x_{3d} + \beta * sign(\dot{x}_{1d}), \quad \forall \dot{x}_{1d} \neq 0$$

Additionally, it is assumed that the motor and load's initial angular position and speed load are taken as 0, which means that a pair of mating gears is as shown in Figure 1, where the size of β between the teeth on the two sides is 0.1 *rad*.

Figure 2 shows the simulation result of the accurate tracking of the load angular position to its reference x_{3d}, which represents the goal of the proposed control system. This figure also shows that the system is started in the backlash mode (BM) with a gap β = 0.1 *rad*; however, the control torque, which actuates the motor side, enforces the motor angular position to follow the desired reference x_{1d}, as shown in Figure 3. Figure 3 also shows that the BM controller makes the motor's angular position follows the load side angular position and return the gears to the contact mode (CM). Despite the switching between the CM and BM controllers, it can be noted from Figure 4 that the load side is approximately unaltered, where it needs less than 0.6 seconds to re-follow x_{3d}. Also, an accurate following to its reference can be easily seen, where the turning to the CM is clarified in the zoomed view in Figures 3 and 4. In addition, the angular velocity of the load side is presented in Figure 5, where it tracks the angular velocity reference $\dot{x}_{3d}$, and again the BM can be easily seen and distinguished from the CM. The sliding variable, the main output that the control system works to regulate it to zero levels (sliding manifold), is shown for the BM and the CM modes in Figure 6.

The switching between the BM and CM controller was made according to the contact condition presented in subsection 2.3. The switching between the BM and CM was reported for the proposed controller in Figure 7. Finally, the control action $T_m(t)$ is presented in Figure 8 for the two mass-system connected by a gearbox using the proposed sliding mode controller, where it can be shown that the maximum motor torque is not exceeding 0.4 $N{\cdot}m$. Moreover, the control efforts are clarified in the zoomed region during the transition from CM to BM and then returning to the CM. The motor torque during the BM enforces the sliding variable s_{bm} to regulate to the zero levels asymptotically, and then after re-contact in the CM, it enforces the sliding variable s_{cm} to regulate to zero. As a result, the control system is again able to make the load angle position tracks the desired reference x_{3d}. Many observations can be deduced from Figure 4 and Figure 7. When the system is in the contact mode, and the motor reverses its direction due to periodic reference motion, the system traverses 2β (0.2 rad) in the BM to return to CM. Another note from Figure 4in the zoomed view is that during the time interval between 12.878 sec to 12.88 sec it takes 0.242 sec to re-contact the gears with an additional 0.002 sec required to asymptotically approach the load side from the motor side. This step was suggested in the present work to prevent the impact load on the gear's load side, caused by the motor gear side during the BM and just before the re-contact.

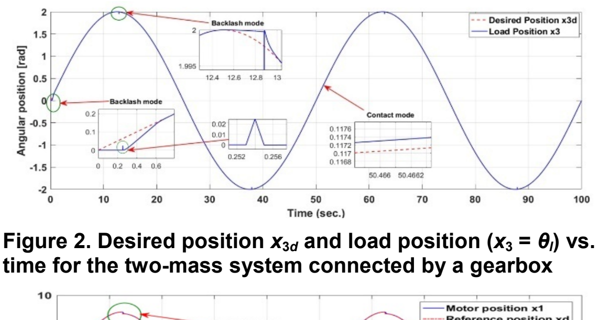

Figure 2. Desired position x_{3d} and load position (x_3 = θ_l) vs. time for the two-mass system connected by a gearbox

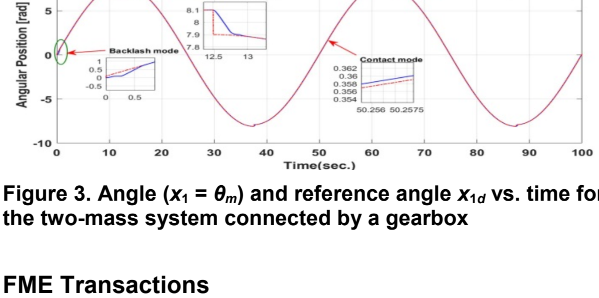

Figure 3. Angle (x_1 = θ_m) and reference angle x_{1d} vs. time for the two-mass system connected by a gearbox

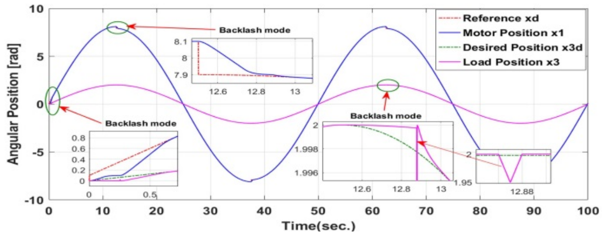

Figure 4. Reference position x_d, motor position (x_1 = θ_m) desired position (x_{3d}), and load position ((x_3 = θ_l) vs. time for the two-mass system connected by a gearbox

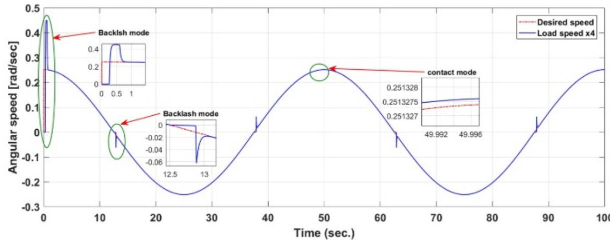

Figure 5. Desired speed $\dot{x}_{3d}$ and load speed ($x_4 = \dot{\theta}_l$) vs. time for the two-mass system connected by a gearbox

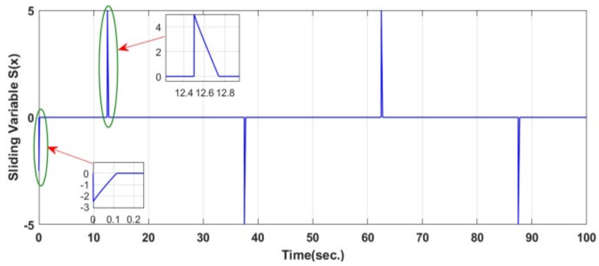

Figure 6. Sliding variables vs. time for both modes

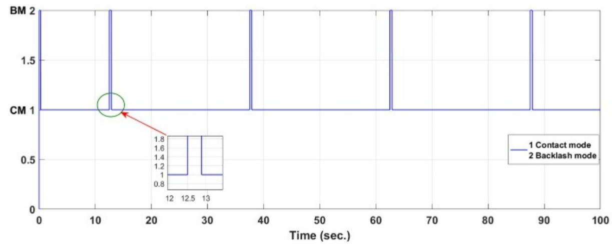

Figure 7. Backlash and contact modes of the system behavior

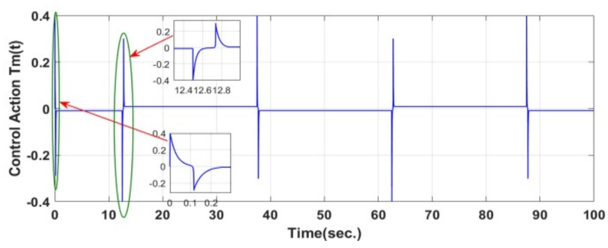

Figure 8. Control action $T_m(t)$ vs. time for the two-mass systems connected by a gearbox

5.3 Test No. 2

Reference angle for the load angular position is a piecewise constant.

$$
\begin{aligned}
x_{3d} &= 1\ rad, \text{ for } 0 \le t \le 15 \\
&= -2\ rad, \text{ for } 15 < t \le 35 \\
&= 0.5\ rad, \text{ for } 35 < t \le 55 \\
&= -1.5\ rad, \text{ for } 55 < t \le 75 \\
&= -0.75\ rad, \text{ for } 75 < t\ \theta \le 100
\end{aligned}
$$

While the motor angle position reference is given by

$$x_{1d} = N * x_{3d} + \beta * sign\left(x_d\right)$$

The ability and the effectiveness of the proposed controller to achieve tracking the desired position (x_{3d}) is again tested for a piecewise constant reference that is given above. The simulation results are presented in Figures 9, 10, and 11 to show the ability of the proposed SMC for the two modes to lead the motor and load positions (x_1, x_3) to track the desired positions. For the first backlash gap, which equals to β, the proposed control system in the BM takes less than 0.042 sec for the transition to the CM, while it takes 0.086 sec for the other backlash's gap, which equals 2β, as shown in Figure 10. The sliding variables s_{bm} and s_{cm} for the two modes, the plot of the BM and CM intervals, and the control action $T_m(t)$ are plotted with time in Figures 12, 13, and 14, respectively.

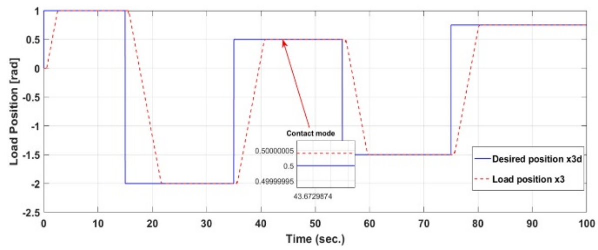

Figure 9. Desired position x_{3d} and load position ($x_3 = \theta_l$) vs. time for the two-mass system connected by a gearbox

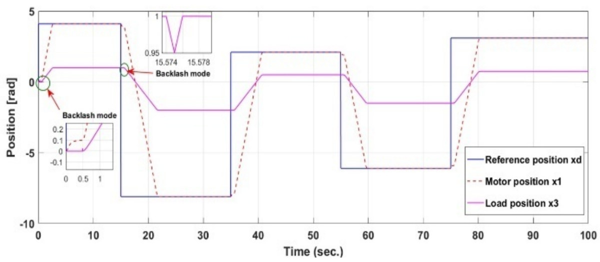

Figure 10. Reference position x_d, motor position ($x_1 = \theta_m$), and load position ($x_3 = \theta_l$) vs. time for the two-mass system connected by a gearbox

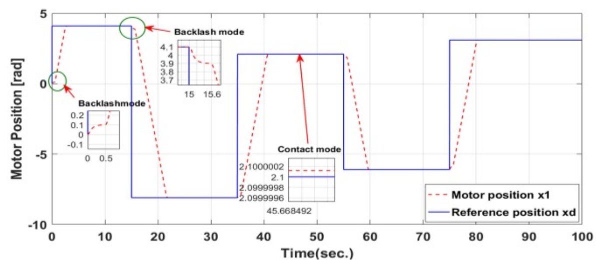

Figure 11. Angle ($x_1 = \theta_m$) and reference angle x_d vs. time for the two-mass system connected by a gearbox

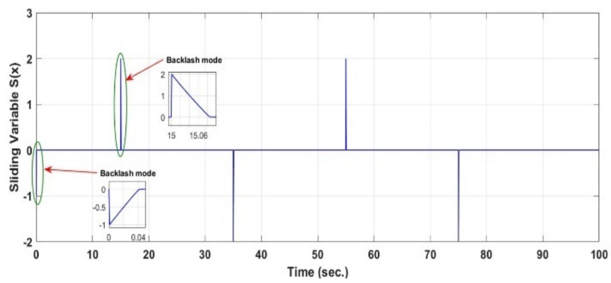

Figure 12. The sliding variables vs. time for both modes

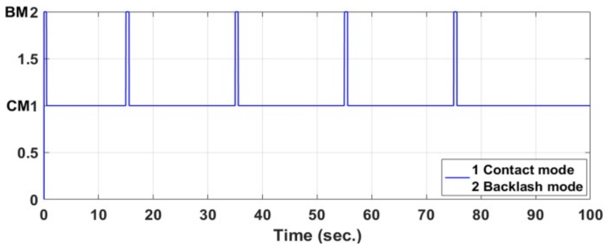

Figure 13. Backlash and contact modes of the system behavior

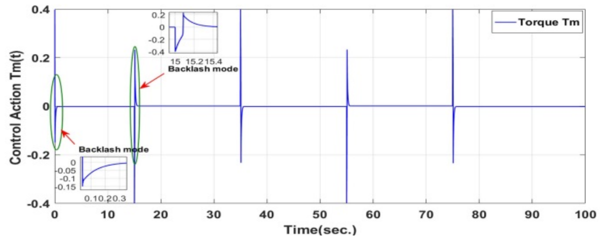

Figure 14. Control action $T_m(t)$ vs. time for the two-mass systems connected by a gearbox

6. CONCLUSIONS

In this work, a hybrid control system was designed for a two-mass system connected by a gearbox consisting of contact and backlash modes. The proposed control method in this case study is based on designing two controllers; the first controller was designed for the contact mode, while the second controller was proposed for the backlash mode. Then, a switching condition was used for the overall controller to switch between the two designed controllers. The hybrid control system was implemented considering two different desired references: sinusoidal signal and piecewise signal, and in the presence of external load torques. The simulation results showed the robustness and ability of the proposed controllers to achieve the tracking in the contact mode and realize the stability in the backlash mode with a smooth transition from the BM to the CM.

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NOMENCLATURE

B_m	Coefficient of Motor Viscosity
B_l	Coefficient of Load Viscosity
e_{cm}	The error of the Contact Mode
e_{bm}	The error of the Backlash mode
J_m	Motor inertia
J_l	Load inertia
k_c	The positive gain for Contact Mode
k_b	The positive gain for backlash mode
N	Gear ratio
SMC	Sliding Mode Control
S_{cm}	Sliding mode variable for the contact mode
S_{bm}	Sliding mode variable for the backlash mode
T_L	Load-torque perturbations
T_m	The applied Torque of the Motor
u_c	Controller of the contact mode
u_{cn}	Nominal term of the controller for contact mode
u_{cs}	The sliding mode controller for the contact mode
u_m	Controller of the backlash mode
u_{mn}	Nominal term of the controller for the backlash mode
u_{ms}	The sliding mode controller for the backlash mode
V_{cm}	Lyapunov function for the contact mode
V_{bm}	Lyapunov function for the backlash mode
x_{1d}	Desired input
$\dot{x}_{1d}$	The first derivative of the desired input
$\ddot{x}_{1d}$	The second derivative of the desired input
ω_l	Angular velocity of load
θ_m	The angular position of the motor
θ_l	The angular position of the load

Greek symbols

β	Backlash gap size
δ_c	Perturbation term for the Contact Mode
δ_1, δ_2	Perturbation terms for the Backlash Mode
λ_{cm}	Positive design parameter for contact mode
λ_{bm}	Positive design parameter for backlash mode

Superscripts

cm	contact mode
bm	Backlash mode

ДИЗАЈН РОБУСТНОГ КОНТРОЛЕРА ЗА ДВОМАСЕНИ СИСТЕМ ПОВЕЗАН МЕЊАЧЕМ НА ОСНОВУ ХИБРИДНОГ МОДЕЛА

А.М. Мохамед, Ш.А. Ал-Самараје, А.А. Џабер

Одбитак је уобичајена нелинеарна појава у меха–ничким системима, која доводи до нежељеног понашања као што су непрецизности и осцилације. Нестабилност на тај начин може ограничити брзину и положај у индустријским роботима, аутомобилској и другим апликацијама. У овом раду, систем са две масе повезан мењачем је моделован као хибридни систем заснован на приступу два режима. Прво, претпоставља се да је позната величина зазора; тако, када су мотор и оптерећење у негативном или позитивном контакту, систем постаје еквивалентан систем (круто тело) и може се моделовати као један степен слободе, који се описује као систем другог

реда, овај режим се назива контакт Режим. Друго, када мотор обрне свој правац, систем се понаша као два одвојена подсистема тако да се сваки подсистем може моделовати као један степен слободе; овај режим се зове Повратни Режим. За ова два режима у овом раду предложен је контролер клизног режима (СМЦ). Стога су дизајнирана два клизног режима контролера, један за контактни режим да би се постигао перформансе положаја праћења, док је други за режим зазора ради постизања стабилности. Коначно, два контролера су повезана дизајнирањем режима контроле преклапања на основу услова и величине празнина. Предложени систем контроле се тестира узимајући у обзир две различите жељене референце. Резултат симулације је доказао спо–собност и робусност дизајнираних СМЦ контролера да приморају положај оптерећења да прати жељену референтну позицију и превазиђу нелинеарности и недостатке СМЦ-а, као што је трепетање.