Analysis of the Performance and Kinematics of the Movement of UAV

Marina Miloš
Teaching Assistant
University of Belgrade
Faculty of Transport and Traffic Engineering

Petar Mirosavljević
Full Professor
University of Belgrade
Faculty of Transport and Traffic Engineering

The use of drones today has become an integral part of modern life to the extent that the level of drone utilization determines the quality of life.The expansion of drone applications has grown exponentially. This growth is primarily due to their low acquisition and maintenance costs, as well as their versatility. First of all, their expansion is contributed by the low costs of both acquisition and maintenance, as well as the possibility of various applications. The paper aims to present drones and their advantages compared to traditional aircraft as a means of transportation. The paper includes an analysis of unmanned aircraft performance, an examination of the kinematics of unmanned aircraft movement, a discussion of the most common structure used in unmanned aircraft (the quadcopter), and a comprehensive assessment of the risks associated with unmanned aircraft and their potential integration into civil air traffic. The quadcopter structure, as the dominant solution for the production of unmanned aircraft, is discussed in detail. At the very end, an overall analysis of the risks posed by unmanned aircraft and, as such, whether they can be integrated into civil air traffic is given.

Keywords: *UAV, drone, quadcopter, performance, kinematics, ground effect.*

1. INTRODUCTION

The paper's topic is the analysis of the performance and kinematics of small unmanned aircraft. It is estimated that there will be approximately 82.1 million units of unmanned aerial vehicles in operation by 2025[1]. Taking into account the estimated growth of transport air traffic by 3% on an annual basis, it can be concluded that airspace will change drastically in the next decade. With the increase in the number of unmanned aircraft, the problem of safe integration of unmanned aircraft operations into the air traffic burial system arises.

The first part of the paper examines unmanned aircraft's development and structural characteristics, providing essential background information for analyzing their performance and kinematics. In order to adequately analyze the performance, it is necessary to become familiar with the construction of unmanned aircraft and the kinematics of their movement. The central part of the paper delves into the performance of unmanned aircraft, with a particular emphasis on quadcopters, which are the dominant construction method today. Depending on flight phases, thrust ratio, rotation speed, and propeller coordination are shown. The paper also outlines the differences between classic unmanned aircraft with fixed and rotating wings and hybrid aircraft, which are currently less common. Section of the paper explores dynamic models to understand the variations in the dynamics of different types of unmanned aircraft. Finally, the paper concludes with a safety risk assessment of integrating unmanned aircraft into global air traffic.

The paper presenting a study on the performance of UAVs compared to previous studies offers several distinct advantages. Firstly, it contributes to the body of knowledge by providing an updated and comprehensive analysis of UAV performance, which can be crucial for both researchers and industry professionals seeking the latest insights. Secondly, it allows for a comparative assessment, enabling a clear understanding of advancements and trends in UAV technology and capabilities over time. Thirdly, such a paper can aid in identifying gaps or areas of improvement in previous studies, directing future research efforts more effectively. Additionally, it provides a valuable reference for policy-makers and stakeholders in fields such as agriculture, surveillance, and environmental monitoring, supporting informed decision-making. Lastly, it fosters collaboration and knowledge sharing within the research community, promoting a collective effort to enhance UAV technology and applications.

2. STRUCTURES OF UNMANNED AIRCRAFT

Like the aircraft themselves, unmanned aircraft have developed and advanced over time. The constructions of the first aircraft were made of wood, transitioning to metals, and today, composite materials are the main choice for a significant number of components. This work focuses on small unmanned aircraft, and it will primarily present their dominant design solutions.

The main division of unmanned aerial vehicles by construction is related to the method of generating lift force. Based on this criterion, unmanned aircraft can be categorized into:

1. Fixed-wing unmanned aircraft,

Received: September 2023, Accepted: October 2023
Correspondence to: Milos Marina
Faculty of Transport and Traffic Engineering,
Vojvode Stepe 305, 11000 Belgrade, Serbia
E-mail: m.marina@sf.bg.ac.rs
doi: 10.5937/fme2304627M

2. Unmanned aircraft with a rotary wing,

3. Unmanned aerial vehicles with a hybrid way of generating buoyancy force – hybrid.[15]

2.1 Fixed wing

The fixed-wing concept generates lift force by utilizing the flow of air over a fixed wing. To generate lift force, a specific airspeed is required. For the take-off, fixed-wing unmanned aircraft typically use launch pads or organized runways. The primary advantages of this species are its long-distance capabilities and durability. Figure 1 illustrates this concept of constructing unmanned aircraft.

Figure 1. FeiyuTech fixed-wing Unicorn UAV [2]

2.2 Rotary wing

Movable-wing aircraft have the ability to climb and land vertically, so their space for that part of the operation is significantly smaller than that of fixed-wing aircraft. The basic division of rotary-wing aircraft is as follows:

1. Helicopters
2. Multicopters

The continuation of the work is dedicated to multicopters with four wings that create lift, i.e., quadcopters. The main reason for their analysis is the very specificity of the concept, as well as their dominance in the market.

2.3 Quadcopters

Most drones feature a quadcopter structure, with four motors arranged in the same plane to form the vertices of a square (Figure 2). In this way, the buoyancy force is realized by means of four vertically oriented propeller electric motors, which are powered by a large spectrum of batteries. [3]

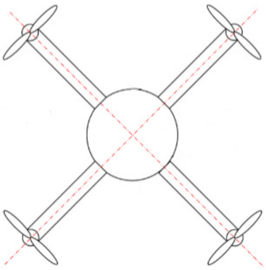

Figure 2. Conceptual design solution of a quadcopter

This concept of aircraft construction dates back to the 20s of the last century. This concept couldn't be developed in the past due to inefficient coordinated engine control.Advances in technology, particularly sensors and microcontrollers, have enabled more efficient use of quadcopters.

When designing the construction of the quadcopter, it is necessary to take care that the plane of the rotor is above the plane where the center of gravity is located. If the center of gravity were in the plane of the rotor, any maneuver around the horizontal plane (Pitch) could cause the quadcopter to roll over. Due to the overturning of the quadcopter, there is a loss of buoyancy force as well as its inevitable fall. So, for the drone's maximum mobility, it is necessary for the center of gravity to be as low as possible because when the drone tilts, the reference plane of the propeller "drops". Therefore, it is not surprising that the additional load, i.e., the equipment, is placed under the very base of the drone.

This type of construction consumes far less energy just to start it, which leaves more possibilities for carrying loads.

It's essential that the vertical axis of all engines must lie in the same circle with the center located on the central axis of the center of gravity of the structure, while the way of connection affects the very aerodynamic properties. Quadcopters that do not meet this condition are highly unstable and challenging to control.

Quadcopters can be constructed in the following two ways:

- QUAD + : a configuration in which the motors are connected to the hub of the quadcopter so that they form a + sign in relation to the main direction of movement.
- QUAD x : a configuration in which the motors are connected to the hub of the quadcopter so that they form an x sign in relation to the main direction of movement.

Unmanned aerial vehicles constructed in this manner are typically small in size. In foreign literature, they can be found under the name MAV - Micro Unmanned Aerial Vehicle. One of the most commonly used drones of this type is the DJI Inspire 2 (Figure 3).

Figure 3. DJI Inspire 2 [10]

2.4 Hybrid UAV

Unmanned aerial vehicles can achieve greater specific capabilities by combining fixed and rotary wing concepts. Aircraft that utilize both of these technologies are referred to as hybrid aircraft. The advantage of the hybrid concept lies in its ability to incorporate the advantageous features of both basic concepts. For instance, hybrid drones offer an extended range and the capability to swiftly transition between flight mode and landing, as well as hovering. Over the course of a century of unmanned aircraft development, a satisfactory level of flight behavior in response to given commands has been achieved. However, the hybrid concept is relatively recent, and the comprehensive control of both rotary and fixed-wing aircraft is still in the developmental stage [11].

Hybrid aircraft can be categorized into two major groups:

- Fully convertible drones,
- Partially convertible unmanned aircraft.

The primary characteristic of convertible aircraft is their capacity to alter a portion of the structural design. These aircraft must exhibit seamless coordination across all structural modes. They are typically categorized based on their construction modifications, which include the following types:

- Tilt-rotor,
- Tilt-wing,
- Dual-system,
- Rotor-wing.

As a portion of the work focuses on quadcopters, a more detailed division is presented below. Tilt-rotor UAVs have the capability to tilt some or all of their engines to redirect thrust, allowing them to generate both vertical and horizontal thrust. One of the most successful models of this type is the unmanned aircraft Bell Eagle Eye (Figure 4a), developed in 1993 by BHTI. This aircraft is historically significant as the first unmanned aircraft with a tilting rotor. The model is based on a compact fuselage with a wing featuring a rotating mechanism and two engines. Another example of this type is Project Zero (Figure 4b), which was initially touted as a competitor to conventional aircraft in terms of performance and potential applications. Unfortunately, Project Zero did not make it to serial production.

Figure 4. Models of tilt-rotor unmanned aircraft [11]

Tilt-wing and tilt-rotor aircraft are quite similar, with the main difference being that tilt-wing aircraft not only tilt the engines but also the wings. This type of aircraft has been under development since 1957, and one example is the Unmanned Quad-Tilt Rotor (Figure 5a). This model is constructed as a quadcopter with wings aligned in relation to the engine's thrust direction. It's known for its rapid transition from vertical to horizontal movement and rotation. Another notable representative of this type is the GL-10 (Figure 5b), developed by NASA. This model features ten engines, with eight located on the wings and the remaining two in the tail section. Both the wings and tail surfaces have the ability to tilt and change thrust direction. In hover mode, the wings are positioned vertically, making them sensitive to side gusts of wind. However, modern systems have made it possible to stabilize these types of aircraft even in strong gusts of wind.

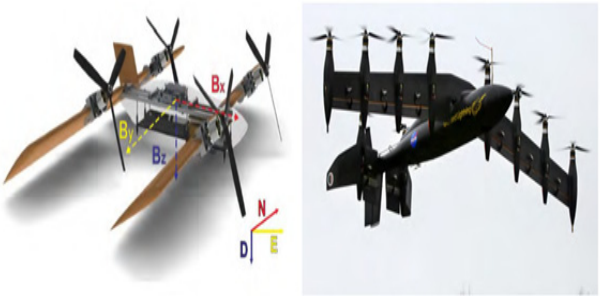

Figure 5. Models of tilt-wing unmanned aircraft [11]

The cost of building a tilt-rotor UAV is significantly higher than that of constructing fixed-wing and rotary-wing UAVs. The complexity involved in designing and maintaining tilt-rotor UAV systems is more intricate and necessitates substantial financial investments. However, these very complexities and higher costs set them apart from other types.

3. PERFORMANCE OF UNMANNED AIRCRAFT

As already discussed, the quadcopter structure is dominant for unmanned aerial vehicles. In the continuation of the work, the focus will be shifted to the quadcopter structure and to the calculation of the performances of such unmanned aircraft.

In order for the drone to be able to fly, it is necessary to generate thrust. By the flow of air through the rotor blades, a certain amount of air is pushed down through the plane of the rotor. This flow can be described analytically through mass flow equations, continuity equations, and conservation of energy. This approach can be found in the literature under the name of Momentum theory.

3.1 Fundamentals of quadcopter movement in space

In the case of quadcopters, the drive motors, at the ends of which there are ailerons, are fixed and vertically oriented. The propellers are designed in such a way that they do not change the pitch of the propeller arm. This is generally the case with most drones, with the exception of some new models that are still in the design and testing stages. Each propeller has the ability to change the rotation speed by reducing or increasing the number of revolutions of a single engine, independently of the others. Sensors and microcontrollers are responsible for their coordination and organization. This kind of organization of propellers enables the drone to perform the entire spectrum of movement, i.e., it gives the aircraft the ability to move with six degrees of freedom. In addition to this security, it is very easy and quick to change the desired flight course.[19]

Before moving on to the actual movement of drones, it should be noted that they do not have the problem of torque and that a certain additional motor does not have to be engaged in order to respond to the reaction force that arises as a result of the rotation of the main rotors. Quadcopters are designed so that each of the four rotors rotates in opposite directions relative to the two adjacent ones. It can be said that rotors located opposite each other move in the same direction. The display of the direction of rotation of the rotor is shown in Figure 6. [4]

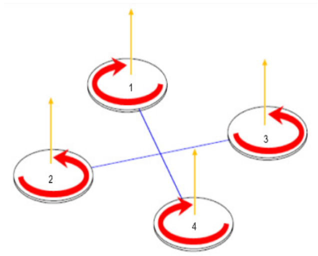

Figure 6. The direction of rotation of the quadcopter rotor

From Figure 6, it can be seen that the rotors numbered 1 and 4 rotate in the same direction, and then the construction moment occurs in the other direction, but since rotors 2 and 3 rotate in the same direction, which is opposite to the direction of rotation of rotors 1 and 4, there is a construction moment that is out by the construction momentum caused by the action of rotors 1 and 4.

Hovering is a drone flight mode in which a constant position is maintained above a certain point, usually at some lower height above the ground. It can be said that hovering is the primary mode of flight and is largely a prerequisite for take-off and landing. In this flight mode, the driving force of buoyancy is equal to the weight of the drone (Figure 7) [5].

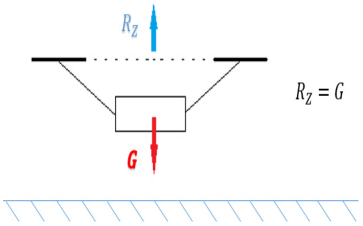

Figure 7. The direction of rotation of the quadcopter rotor

The lift driving force is created as a sum of the aerodynamic lift forces of four individual rotors. If the relation $R_Z = G$ were to be violated, it would lead to a vertical rise, that is, descent. Vertical ascent would occur with the condition that $R_Z > G$, that is, descent with the condition that $R_Z < G$.

As already mentioned, drones do not have rotors with variable pitch propellers; therefore, their lateral and rotational movement is solely responsible for the number of revolutions of the drive rotors. To ensure lateral movement, it is necessary that the three rotors have the same speed, thus creating equal direction, direction, and intensity of aerodynamic forces, while the fourth rotor needs to have a different number of revolutions compared to the others. In this way, a potential difference of forces, i.e., a moment, is created, which tilts the drone in a certain direction and ensures its lateral movement. The lateral movement of the drone is shown in Figure 8, where a) shows the lateral movement and turning moment around the x-axis - Roll, and under b) shows the lateral movement and turning around the y-axis - Pitch.

As for the forces, they are divided into two components, so that in the vertical plane, we get the buoyant force, and in the horizontal plane, the propulsive force that enables it to move along that plane.

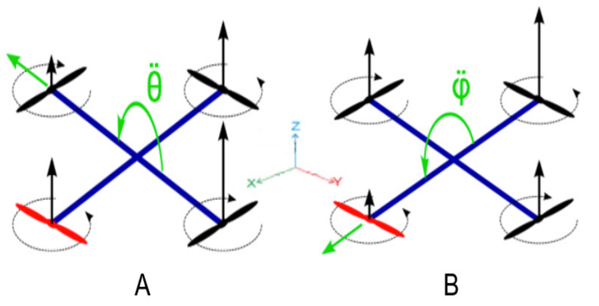

Figure 8. The influence of forces and moments in the lateral movement of the drone [6]

The drone can certainly rotate around the z-axis - Yaw. Since two of the four rotors turn oppositely in the same direction, the rotational movement of the drone is obtained by reducing their number of revolutions in relation to the other two. The drone will rotate in the direction in which the rotors rotate with a lower number of revolutions because, in that case, there is a greater moment of the structure created by the action of the faster rotors, opposite to their direction of rotation. Figure 9 shows the rotation of the drone around the z-axis. It is observed that rotors 2 and 3 have a lower rotation speed, thus creating a lower aerodynamic lift force, and that the drone rotates in the direction of rotation of those rotors.

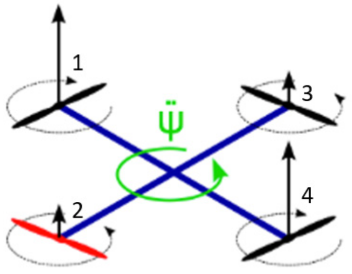

Figure 9. The influence of forces and moments in the rotational movement of the drone [6]

Among the basic characteristics that manufacturers put, such as maximum height, speed, and duration of flight, there is also the maximum tilt of the drone. This tilt gives the drone mobility in the xy plane, i.e. its lateral movement. Exceeding this slope would probably cause the drone to stall and inevitably crash. The reason for this is primarily that when the drone tilts, the buoyancy force is divided into two components, and if the vertical component of the force is not enough to overcome its weight, it will decrease the height.

In this part, first of all, the basic way of moving the drone quadcopter is presented. It should be kept in mind that the drone will behave like this in conditions without the existence of wind, while in conditions with wind, it is possible to correct the speed of the rotor more or less in order to achieve the desired movement.

There are models on the market that have the ability to activate the automatic hover command. After activating that command, the user has the ability to focus more on the work task.

3.2 Thrust in different phases of flight

This part of the paper is devoted to the calculation of thrust of quadcopter propulsion groups. At the very beginning, the thrust itself was defined, and later, its calculation through certain phases of flight. First of all, an explanation of the thrust in the phases of take-off and landing, and then in the phase of hovering, is given. [17]

Thrust is a force perpendicular to the plane of the propeller. It is created by rotating the rotor at a certain speed. Simply put, it accelerates the quadcopter in the direction of its action. The mathematical expression of the thrust force can be seen in equation (1). Equations 1 to 3 are valid under the assumption of an ideal rotor disk, and the net T means that the ratio of thrust and weight is equal to 1. In this and other equations, the "i" indicates which quadcopter rotor it refers to.

$$\left|\vec{T}\right| = \rho A v_i^2 \tag{1}$$

where: v_i -rotor rotation speed, ρ – air density, and A – rotor cross-sectional area.

When applying in practice, it is necessary to read the air density in real time because the air density is correlated with external factors that are not always constant. As a result, the rotor speed required to get the quadcopter to a certain height or maintain it is also variable due to coupling effects. If the air density were not read in real-time, there would be a degradation of performance and a risk of compromising the total thrust. As for the cross-section of the propeller, it is always constant and does not change with time, but the amount of thrust generated in total depends on the surface area of the propeller.

When the quadcopter is in the take-off phase, its rotors and all propellers turn clockwise. This way of turning the propeller contributes to a positive net thrust and enables translational movement in the direction of the Z axis, i.e., vertical lifting. In landing mode, all four rotors rotate counterclockwise, thus creating a negative net thrust. Equation (2) gives the net thrust provided that all engines rotate in the same direction and at the same rotational speed.

$$netT = \rho A \sum_{i=1}^{4} \left[v_i \right]^2 \tag{2}$$

The condition in order to achieve hovering, i.e., the state of maintainig a constant altitude, the net thrust of all rotors is equal to zero. The direction of rotation of the opposite rotors is the same, while the adjacent ones are different in order to cancel the thrust. The speed of rotation of all rotors must be equal in order to create the same intensity of thrust. Equation (3) represents the net thrust during hovering.

$$netT = \rho A \sum_{i=1}^{4} \left[v_i \right]^2 - \rho A \sum_{i=1}^{4} \left[v_i \right]^2 \tag{3}$$

3.3 Orientation

Maneuvering the quadcopter is done by adjusting the angular orientation, that is, by reducing or increasing the number of revolutions of certain engines, as explained above. Figure 10 shows the rotations that the quadcopter can perform. The angle φ represents the angle of rotation in relation to the X axis in terminology known as Roll, while the angle θ is the angle of rotation around the Y axis in terminology known as pitch. Table 1 shows the basic quadcopter moves with the position of significant angles and axis orientations. In the above table, we distinguish 4 cases:

I – Floating,
II – Movement back and forth,
III – Left-right movement,
IV – Rotation around the X and Y axes.

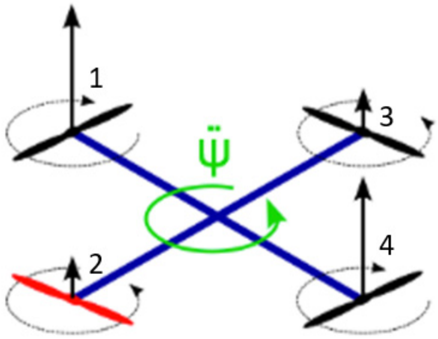

Figure 10. Quadcopter rotation [7]

Table 1. Positions of the quadcopter in relation to the axes at different phases of flight

C	Orientation	θ	φ	X	Y	Z
1	Hovering	θ=0°	φ=0°	0	0	$\pm T_Z$
2	Forward	0°<θ<90°	φ=0°	T_X	0	$\pm T_Z$
	Backward	-90°<θ<0°	φ=0°	$-T_X$	0	$\pm T_Z$
3	Left	θ=0°	0°<φ<90°	0	T_Y	$\pm T_Z$
	Right	θ=0°	-90°<φ<0°	0	$-T_Y$	$\pm T_Z$
4	Pitch + Roll	0°<θ<90°	0°<θ<90°	T_X	T_Y	$\pm T_Z$
	Pitch + Roll	-90°<θ<0°	-90°<φ<0°	$-T_X$	$-T_Y$	$\pm T_Z$

4. KINEMATICS OF QUADCOPTER MOVEMENT

In this part of the work, the basic kinematic equations are presented, which can be used to describe the movement of the quadcopter in space. In addition to the equations, the way in which the variation of the rotation of the pogo groups can be established is the connection with the movement of the quadcopter itself. At the very end of this section, there is a relevant example of the calculation of kinematics movement [7].

4.1 Equations of motion

Based on the law of conservation of energy, equation (4) is obtained; that is, the required speed is obtained for the quadcopter to climb to the desired height, regardless of its orientation. The condition for the quadcopter to move vertically is that the thrust-to-weight ratio must be greater than 1; in this case, the ratio is 2.

$$v^2 = 4g\left(h_f - h_0\right) \tag{4}$$

where:v – represents the difference in speed, i.e. the required increase, h_f – the desired height to which the quadcopter should climb h_0 – the height at which the quadcopter is located

The following equation (5) shows how much thrust needs to be generated to maintain the quadcopter at a certain height depending on the mass and the angle of inclination of the quadcopter.

$$\left|\vec{T}\right| = \frac{\rho A 4g\left(h_f - h_0\right) + mg}{\cos(\theta)\cos\varphi} \tag{5}$$

The following equations (6-8) show the possibility of dividing thrust along the X, Y, and Z axes. Decomposition of thrust is on three vectors, oriented according to the already established coordinate system.

$$\left|T_x\right| = \sqrt{-\left|T\right|^2 \cos(\theta)^2 \left(1 - \frac{1}{\cos(\theta)}\right)} \tag{6}$$

$$\left|T_y\right| = \left|T\right| \cos(\theta)\cos(\varphi) \tag{7}$$

$$\left|T_z\right| = \left|T\right| \cos(\theta)\cos(\varphi) \tag{8}$$

From equation (8) it can be concluded that the altitude can be kept constant only if the total thrust T_z is constant. The vector thrust of the motor can be obtained using equation (9).

$$\vec{T} = T_X \hat{i} + T_Y \hat{i} + T_Z \hat{k} \tag{9}$$

The previous equation is only valid under the following conditions:

- The center of gravity of the quadcopter coincides with the geometric center of the structure,
- The cross-section of all four propellers is in the same plane,
- Angles θ and φ should be less than 90°,
- The arms of the quadcopter are perpendicular to each other.

4.2 Changing the propeller speed

As already mentioned, in order for the quadcopter to be controllable, it is necessary to achieve a certain rotation speed for the propeller. If the speed of the engine, and thus of the propeller, are variable, then a different clutch of moments is achieved, which enables the quadcopter to be moved in the desired direction and direction. The thrust ratio generated on the rotors is equal to the ratio of the angles θ and φ, which is shown in equation (10).

$$\theta : \varphi = \left|\vec{T}\right|_{\theta} : \left|\vec{T}\right|_{\varphi} \tag{10}$$

Table 2. Amount of thrust required depending on flight phase

$\vec{T}_i$		Case			
		1	2	3	4
Rotor	$\vec{T}_1$	$+\frac{Z}{4}$	$+\frac{Z}{4}$	$+\frac{Z}{4}$	$+\frac{Z}{4}$
	$\vec{T}_2$	$-\frac{Z}{4}$	$-\frac{Z}{4}$	$-(\lvert T\rvert - Z) + \frac{Z}{4}$	$-\left(\lvert T\rvert_{\theta} - \frac{Z}{2}\right) + \frac{Z}{4}$
	$\vec{T}_3$	$-\frac{Z}{4}$	$-\frac{Z}{4}$	$-\frac{Z}{4}$	$-\frac{Z}{4}$
	$\vec{T}_4$	$+\frac{Z}{4}$	$+(\lvert T\rvert - Z) + \frac{Z}{4}$	$+\frac{Z}{4}$	$+\left(\lvert T\rvert_{\theta} - \frac{Z}{2}\right) + \frac{Z}{4}$

Using equation (10) and the data from Table 1, the calculations that are expressed in Table 2 are arrived at. The amount of thrust that must be created by each rotor in order for the quadcopter to be placed in the appropriate position, depending on the angles θ and φ is shown. The four cases shown in Table 1 are also shown in Table 2.

Table 3 shows the partial components of thrusts T_X, T_Y and T_Z generated by each propeller, whose addition determines the total thrust vectors T_X, T_Y and T_Z as in equation (9).

Table 3. Partial thrust components

Rotor	1	$T_{X1}+$	$T_{Y1}+$	T_{z1}
	2	$T_{X2}+$	$T_{Y2}+$	T_{z2}
	3	$T_{X3}+$	$T_{Y3}+$	T_{z3}
	4	$T_{X4}+$	$T_{Y4}+$	T_{z4}
	$T =$	T_X +	T_Y +	T_Z

4.3 Dependence of the thrust on the number of revolutions

Most quadcopters in use today use constant-speed propeller motors. These motors are brushless, allowing for variable motor revolutions. This solution is used because of its efficiency, reliability, and low main–tenance costs. The power of these motors is expressed in Kv; that is, it represents the number of revolutions of the motor and the voltage. In the literature, such motors are called constant motors. The following equation (11) shows the dependence of the speed of rotation of the engine and the thrust itself, which is created depending on the parameters of the propeller and the external environment [13].

$$v_i = \pm\sqrt{\frac{\left|\vec{T}\right|_i}{\rho A}} \tag{11}$$

Equation (12) provides the possibility of calculating the required voltage for turning the motor at a certain number of revolutions of the motor at linear speed.

$$volt_i = \frac{1}{motor\,cons\tan t} * \frac{60 v_i}{2\pi r} \tag{12}$$

r-diameter of the propeller

5. DYNAMIC MODELS OF QUADCOPTERS

As already mentioned, fixed-wing quadcopters have propellers that cannot be rotated along the longitudinal axis. In this case, quadcopters gain a degree of freedom of movement based on the number of individual motors attached to the ends of the drone and their coordination. In the case of unmanned aircraft with the ability to rotate the wings around the longitudinal axis, changing the degrees of freedom is done in a simpler way. Due to the difference in the degree of freedom between these two types, the dynamics of the models themselves also differ. In the first part of this section, a dynamic model of a fixed-wing quadcopter is presented, while a dynamic model of so-called Tilting quadcopters will be presented later [8].

5.1 Dynamic model of classic quadcopters

Figure 11 shows the adopted coordinate system and the forces acting on the quadcopter. In the three-dimensional coordinate system, the body of the quadcopter and the arms holding the motors at the ends are represented. At the ends, there are vertical forces that arise as a result of the rotation of the propeller. Each of the propellers creates a torque opposite to the direction of rotation of the propeller as a product of inertia. In order to cancel the torques, the counter rotors rotate in the same direction while the adjacent rotors rotate in different directions. The angles φ, θ, and ω represent the rotation angles around the axes X, Y, and Z. Below is the rotation matrix created by combining all the modulating free positions in which the quadcopter can be found by grasping the above-mentioned angles [12,14].

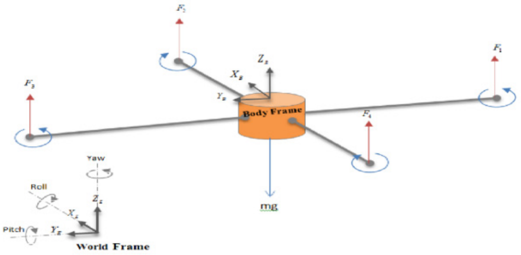

Figure 11. Quadcopter of classic construction in the adopted coordinate system [8]

$$R_{EB} = \begin{bmatrix} c\varphi c\theta & c\varphi c\theta s\omega - s\varphi c\omega & c\varphi c\theta c\omega - s\varphi s\omega \\ s\phi c\theta & s\varphi c\theta s\omega - c\varphi c\omega & s\varphi c\theta c\omega - c\varphi s\omega \\ -s\theta & c\theta s\omega & c\theta cw \end{bmatrix} \tag{13}$$

c-cosines of angles, s-sines of angles.

By obtaining the vertical forces acting on the quadcopter and applying Newton's second law for the X, Y, and Z axes, the following formulas can be written:

$$\begin{aligned} m\ddot{x} &= \sum F_i\left(s\varphi s\omega + c\varphi s\theta c\omega\right) - C_1\dot{x} \\ m\ddot{y} &= \sum F_i\left(s\varphi s\theta c\omega - s\varphi s\omega\right) - C_2\dot{y} \\ m\ddot{z} &= \sum F_i\left(c\theta c\omega\right) - mg - C_3\dot{z} \end{aligned} \tag{14}$$

m – the total mass of the quadcopter, g - acceleration of the earth's gravity, Ci– movement resistance coefficients (at low speeds, they are negligible)

The forces produced by the rotors are expressed as follows:

$$F_i = K_f \omega_i^2 \tag{15}$$

ω_i – angular velocity of the ith propeller, K_f – constant

If Euler's equations were to be applied, the following angular acceleration of the quadcopter would be obtained:

$$\begin{aligned} I_x\ddot{\varphi} &= l\left(F_3 - F_1 - C_1\dot{\varphi}\right) \\ I_y\ddot{\theta} &= l\left(F_4 - F_2 - C_2\dot{\theta}\right) \\ I_x\ddot{\omega} &= M_1 - M_2 + M_3 - M_4 - C_3\dot{\omega} \end{aligned} \tag{16}$$

l – the distance of each rotor from the center of gravity of the quadcopter, Ii – moments of inertia in the direction of the Z axis, Ci – coefficients of resistance to rotational movement

The torque created as a result of the rotation of the propeller and its angular velocity can be represented as follows:

$$M_i = K_m \omega_i^2 \tag{17}$$

ω_i – angular velocity of the *i* rotor, K_m – constant.

During the hovering phase, the quadcopter does not accelerate, has no speed, and all rotation angles are equal to zero. In normal hover conditions, the force produced on each of the propellers must satisfy the following conditions:

$$F_i = \frac{1}{4}\left(mg\right) \tag{18}$$

and engine rotation speeds are given by the following equation:

$$\omega_i = \omega_h = \sqrt{\frac{mg}{4k_f}} \tag{19}$$

5.2 Dynamic model of tilting quadcopters

For Tilting quadcopters, i.e., quadcopters that have variable blade propellers, the change of 4 angles for 4 variable blades on the propellers must be added to the existing equations. By adjusting the propeller arms, greater controllability and better-hovering ability at inclined angles θ,φ are enabled [8,16,20].

In order to more easily describe this movement of the quadcopter, the illustration found in Figure 12 is necessary. In the figure, you can see a schematic representation with forces and moments that act with the coordinate system that was used during modeling.

Propellers can be tilted along their axis. Dashed lines mark the original positions of the quadcopter, which correspond to the zero deflection angles of the propeller arms. Similarly, quadcopters with solid lines are those with inclined planes of rotation θi (i=1,2,3,4) of the respective propellers. It is seen that the lift forces generated are perpendicular to the plane of rotation of the propeller blades. Using the rotation matrix (13), the equations of motion of such systems can be written as follows:

$$\begin{aligned} m\ddot{x} &= F_1 s\theta_1 c\varphi c\theta - F_3 s\theta_3 c\varphi c\theta - F_4 s\theta_4 c\varphi s\theta s\omega \\ &+ F_4 s\theta_4 c\varphi c\omega + F_2 s\theta_2 c\varphi s\theta s\omega - F_2 s\theta_2 s\varphi c\omega \\ &+ F_1 c\theta_1 c\varphi s\theta c\omega + F_4 s\theta_4 c\varphi s\theta c\omega + F_4 c\theta_1 s\varphi c\omega \\ &+ F_3 c\theta_3 c\varphi s\theta c\omega + F_3 c\theta_3 s\varphi c\theta + F_4 c\theta_4 s\varphi s\omega \\ &+ F_2 s\theta_2 s\varphi c\omega + F_3 s\theta_3 s\varphi c\omega - F_4 s\theta_4 s\varphi s\theta s\omega - C_1\dot{x} \\ m\ddot{y} &= F_1 s\theta_1 c\varphi c\theta - F_3 s\theta_3 c\varphi c\theta - F_4 s\theta_4 c\varphi s\theta s\omega \\ &+ F_2 s\theta_2 s\varphi s\theta_2 \omega - F_4 s\theta_4 c\varphi c\omega + F_2 s\theta_2 c\varphi c\omega \\ &+ F_1 c\theta_1 s\varphi s\theta c\omega + F_2 s\theta_2 s\varphi s\theta c\omega \\ &+ F_3 s\theta_3 s\varphi s\theta s\omega + F_4 c\theta_4 s\varphi s\theta c\omega - F_1 c\theta_4 c\varphi c\omega \\ &- F_2 c\theta_2 c\varphi s\omega - F_3 c\theta_3 c\varphi c\omega - F_4 c\theta_4 c\varphi s\omega - C_2\dot{y} \\ m\ddot{z} &= -F_1 s\theta_1 s\theta + F_3 s\theta_3 s\theta - F_4 s\theta_4 c\theta s\omega \\ &+ F_2 s\theta_1 c\theta c\omega + F_1 c\theta_1 c\theta c\omega - F_2 c\theta_2 c\theta c\omega \\ &- F_3 c\theta_3 c\theta c\omega + F_4 c\theta_4 c\theta c\omega - mg - C_3\dot{z} \end{aligned} \tag{20}$$

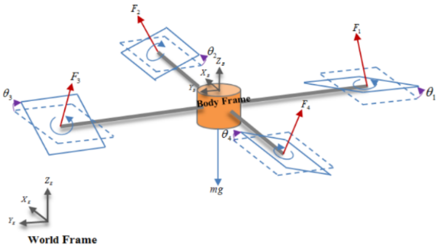

Figure 12. Coordinate system and diagram of a quadcopter with variable propeller blades [8]

In a similar way, the angular accelerations can be determined using Euler's equation:

$$\begin{aligned} I_x\ddot{\varphi} &= l\left(F_3c\theta_3 - F_1c\theta_1 - C_1'\dot{\varphi}\right) + \\ &+\left(M_3s\theta_1 - M_3s\theta_3\right) + \left(M_2' + M_4'\right) \\ I_y\ddot{\theta} &= l\left(F_4c\theta_4 - F_2c\theta_2 - C_2'\dot{\theta}\right) + \\ &+\left(M_4s\theta_4 - M_2s\theta_2\right) + \left(M_1' + M_3'\right) \\ I_z\ddot{\theta} &= l\left(F_1s\theta_1 + F_2s\theta_2 + F_3s\theta_3 + F_4s\theta_4 - C_3'\dot{\varphi}\right) + \\ &+\left(M_1c\theta_1 - M_2c\theta_2 + M_3c\theta_3 - M_4c\theta_4\right) \end{aligned} \tag{21}$$

Mi, - (i=1,2,3,4) are the pitch moments generated by the four servo motors attached to the end of the propeller, in order to change the wing of the same

Assuming $\theta_1 = -\theta_3$ and $\theta_2 = -\theta_4$, all the given moments will cancel.

Based on this dynamic model, the following theorems can be established: [8]

Theorem 1: Taking into account the dynamics of a rotor with a variable arm in a quadcopter given by equations (20) and (21) and assuming that the ratio of the rotor pitch angles is $\theta_1 = -\theta_3$ and $\theta_2 = -\theta_4$, with the same rotation speed on all propellers at the equilibrium state of hovering, the "roll"-inclination angle $\varphi = \theta_1/2$ under the condition that the "pitch"-rolling angle is equal to zero as well as the "pitch"-rolling angle $\theta = \theta_2/2$ under the condition that the "roll"-angle is slope equal to zero.

Theorem 2: Taking into account the dynamics of the pitch of the quadcopter rotor given in equations (19) and (20) and assuming that the pitch angles of the rotor are in the following correlations: $\theta_1 = -\theta_3$ and $\theta_2 = -\theta_4$, the speed that the quadcopter must have when hovering at a certain angle of inclination can be expressed as:

a) When θ=0:

$$\omega_i = \omega_h\sqrt{\frac{mg}{4k_f c\frac{\theta_1}{2}}} \tag{22}$$

b) When φ=0

$$\omega_i = \omega_h\sqrt{\frac{mg}{4k_f c\frac{\theta_2}{2}}} \tag{23}$$

6. DYNAMIC ANALYSIS OF THE GROUND EFFECT AT LOW SPEEDS

The angular effect of the ground effect was almost thoroughly processed only for helicopters. There was almost no research on drones related to this topic. One study conducted by Bernard et al. is about the effect of the ground effect in drones. [9]

First of all, it is necessary to say something about the self-effect of the ground effect. This effect means that the rotor blades near the ground are not able to fully develop downward thrust. Thus, a significant increase in lift occurs, and less engine power is required to create the aerodynamic lift force to overcome the weight of the aircraft.

Bernard et al. conducted an experiment with a drone-quadcopter in laboratory conditions. The drone was above a certain platform, and varying the height was accompanied by a change in thrust and torque.

Two cases were considered. The first case referred only to the dynamic response of one rotor for different heights from the ground, while the second case studied the dynamic response of four rotors working together on a quadcopter body. The change in thrust and torque by varying the number of revolutions on each engine was analyzed. In the case of a single rotor, the thrust response is almost instantaneous and proportional to the change in the number of revolutions, so a linear relationship between the thrust response and the change in the engine revolutions can be established. While there is a certain delay in torque that the researchers believe is related to the torsional flexibility of the structure connecting the drone and the sensors to read the measuring device:

$$F_Z = A_0 + \sum_{i=1}^{n} step\left(t - T_i\right) A_i e^{\frac{t-T_i}{T_i}} \tag{24}$$

where A_0 is the static thrust, T_i is the start time of the next step, A_i is the amplitude, and T_i is the time constant to be identified.

Taking into account the estimated values of the time constants of the response of the thrust to the variation of the engine speed for each step at different heights, the difference between positive and negative time constants can be seen. The main reason is that most drones use ESC (electronic speed control), and motors of this type do not have active brakes. So, the time constants of the thrust response to a decrease in the engine speed are greater than the time constants of the thrust response to an increase in the engine speed. It was found that with the increase in the number of revolutions of the engine, the response accelerates so that for a higher number of revolutions, the thrust response is faster. In simpler terms, the dynamics of the response depends on the number of revolutions of the engine. [17]

From everything presented, the researchers came to the conclusion that there is no dependence on the time constant of the number of revolutions of the engine thrust in relation to the height from the ground when there is only one active rotor.

The same tests conducted on one rotor were repeated for four rotors mounted on the chassis to form a quadcopter. The same behavior as for a single rotor was observed. It was also not possible to recognize the evident dependence of the time constant of the response in relation to the height from the ground. It can be concluded that the influence of the air cushion effect and any creation of turbulence does not appear with quadcopters. Moreover, by comparing the dynamics of a single-rotor case with a four-rotor case, it was determined that no aerodynamic interactions between the rotor and the quadcopter's structure appear to affect the rotor dynamics. At the static level, a certain interaction between the engine and the structure itself was observed. Until now, the analysis only referred to the thrust and torque of the engine, and in that case, there was no influence on the change in height. If the variation of the position of the quadcopter itself is also considered, certain assumptions are made about the impact of the ground effect at low altitudes.

From all that has been stated, the conclusion is reached that the effect of the air cushion in quadcopters is also present as it is present in helicopters. However, with drones, this effect is far less pronounced. First of all, this effect mainly occurs at heights equal to half the diameter of the rotor. As drones generally contain four rotors, the spans of the rotors are smaller, and the height at which this effect occurs is also lower. The construction of drones itself should be taken into account, where the center of gravity tends to be far below the plane of the rotor. So, this effect will generally not even be felt in drones because the rotors are generally placed above the ground, much higher than half of their rotor diameter. The effects of the ground effect that can occur are the influence on the position of the drone in relation to the ground, but these influences have been poorly researched so far. These influences are evident, but they are not as pronounced as, for example, with helicopters. When building a platform for taking off and landing drones, it is not necessary to take into account the turbulence caused by the effects of the ground effect and the measures to break it [18].

7. CONCLUSION

Today, UAVs have become an everyday thing, and to a large extent, they tend to change the air traffic system as we know it today. For now, they have primitive recording and monitoring roles, but the possibility of their application in various spheres of our lives is very great.

A big problem is the integration of unmanned aircraft, which most drones are, into civil air traffic. In order to achieve this in a valid way, it is necessary to carefully study all the elements of their management and the elements that can contribute to the violation of security. The paper contains a detailed analysis of the performance of quadcopters, today's dominant unmanned aircraft in civilian use, with all elements of kinematics and dynamics of their movement. One part of the work is dedicated to newer structural solutions, the control and management of which is much easier than with the quadcopter structure.

The paper contains the current considerations presented so far on UAVs. This type of aircraft has not been sufficiently researched and remains a great possibility for further research. First of all, in the study of the ground effect and its potential impact on UAVs. The fact that its impact is insignificant is simply fascinating, which gives drones a great possibility of use because special places for their take-off and landing do not have to be used.Taking into account the absence of problems during the transition of drones from the hovering phase to progressive flight, as well as the almost negligible impact of the air cushion effect, drones open up great possibilities for application in air traffic and transport.

Understanding the kinematics of unmanned aircraft is pivotal for advancing the development of autonomous vehicles in space. This comprehension enables precise navigation and control, crucial for maneuvering in the complex environment of space. Autonomous space vehicles benefit from optimized trajectories, fuel-efficient propulsion, and collision avoidance strategies, all grounded in kinematic principles. Kinematic models serve as the foundation for autonomous control algorithms, supporting real-time decision-making, mission planning, and space traffic management. By leveraging this understanding, we can enhance the efficiency, safety, and success of space exploration and missions while also paving the way for future innovations in the realm of autonomous space vehicles.

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АНАЛИЗА ПЕРФОРМАНСИ И КИНЕМАТИКЕ КРЕТАЊА БЕСПИЛОТНИХ ВАЗДУХОПЛОВА

М. Милош, П. Миросављевић

Употреба дронова данас је постала саставни део савременог живота до те мере да је квалитет живота одређен степеном искоришћености дронова. Експанзија апликација за дронове је експо–ненцијално порасла. Овом расту првенствено доприносе ниски трошкови набавке и одржавања, као и њихова свестраност. Пре свега, њиховој експанзији доприносе ниски трошкови набавке и одржавања, као и могућност различитих примена. Рад има за циљ да представи дронове и њихове предности у односу на традиционалне авионе као превозно средство. Рад обухвата анализу перформанси беспилотне летелице, испитивање кинематике кретања беспилотне летелице, дискусију о најчешћим структурама које се користе у беспилотним летелицама (квадрокоптерима), као и свеобухватну процену ризика повезаних са беспилотним летелицама и њиховог потенцијала. интеграцију у цивилни ваздушни саобраћај. Детаљно је размотрена структура квадрокоптера, као доминантног решења за производњу беспилотне летелице. На самом крају дата је свеобухватна анализа ризика које носе беспилотне летелице и, као такве, могу ли се интегрисати у цивилни ваздушни саобраћај.