

Investigations on MIT and Lyapunov Rule-Based Modified MRAC for Non-interacting and Interacting Two-Tank Coupled Systems

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The adaptive control method is a technique that measures the dynamic characteristics of the plant automatically and continuously to make a comparison with its required output. It utilizes the difference between plant output and reference output to compute adaptable system parameters to maintain optimal performance regardless of the system variations. The behavior of the adaptation rule is significantly affected by the adaptation gain value. This paper describes the design of MIT (Massachusetts Institute of Technology) and Lyapunov rule-based modified Model Reference Adaptive Controller (MRAC) to stabilize a non-interacting and interacting two-tank process system. Investigation of variation in adaptation gain has also been done. Initially, a traditional MIT and Lyapunov rule-based MRAC is designed to stabilize the non-interacting and interacting tow-tank coupled system and it is found that both the systems are stable only for a few values of adaptation gain. To overcome this problem the modified MRAC is planned to stabilise and improve the response of the system. The modified MRAC scheme is just the PD (Proportional Derivative) controller superimposed on the MRAC control method. Now with the modified MRAC, the systems have been stabilized and their response has been improved for the wide range of adaptation gains. The comparative analysis of traditional and modified MRAC has also been presented. The performance analysis in terms of rise time, settling time, and peak overshoot has been carried out by comparing the results obtained for all the mentioned rules with the variations in the adaptation gain, on the MATLAB/Simulink platform. The obtained results present encouraging outcomes.

Keywords: Adaptive Control, MRAC, Adaptation gain, MIT Rule, Lyapunov Rule, Non-interacting Tow-Tank Coupled System, Interacting Tow-Tank Coupled System.

1. INTRODUCTION

The primary goal of process control [1], which is a subfield of both chemical engineering and control engineering, is to maintain a predetermined range for a given quantity or parameter. The oil and petrochemical industries, the food and beverage industry, the bottling industry, the pulp and paper industry, the chemical industry, the power industry, the biotechnology and pharmaceutical industries, and many more all use process control in some capacity. Maintaining a steady liquid level and flow rate in a series of tanks is a common challenge in the process industries [2-7]. It is common practice to pump liquid from one tank into another tank or tank [8]. These tanks can be placed in a non-interacting or interacting series. Using a two-tank system as an example, in a non-interacting configu-

ration, the liquid level in tank 2 does not affect the liquid level in tank 1. However, in interacting configuration [9], the liquid levels in both tanks influence one another. To preserve the reaction balance and prevent spilling or equipment damage, the volume and velocity of the liquid contained in such tanks must be controlled. [10, 11].

Open-loop control, closed-loop control, feed-forward control, etc. are a few examples of the various control systems used in industry. The control action in open-loop control is independent of the system's output and is instead based on the input signal. For simulating the plant or process in advance, feed-forward control can anticipate and mitigate the effects of measured disruptions. To implement feedback control [12], a feedback loop is added to an existing system, resulting in a control action that guides the system to the desired output.

For MRAC systems, the MIT rule is a gradient method of analysis and design. The process begins with establishing an error signal, which is the relative deviation between the plant's output and the reference model's output. For constructing an objective function

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based on this error signal, we may fine-tune the controller's parameters. The controller's parameters are adjusted so that the objective function has a negative gradient. A stable adaptive control law is however ensured by the Lyapunov method. A Lyapunov function of the output error (or state error) and the parameter error is used to implement the Lyapunov technique. A negative definite derivative of the Lyapunov function ensures the stability of the control loop. [13, 14]. The parameters in the control law are adjusted with the help of an adaptation mechanism developed using the MIT and Lyapunov techniques. Resetting the parameters to their optimal values, the system is made error-free and stable [15]. The gain from the adaptation mechanism has an effect on the system's performance, when the adaption gain is high, the system can become unstable [16, 17]. A modified MRAC was developed as a solution, and it proved successful in stabilizing the system even at high adaption gain.

The objective of the research is to stabilize the MRAC systems for optimal performance of the non-interacting two-tank and interacting two-tank systems and to identify the impact of adaptation gain on these systems. In this paper, MIT and Lyapunov rule-based traditional MRAC and modified MRAC have been designed and their comparative analysis has been done. First, both non-interacting and interacting configurations of a coupled two-tank system are modeled. To control the connected two-tank systems, we first derive transfer function representations for both models and then utilize the MIT and Lyapunov techniques to design the conventional MRAC and the modified MRAC systems. Adaptation gain (α), rising time (T_r), settling time (T_s), and peak overshoot (M_p) are all measured and compared across MRAC-optimized systems based on both methods. MATLAB/Simulink is used to run the simulation.

The main practical applications of the research lie in the field of control systems engineering, particularly in the control and optimization of two-tank coupled systems. Here are some potential practical applications and contributions:

Process Control in Chemical Engineering: Two-tank systems are commonly used in chemical engineering for processes such as liquid level control, temperature control, or flow control. The research provides insights into utilizing Model Reference Adaptive Control (MRAC) techniques for effectively regulating these processes. This could lead to more efficient and precise control of chemical processes, improving productivity and reducing waste.

Water Management Systems: Two-tank systems can also represent water management systems, such as water storage tanks in municipal water supply networks. By applying the modified MRAC techniques studied in the research, engineers can better manage water levels, ensure consistent supply, and optimize distribution, contributing to more reliable and sustainable water management practices.

Renewable Energy Systems: In renewable energy applications like solar thermal systems or energy storage systems, two-tank configurations are used for heat storage or energy buffering. Implementing advanced

control strategies based on the findings of the research can enhance the performance and stability of these systems. It could lead to better utilization of renewable energy sources, improving overall system efficiency and reducing environmental impact.

Industrial Automation: Two-tank systems are prevalent in industrial processes for tasks such as mixing, blending, or batching. By incorporating the modified MRAC techniques investigated in the research, industrial automation systems can achieve tighter control over these processes, resulting in higher product quality, reduced waste, and increased throughput.

Scientific Merit and Engineering Practice Contribution:

Theoretical Advancements: The research contributes to the advancement of control theory by investigating the application of modified MRAC techniques, incorporating Lyapunov-based stability analysis, for complex two-tank systems. This contributes to a deeper understanding of adaptive control methodologies and their applicability to real-world engineering systems.

Practical Relevance: By focusing on practical systems like two-tank setups, the research ensures its findings have direct relevance to real engineering problems. Engineers can directly apply the insights gained from this research to improve the performance, efficiency, and robustness of various industrial and process control systems.

Performance Enhancement: The application of modified MRAC techniques has the potential to enhance the performance of control systems in terms of stability, tracking accuracy, and disturbance rejection. This can lead to tangible benefits such as improved product quality, reduced energy consumption, and increased system reliability in engineering practice.

The research makes valuable contributions to both the theoretical understanding and practical implementation of adaptive control strategies, particularly in the context of non-interacting and interacting two-tank coupled systems, with implications for a wide range of engineering applications.

After the brief introduction in section 1, section 2 gives the mathematical modeling of the NITT (Non-inverting Two-Tank) and ITT (Inverting Two-Tank) systems. The designing procedure of the MIT and Lyapunov rule-based normal MRAC and modified MRAC have been discussed in sections 3 and 4 respectively. Performance evaluation and simulation results are given in section 5. The last section 6 gives the overall conclusions of this work.

2. MATHEMATICAL MODELING OF TWO-TANK SYSTEM

In this paper, the two-tank liquid level system is selected as a plant to be controlled, because it is a non-linear inherently unstable system. The Two-tank liquid level system is arranged in non-interacting and interacting modes as shown in Figure 1.

The parameters used in Figure 1 are defined as A_1 and A_2 is the cross-sectional area of tank-1 and tank-2 respectively, Q_{in} is the inflow rate of liquid in tank-1, Q_1 , and Q_2 are the outflow rate of liquid in tank-1 and

tank-2 respectively, H_1 and H_2 are the height of liquid in Tank-1 and tank-2 respectively and R_1 and R_2 are the resistances to flow of liquid from Tank-1 and tank-2 respectively.

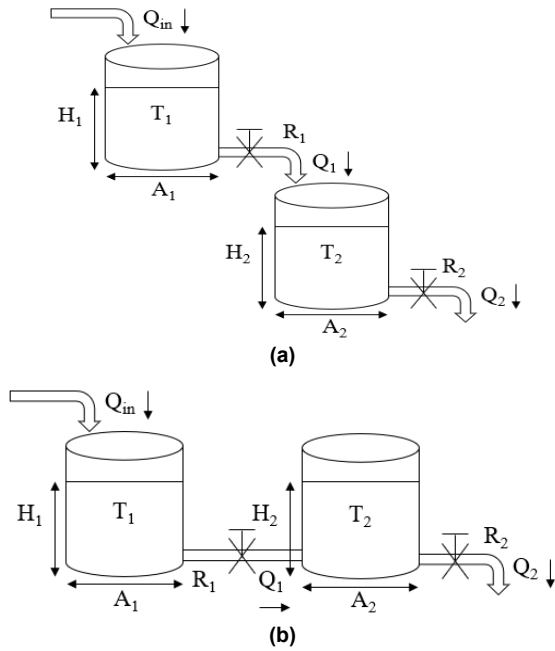


Figure 1. Block Diagram of (a) Noninteracting Two-Tank Liquid Level System (b) Interacting Two-Tank Liquid Level System

2.1 Noninteracting

The transfer function of the non-interacting two-tank coupled system depicted in Figure 1(a) is determined by solving the mass balance equation,

(Mass inflow rate of the first tank) - (Mass outflow rate of the first tank) = rate of change of mass inside the tank

$$\rho Q_{in} - \rho Q_1 = \frac{d}{dt}(\rho A_1 H_1) \quad (1)$$

$$Q_{in} - Q_1 = A_1 \frac{d}{dt}(H_1) \quad (2)$$

Assuming linear resistance to flow we have,

$$Q_1 = \frac{H_1}{R_1} \quad (3)$$

Now, put the Q_1 from Eq. (3) to Eq. (2),

$$Q_{in} - \frac{H_1}{R_1} = A_1 \frac{d}{dt}(H_1) \quad (4)$$

Taking Laplace Transform of Eq. (4)

$$Q_{in}(s) - \frac{H_1(s)}{R_1} = A_1 s H_1(s) \quad (5)$$

$$H_1(s) = \frac{R_1}{A_1 R_1 s + 1} Q_{in}(s) \quad (6)$$

Similarly, the mass balance equation for tank 2,

$$Q_1 - Q_2 = A_2 \frac{d}{dt}(H_2) \quad (7)$$

$$\frac{H_1}{R_1} - \frac{H_2}{R_2} = A_2 \frac{d}{dt}(H_2) \quad (8)$$

Taking Laplace Transform of Eq. (8),

$$\frac{H_1(s)}{R_1} - \frac{H_2(s)}{R_2} = A_2 s H_2(s) \quad (9)$$

Substituting $H_1(s)$ from Eq. (6) to Eq. (9),

$$R_2 \left(\frac{R_1}{A_1 R_1 s + 1} Q_{in}(s) \right) = (A_2 R_1 R_2 s + R_1) H_2(s) \quad (10)$$

$$\frac{H_2(s)}{Q_{in}(s)} = \frac{R_2}{A_1 R_1 A_2 R_2 s^2 + (A_1 R_1 + A_2 R_2) s + 1} \quad (11)$$

$$\frac{H_2(s)}{Q_{in}(s)} = \frac{R_2}{A_1 R_1 A_2 R_2 s^2 + (A_1 R_1 + A_2 R_2) s + 1} \quad (12)$$

Now, Eq. (12) represents the Transfer Function of a non-interacting two-tank coupled system.

2.2 Interacting

The transfer function of the interacting two-tank coupled system depicted in Figure 1(b) is determined by solving the mass balance equation,

(Mass inflow rate of the first tank) - (Mass outflow rate of the first tank) = rate of change of mass inside the tank

$$\rho Q_{in} - \rho Q_1 = \frac{d}{dt}(\rho A_1 H_1) \quad (13)$$

$$Q_{in} - Q_1 = A_1 \frac{d}{dt}(H_1) \quad (14)$$

Assuming linear resistance to flow we have,

$$Q_1 = \frac{H_1 - H_2}{R_1} \quad (15)$$

Now, put the Q_1 from Eq. (15) to Eq. (14),

$$Q_{in} - \frac{H_1 - H_2}{R_1} = A_1 \frac{d}{dt}(H_1) \quad (16)$$

$$Q_{in} R_1 - H_1 + H_2 = A_1 R_1 \frac{d}{dt}(H_1) \quad (17)$$

Taking Laplace Transform of Eq. (17),

$$Q_{in}(s) R_1 - H_1(s) + H_2(s) = A_1 R_1 s H_1(s) \quad (18)$$

$$H_1(s) = \frac{R_1 Q_{in}(s) + H_2(s)}{A_1 R_1 s + 1} \quad (19)$$

Similarly, the mass balance equation for tank 2,

$$Q_1 - Q_2 = A_2 \frac{d}{dt}(H_2) \quad (20)$$

$$\frac{H_1 - H_2}{R_1} - \frac{H_2}{R_2} = A_2 \frac{d}{dt}(H_2) \quad (21)$$

Taking Laplace Transform of Eq. (21),

$$\frac{H_1(s) - H_2(s)}{R_1} - \frac{H_2(s)}{R_2} = A_2 s H_2(s) \quad (22)$$

$$R_2 H_1(s) = A_2 R_1 R_2 s H_2(s) + R_1 H_2(s) + R_2 H_2(s) \quad (23)$$

Substituting $H_1(s)$ from Eq. (19) to Eq. (23),

$$R_2 \left(\frac{R_1 Q_{in}(s) + H_2(s)}{A_1 R_1 s + 1} \right) = (A_2 R_1 R_2 s + R_1 + R_2) H_2(s) \quad (24)$$

$$\frac{H_2(s)}{Q_{in}(s)} = \frac{R_2}{A_1 R_1 A_2 R_2 s^2 + (A_1 R_1 + A_2 R_2 + A_1 R_2) s + 1} \quad (25)$$

Now, Eq. (25) represents the Transfer Function of the interacting two-tank coupled system. Table 1 lists the specifications of the two-tank coupled system [18].

Table 1: Parameters Specifications to Model both Two-Tank Coupled Systems

Parameters	Values
A_1	0.025 m ²
A_2	0.025 m ²
H_1	0.3 m
H_2	0.15 m
R_1	100 s/m ²
R_2	200 s/m ²

The transfer functions of the non-interacting and interacting two-tank coupled systems are given by Eqs. (26) and (27) when the parameters from Table 1 are substituted into the formulas of Eqs. (12) and (25) respectively,

$$\frac{H_2(s)}{Q_{in}(s)} = \frac{200}{12.5s^2 + 7.5s + 1} \quad (26)$$

$$\frac{H_2(s)}{Q_{in}(s)} = \frac{200}{12.5s^2 + 12.5s + 1} \quad (27)$$

The two-tank system's linearized model is a second-order model. As a result, the linear reference model can be constructed as a standard second-order transfer function, as shown below,

$$\frac{y_m(s)}{u_c(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (28)$$

In MRAC, the first step is to select the reference model depending on the requirement. After that, the control algorithm's design is done to update the controller's adjustable parameters. The reference model, which describes the desired performance specifications, i.e., rise time (T_r), settling time (T_s), and peak-overshoot (M_p) for system response, is given. For the analysis of this research work, a critically damped ($\zeta = 1$) second-order system, is taken from [18] and represented by Eq. (29) and Eq. (30) for non-interacting and interacting two-tank coupled systems respectively.

$$\frac{y_m(s)}{u_c(s)} = \frac{1}{6.25s^2 + 5s + 1} \quad (29)$$

$$\frac{y_m(s)}{u_c(s)} = \frac{1}{s^2 + 2s + 1} \quad (30)$$

3. MODEL REFERENCE ADAPTIVE CONTROL

MRAC is a form of adaptive control that belongs to the broader category of non-dual adaptive control [19]. A

reference model may define the performance of the system. After comparing the actual output to the modeled output, the feedback controller settings are modified using different approaches like the MIT rule and the Lyapunov rule, which is described below. The MRAC is modeled to manipulate the plant or system output to track the reference model. Model Reference Adaptive System has two types of loops. First is an inner loop, also known as a regulator loop. It is a standard control loop that consists of a regulator and the plant to update the plant parameter through an adaptation mechanism. The second is the outer loop, also known as the adaptation loop. This loop coordinates the regulator parameters to manage the steady state error between the system output and modeled output down to zero. The basic block diagram of MRAC is shown in Figure 2.

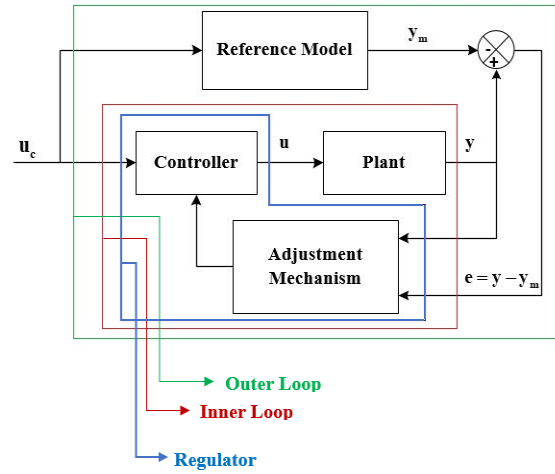


Figure 2. Block Diagram of Model Reference Adaptive Controller

Since the designed mathematical model of the system is a second-order system, hence, we have chosen a standard critically damped second-order system as a reference model to design the controller. Let the equations (31) and (32) given below characterize the system and reference model equation, respectively:

$$\frac{d^2 y(t)}{dt^2} = -a \frac{dy(t)}{dt} - b_1 y(t) + b_2 u(t) \quad (31)$$

$$\frac{d^2 y_m(t)}{dt^2} = -a_m \frac{dy_m(t)}{dt} - b_m y_m(t) + b_m u_c(t) \quad (32)$$

Here, a , b_1 , b_2 , a_m and b_m are constants $y(t)$ is plant output and $y_m(t)$ is reference model output. The equation that describes the control law is shown below by Eq. (33)

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) \quad (33)$$

The difference between the output of the reference model y_m , and the plant output y is defined as the term "error function," It may be represented as follows by Eq. (34):

$$e(t) = y(t) - y_m(t) \quad (34)$$

3.1 MIT RULE

This rule was devised by the Massachusetts Institute of Technology (MIT); therefore, it is commonly known as

the MIT rule. It applies the MRAC scheme [20-22] to real-world systems. For the stability analysis of the system by the MIT rule, we needed a loss function J , often known as the cost function, which may be illustrated using [23-27],

$$J(\theta) = \frac{1}{2} e^2 \quad (35)$$

$$\frac{\partial J}{\partial e} = e \quad (36)$$

where, e is output error, which may be considered as the difference between the output of the plant and the output of the reference model, and θ (i.e., θ_1 and θ_2) is the regulating parameter that is generally recognized as the control parameter. In this case, the loss function is reduced by adjusting a parameter denoted by θ (i.e., θ_1 and θ_2). Therefore, adjusting the parameter so that it moves in the opposite direction as J 's gradient would be appropriate., i.e.,

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J(\theta)}{\partial \theta} \quad (37)$$

$$\frac{d\theta}{dt} = -\gamma e \frac{\partial e(\theta)}{\partial \theta} \quad (38)$$

Here, $\partial e / \partial \theta$ is known as the sensitivity derivative of the plant. This term depicts how the error is affected by modification made in the parameter, denoted by the symbol θ . The γ is adaptation gain [18]. Adaptation gain refers to a tuning parameter used to adjust the adaptation rate of the controller. The adaptation gain aims to strike a balance between fast convergence and stability. A high adaptation gain can lead to quick parameter updates and quicker convergence, but it may also introduce instability and overshoot in the control system. On the other hand, a low adaptation gain can improve stability but may result in slower convergence and reduced tracking performance. The selection of the adaptation gain depends on the characteristics of the controlled system and the desired performance specifications. The best adaptation gain can be determined using a systematic strategy, such as trial-and-error, optimization algorithms, or advanced control design methods. In this paper, the trial-and-error method has been used to find a suitable range of adaption gain.

Since the error is specified by Eq. (34), hence the modification in error w.r.t. time may be written as

$$\begin{aligned} \dot{e} = & \left(-\left(\frac{1}{a} \ddot{y} - \frac{1}{a_m} \ddot{y}_m \right) - \frac{b_m}{a_m} e - \left(\frac{b_2}{a} \theta_2 + \frac{b_1}{a} - \frac{b_m}{a_m} \right) y + \right. \\ & \left. + \left(\frac{b_2}{a} \theta_1 - \frac{b_m}{a_m} \right) u_c \right) \end{aligned} \quad (39)$$

The aim of an adaptive controller is that the y should be the asymptotic trace of y_m and hence from Eq. (39),

$$\frac{1}{a} \ddot{y} - \frac{1}{a_m} \ddot{y}_m = 0 \Rightarrow a = a_m \quad (40)$$

$$\frac{b_2}{a} \theta_1 - \frac{b_{2m}}{a_m} = 0 \Rightarrow \theta_1 = \frac{b_{2m}}{b_2} \quad (41)$$

$$\frac{b_2}{a} \theta_2 + \frac{b_1}{a} - \frac{b_{1m}}{a_m} = 0 \Rightarrow \theta_2 = \frac{b_m - b_1}{b_2} \quad (42)$$

Now, $\dot{e} = -a_{1m} e(t)$ is negative definite and $e \rightarrow 0$ as $t \rightarrow \infty$, hence the system will be stable.

Putting Eq.(33) into Eq. (31) and using the Laplace transform, we get the following:

$$y(s) = \frac{b_2 \theta_1}{s^2 + as + (b_1 + b_2 \theta_2)} u_c(s) \quad (43)$$

Taking Laplace transform of Eq. (32),

$$y_m(s) = \frac{b_m}{s^2 + a_m s + b_m} u_c \quad (44)$$

Take Laplace Transform of Eq. (34) and put Eq. (43) and Eq. (44),

$$e(s) = \frac{b_2 \theta_1}{s^2 + as + (b_1 + b_2 \theta_2)} u_c(s) - \frac{b_m}{s^2 + a_m s + b_m} u_c(s) \quad (45)$$

$$\frac{\partial e}{\partial \theta_2} = \frac{b_2}{s^2 + ac + (b_1 + b_2 \theta_2)} u_c(s) \quad (46)$$

$$\frac{\partial e}{\partial \theta_2} = \frac{-b_2^2 \theta_1}{(s^2 + ac + (b_1 + b_2 \theta_2))^2} u_c(s) \quad (47)$$

$$\frac{\partial e}{\partial \theta_2} = \left(\frac{-b_2}{s^2 + ac + (b_1 + b_2 \theta_2)} \right) \left(\frac{b_2 \theta_1}{s^2 + ac + (b_1 + b_2 \theta_2)} \right) u_c(s) \quad (48)$$

$$\frac{\partial e}{\partial \theta_2} = \left(\frac{-b_2}{s^2 + ac + (b_1 + b_2 \theta_2)} \right) y \quad (49)$$

Put the value of a from Eq. (40) and θ_2 from Eq. (42) into Eq. (46) and (49), we get Eq. (50) and (51)

$$\frac{\partial e}{\partial \theta_1} = \frac{b_2}{s^2 + a_m s + b_m} u_c \quad (50)$$

$$\frac{\partial e}{\partial \theta_2} = \frac{-b_2}{s^2 + a_m s + b_m} y \quad (51)$$

Eq. (38) can be written as,

$$\frac{d\theta_1}{dt} = -\gamma e \frac{\partial e(\theta)}{\partial \theta_1} \quad (52)$$

$$\frac{d\theta_2}{dt} = -\gamma e \frac{\partial e(\theta)}{\partial \theta_2} \quad (53)$$

By substituting the values of $\frac{\partial e}{\partial \theta_1}$ and $\frac{\partial e}{\partial \theta_2}$ from Eq. (50) and (51) into Eq. (52) and (53), we get Eq. (54) and (55) respectively,

$$\frac{d\theta_1}{dt} = -\gamma e \frac{b_2}{s^2 + a_m s + b_m} u_c \quad (54)$$

$$\frac{d\theta_2}{dt} = \gamma e \frac{b_2}{s^2 + a_m s + b_m} y \quad (55)$$

To absorb the plant parameter b_2 Eq. (54) and Eq. (55) re-defined as Eq. (56) and Eq. (57) respectively

which defined the adaptation laws using the MIT rule. Where $\alpha = \gamma b_2/b_m$. Figure 3 shows the block diagram of MRAC using the MIT rule.

$$\frac{\partial \theta_1}{\partial t} = -\gamma e \frac{b_m}{s^2 + a_m s + b_m} u_c \quad (56)$$

$$\frac{\partial \theta_2}{\partial t} = \gamma e \frac{b_m}{s^2 + a_m s + b_m} y \quad (57)$$

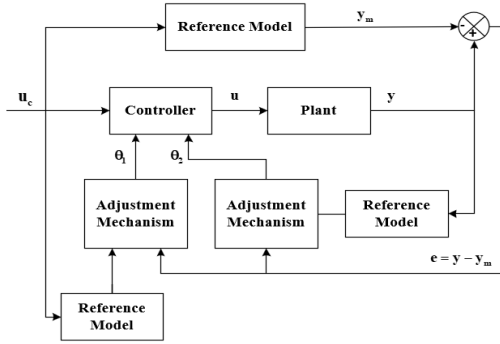


Figure 3. Block Diagram Model for MRAC using MIT rule

3.2 LYAPUNOV RULE

The Lyapunov stability theory may be used to describe the algorithms for adjusting parameters in the MRAC system. Based on the system mentioned above by Eq. (31), the control law defined by Eq. (33), and the error given by Eq. (34). We have chosen the Lyapunov function as [12, 18],

$$V(e, \theta_1, \theta_2) = \frac{1}{2} \left(e^2 + \frac{1}{b_2 \gamma} (b_2 \theta_2 + b_1 - b_m)^2 + \frac{1}{b_2 \gamma} (b_2 \theta_1 - b_m)^2 \right) \quad (58)$$

$$\dot{V} = -a_m e^2 + e^2 + \frac{1}{\gamma} (b_2 \theta_2 + b_1 - b_m) (\dot{\theta}_2 - \alpha y e) + \frac{1}{\gamma} (b_2 \theta_1 - b_m) (\dot{\theta}_1 - \alpha u_c e) \quad (59)$$

where, $\alpha = \gamma b_2/b_m$ and to ensure that \dot{V} is negative definite,

$$\frac{1}{\gamma} (b_2 \theta_2 + b_1 - b_m) (\dot{\theta}_2 - \alpha y e) + \frac{1}{\gamma} (b_2 \theta_1 - b_m) (\dot{\theta}_1 - \alpha u_c e) = 0 \quad (60)$$

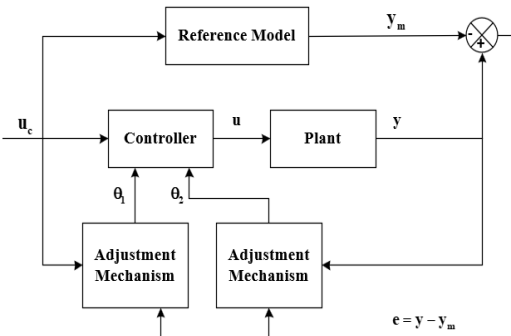


Figure 4. Block Diagram Model for MRAC using Lyapunov Rule

Therefore, Eq. (61) and Eq. (62) define adaptation laws using the Lyapunov rule. Figure 4 shows the block diagram of MRAC using the Lyapunov rule.

$$\frac{d\theta_2}{dt} = \alpha e y \quad (62)$$

4. MODIFIED MODEL REFERENCE ADAPTIVE CONTROL

Now, the modified MRAC has been planned to improve the response of the system. The modified MRAC scheme is just the PD controller superimposed on the MIT and Lyapunov rule-based MRAC control method. Here, we stabilize both systems and follow the intended response by combining the control laws of MRAC using the MIT rule, Lyapunov rule, and PD control law. The PD control law's Proportional and Derivative gains can be modified with the MIT and Lyapunov rule's adaption parameters. Therefore, the controller law is demonstrated as Eq. (63). Figure 5(a) shows the block diagram of the modified MIT rule-based MRAC and Figure 5(b) shows the block diagram of the modified Lyapunov rule-based MRAC.

$$u(t) = \theta_1 r(t) - \theta_2 y(t) - \left(k_p e(t) + k_d \frac{de(t)}{dt} \right) \quad (63)$$

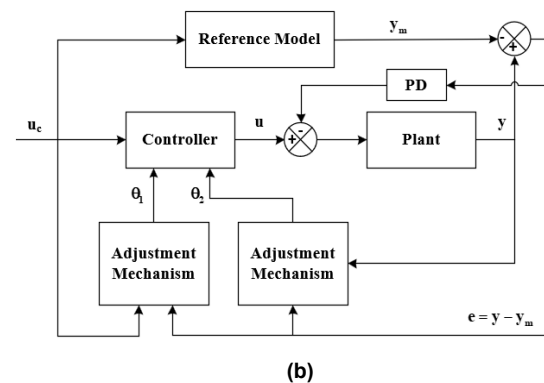
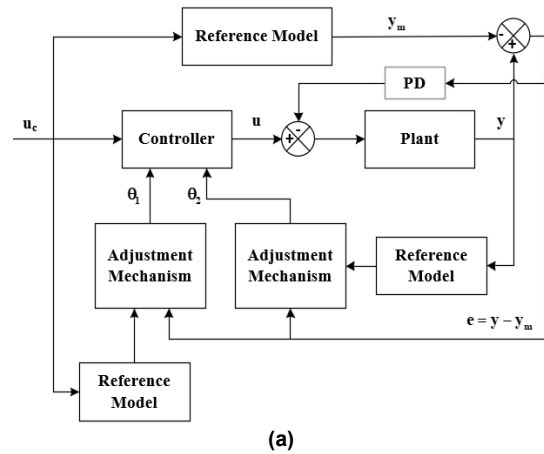


Figure 5. Block Diagram of Modified MRAC using (a) MIT Rule (b) Lyapunov Rule.

5. PERFORMANCE EVALUATION AND SIMULATION RESULTS

This section details a simulation performance evaluation of both adaptive controllers for both systems. The set

point which is the value of height, H_2 , is 0.15m. It investigated how different adaptation gain values affect the system behavior for all adaptation strategies. The adaptive controllers are analyzed using a step input signal by taking a non-interacting system as Eq. (26) and an interacting system as Eq. (27). In MRAC, the first step is to select the reference model depending on the requirement. After that, the control algorithm's design is done to update the controller's adjustable parameters. The reference model regarding the transfer function obtained from the desired performance specifications (i.e., rise time, settling time, overshoot, and steady-state error) is given. In the present work, for the analysis, the reference model of the non-interacting and interacting two-tank coupled system is taken from Dinakin, and Oluseyi, 2021.

5.1 Influence of adaptation gain using conventional MRAC on non-interacting system

Firstly, the adaptation gain α is varied, and the resulting influence on the non-interacting system's (26) time response is analyzed. The results of a simulation with gain values 0.01, 0.1, 1, 5, 10, 100, and 1000 have been shown in Figure 6 with the MIT rule and in Figure 7 with the Lyapunov rule for the reference model (29).

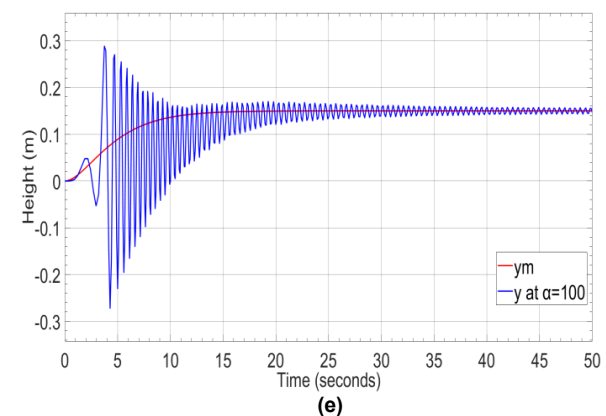
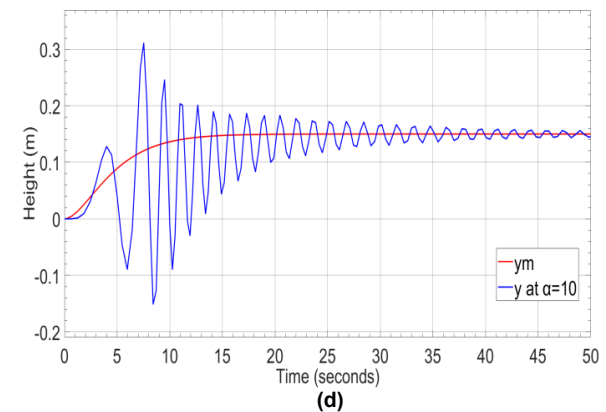
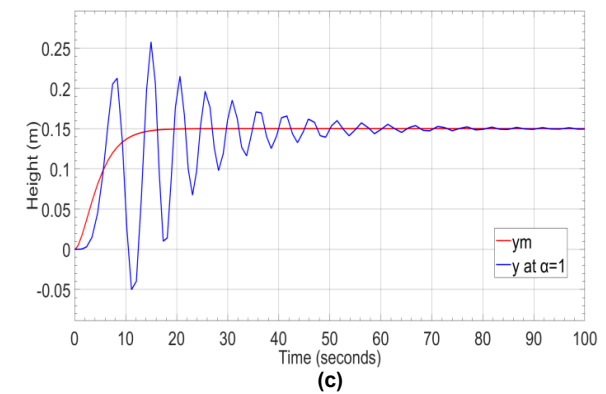
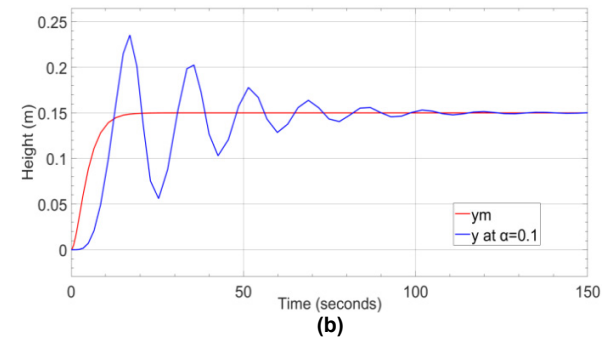
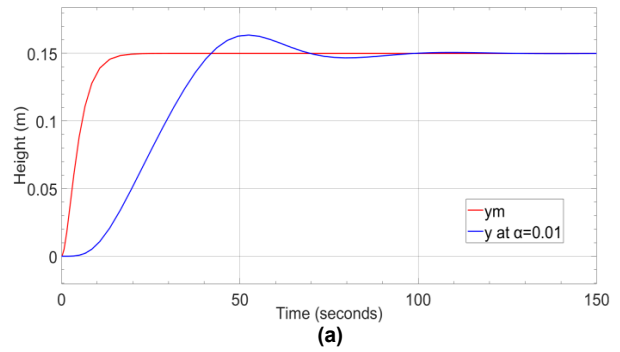
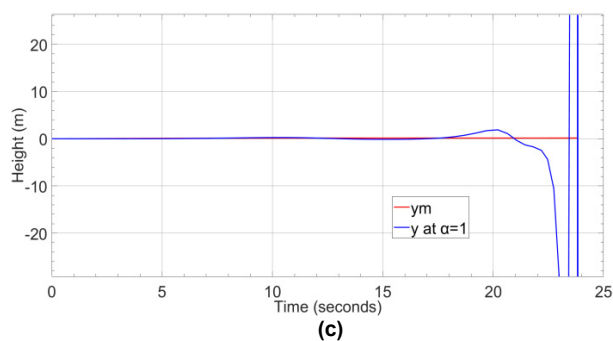
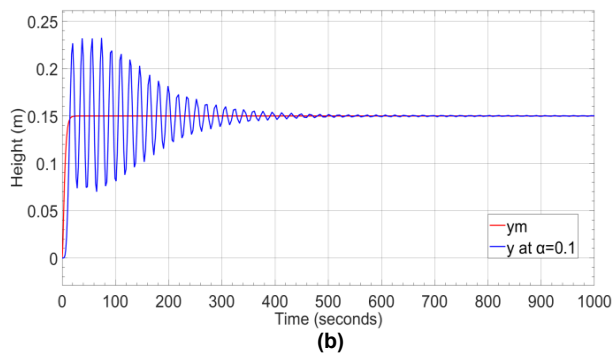
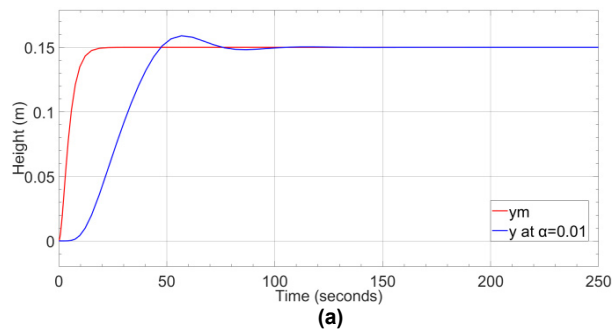


Figure 6: Simulation Results of Conventional MRAC with MIT Rule at (a) $\alpha = 0.01$, (b) $\alpha = 0.1$, (c) $\alpha = 1$ for Non-interacting System

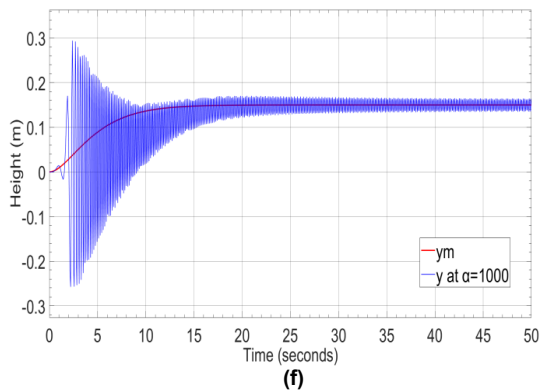
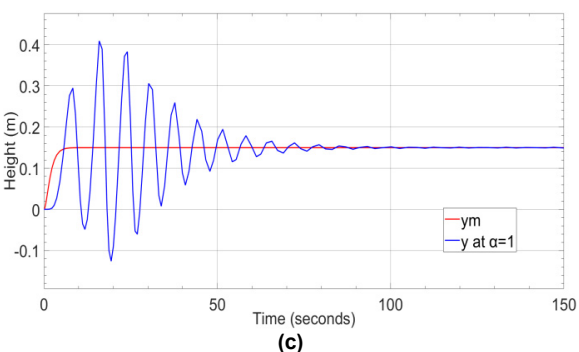
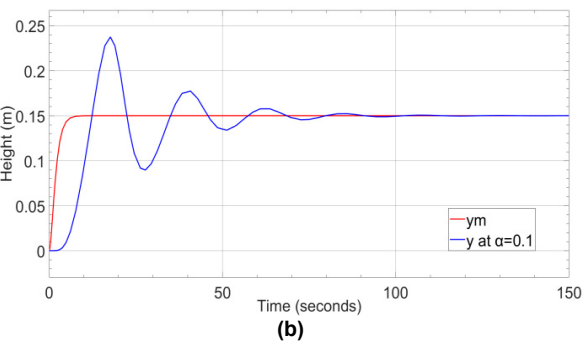
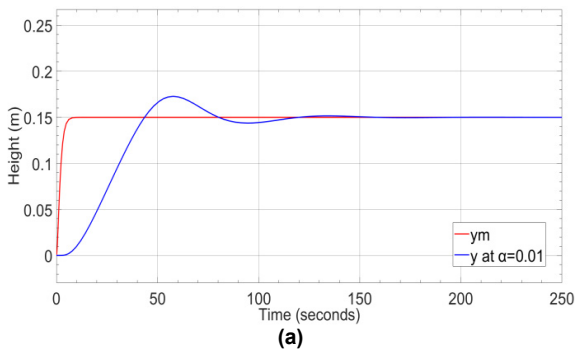


Figure 7: Simulation Results of Conventional MRAC with Lyapunov Rule at $\alpha = 0.01$, (b) $\alpha = 0.1$, (c) $\alpha = 1$ (d), $\alpha = 10$, (e) $\alpha = 100$, (f) $\alpha = 1000$ for Non-interacting System

From Figure 6 and Table 2, it may be observed that as the adaptation gain increases, the system starts osci-

Table 2: Effect of Adaptation Gain on Non-interacting Coupled System with MRAC

Adaptation Law	MIT Rule				Lyapunov Rule				
	Adaptation Gain	Overshoot (%)	Peak Time	Settling Time	Rise Time	Overshoot (%)	Peak Time	Settling Time	Rise Time
	0.01	5.9561	56.7946	70.0150	27.6027	9.1257	52.4903	83.2475	25.3755
	0.1	54.8493	74.0427	422.2048	6.5342	56.7376	17.0250	96.3780	6.1572
	1	0	23.8360	23.8360	7.1054×10^{-14}	72.5685	14.9886	66.8941	2.8020
	10	-	-	-	-	112.8315	7.5277	49.2161	4.8553
	100	-	-	-	-	96.8645	3.7443	49.3982	2.2316
	1000	-	-	-	-	116.4551	2.3791	49.9839	0.8704



lating and at $\alpha = 1$ the system becomes unstable with the MIT rule. But with the Lyapunov rule the system's response is fast but oscillating and the oscillation increases with adaptation gain as shown in Figure 7. The performance of the system w.r.t. rise time, settling time, peak time, and peak overshoot is shown in Table 2.

5.2 Influence of adaptation gain using conventional MRAC on interacting system

In this section, the interacting system's time response is analyzed with varying the adaptation gain α . The results of a simulation with the gain values 0.01, 0.1, 1, 2, 2.1, 5, 10, 100, and 1000 have been shown in Figure 8 with MIT rule the gain values 0.01, 0.1, 1, 10, 100, and 1000 have been shown in Figure 9 with the Lyapunov rule for the reference model (30).

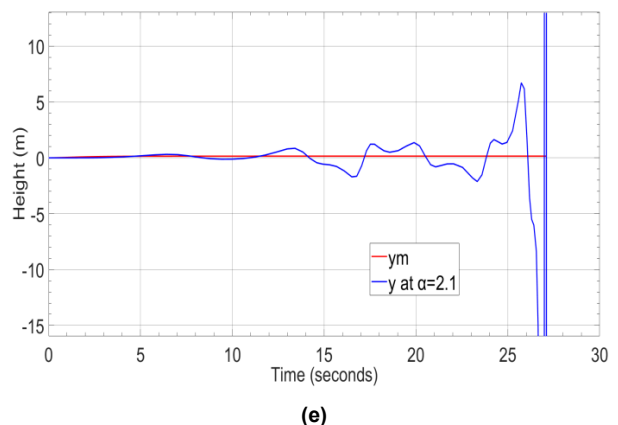
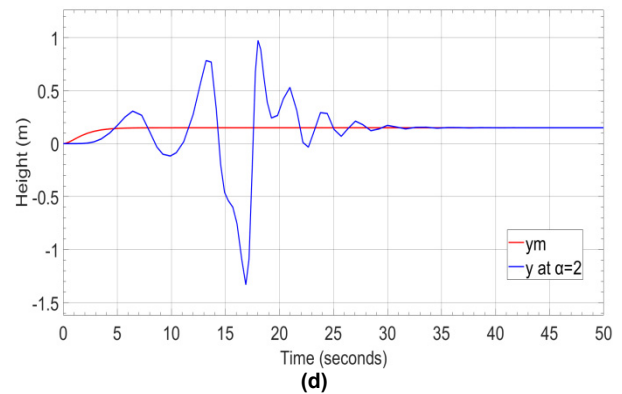


Figure 8: Simulation Results of Conventional MRAC with MIT Rule at (a) $\alpha = 0.01$, (b) $\alpha = 0.1$, (c) $\alpha = 1$, (d) $\alpha = 2$, (e) $\alpha = 2.1$ for Interacting System

From Figure 8 and Table 3, it may be observed that as the adaptation gain increases, the system starts oscillating and at $\alpha = 2.1$, the system becomes unstable with the MIT rule. But with the Lyapunov rule the system's

response is fast but oscillating and the oscillation increases with adaptation gain as shown in Figure 9. The performance of the system w.r.t. rise time, settling time, peak time, and peak overshoot is shown in Table 3. To overcome this problem the modified MRAC has been designed and analysis has been discussed in section 5.3 and section 5.4 for non-interacting and interacting systems respectively.

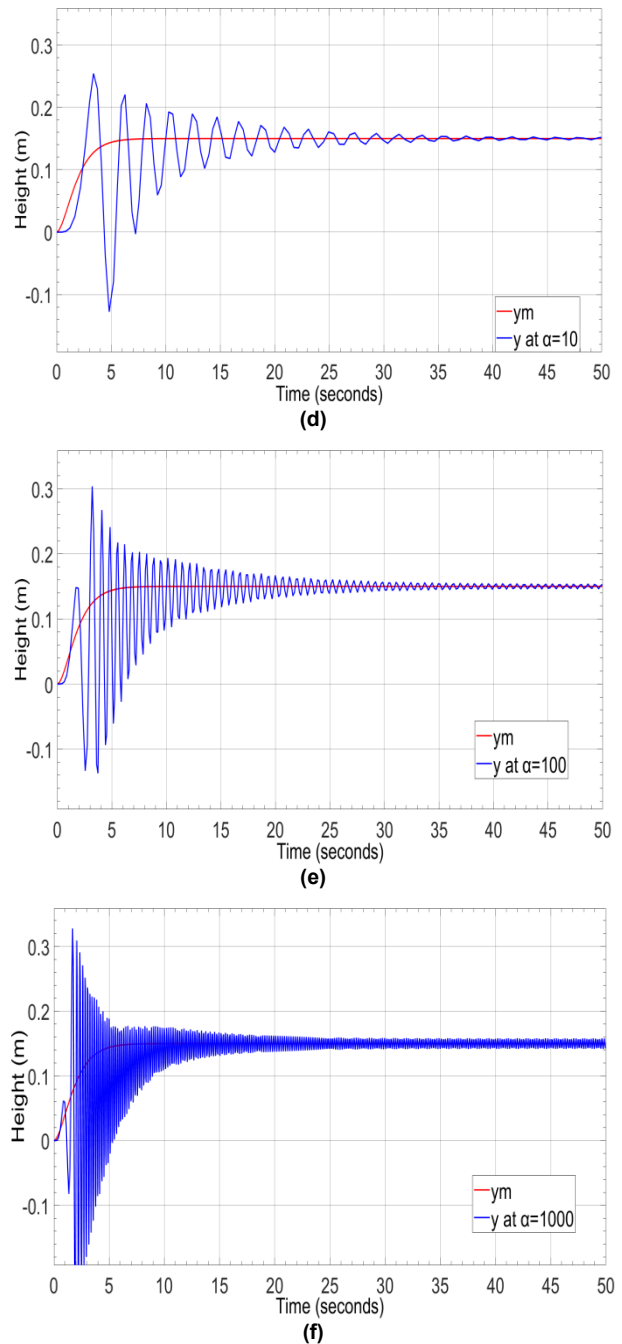
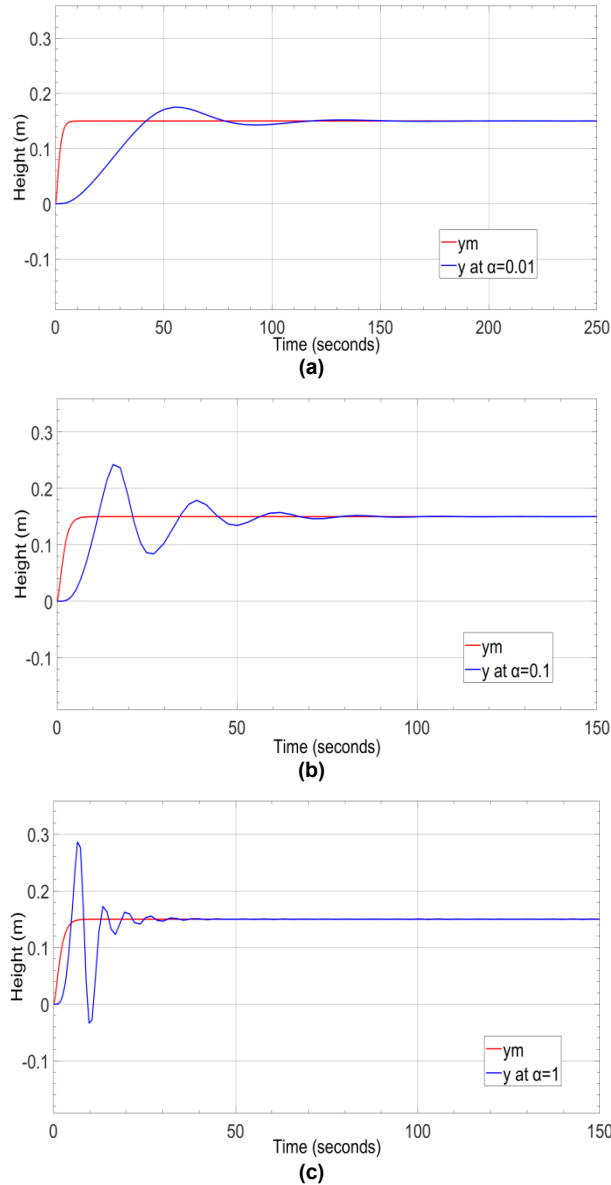


Figure 9: Simulation Results of Conventional MRAC with Lyapunov Rule at (a) $\alpha = 0.01$, (b) $\alpha = 0.1$, (c) $\alpha = 1$ (d), $\alpha = 10$, (e) $\alpha = 100$, (f) $\alpha = 1000$ for Interacting System

Table 3: Effect of Adaptation Gain on Interacting Coupled System with MRAC

Adaptation Law		MIT Rule			Lyapunov Rule			
Adaptation Gain	Overshoot (%)	Peak Time	Settling Time	Rise Time	Overshoot	Peak Time	Settling Time	Rise Time
0.01	15.1789	57.7352	109.4357	27.6781	16.6722	54.7833	108.6629	26.9792
0.1	58.2757	17.6557	76.5372	6.2898	61.5635	15.5906	6.0719	6.0719
1	174.4740	15.9238	79.7763	2.4037	91.1170	6.6162	27.2981	2.2286
2	549.6657	16.8916	27.6988	1.8798	89.7327	5.8287	28.6087	1.8340
10	-	-	-	-	67.4724	3.3769	36.3920	1.1898
100	-	-	-	-	98.2883	3.2040	49.6718	0.7653
1000	-	-	-	-	116.0202	1.6961	49.9601	1.0196

5.3 Influence of adaptation gain using modified MRAC on non-interacting system

Here, we have combined the control law of MRAC using the MIT and Lyapunov rule with the PD control law such that the system became stable and tracked the desired response. The gains of PD control law (Proportional and Derivative) have been fine-tuned using a trial-and-error process. In the process of turning the initial values of K_p and K_d have been considered. These values can be estimated based on knowledge of the system. In this process firstly, we adjusted the proportional gain (K_p) while keeping the derivative gain (K_d) at zero. Therefore, we have increased the value of K_p gradually until the system exhibits some overshoot in response to a step input. Once we have determined an appropriate value for K_p , we go for adding derivative gain (K_d). thereafter we increased K_d gradually while monitoring the system's response. The derivative action helped to reduce overshoot and improved settling time. The process of tuning is continued for the fine-tuning of K_p and K_d until we have achieved the desired control performance. Back and forth process has been adapted for adjusting the K_p and K_d while observing the system's response. The Performance metrics such as rise time, settling time, overshoot, and steady-state error are used to evaluate the controller's performance. After fine-tuning of controller, the gains in the PD controller have been found as $K_p = 5$, and $K_d = 2.5$.

In this section, the non-interacting system's time response is analyzed with modified MRAC by varying the adaptation gain α . The results of a simulation with the gain values 0.1, 1, 10, 50, 100, and 1000 have been shown in Figure 10 and Figure 11 with the MIT rule and in Figure 12 and Figure 13 with the Lyapunov rule for the reference model (29). Figure 10 and Figure 12 show the step response and Figure 11 and Figure 13 shows the error between the system and reference model.

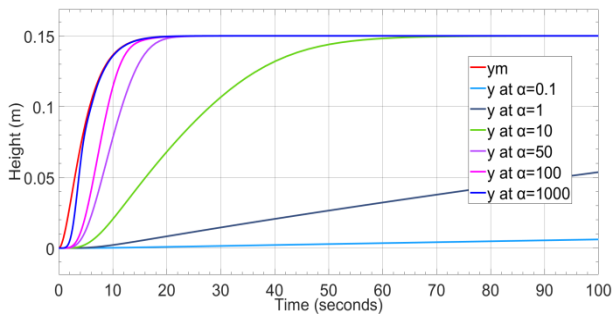


Figure 10: Simulation Results of Modified MRAC with MIT Rule for Various Values of Adaptation Gain (α)

Table 4: Effect of Adaptation Gain on Non-interacting Coupled System with Modified MRAC

Adaptation Law	MIT Rule				Lyapunov Rule				
	Adaptation Gain	Overshoot	Peak Time	Settling Time	Rise Time	Overshoot	Peak Time	Settling Time	Rise Time
	0.1	0	100	98.1122	74.1687	0	100	98.0735	75.6608
	1	0	100	97.9141	74.2825	0	100	97.8782	75.7652
	10	0	100	55.9504	33.0764	0	100	49.5992	29.9893
	50	3.0985×10^{-4}	40.2645	19.0918	10.2749	3.5358×10^{-7}	60.2578	16.2895	9.1021
	100	2.8645×10^{-6}	56.2578	15.7002	7.6511	1.0310×10^{-6}	56.2578	15.3303	7.5825
	1000	6.2089×10^{-7}	56.2578	14.6839	7.2591	4.3198×10^{-8}	62.2578	14.6562	8.1923

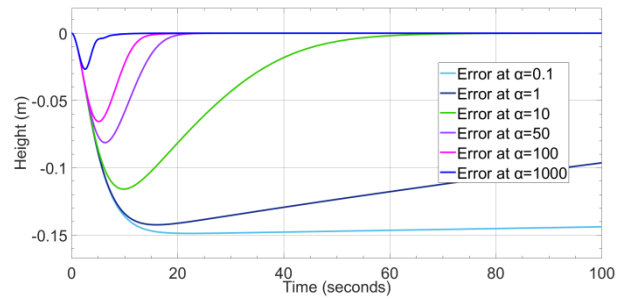


Figure 11: Error in Simulation Results of Modified MRAC with MIT Rule for Various Values of Adaptation Gain (α)

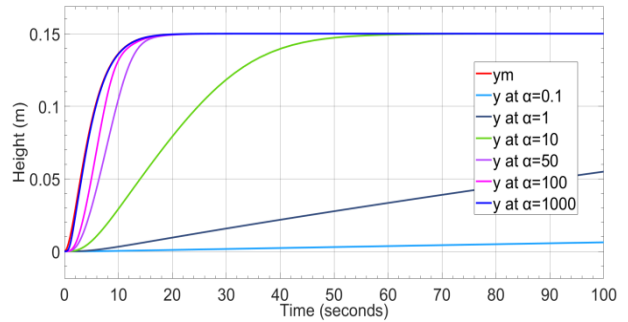


Figure 12: Simulation Results of Modified MRAC with Lyapunov Rule for Various Values of Adaptation Gain (α)

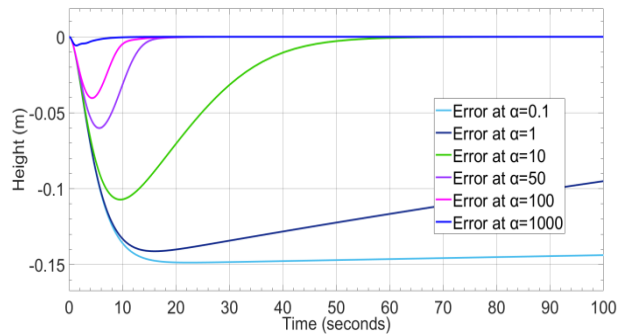


Figure 13: Error in Simulation Results of Modified MRAC with Lyapunov Rule for Various Values of Adaptation Gain (α)

From Figure 10 to Figure 13 and Table 4 it may be observed that by designing a modified MRAC the non-interacting coupled system became stable; overshoot and oscillations are removed for a higher range of adaptation gain values with both the MIT and Lyapunov rule. As the adaptation gain increased, the system response became fast as shown in Table 4. The performance of the system w.r.t. rise time, settling time, peak time, and peak overshoot are shown in Table 4 and it can also be observed that the Lyapunov rule gives a better result than the MIT rule.

Table 5: Effect of Adaptation Gain on Interacting Coupled System with Modified MRAC

Adaptation Law		MIT Rule			Lyapunov Rule			
Adaptation Gain	Overshoot (%)	Peak Time	Settling Time	Rise Time	Overshoot (%)	Peak Time	Settling Time	Rise Time
0.1	0	100	98.0268	77.3924	0	100	98.0113	77.9924
1	0	100	97.8189	77.4866	0	100	97.8045	78.0851
10	0	100	49.1994	30.4553	0	100	46.2938	29.0241
50	0.0244	16.5776	12.3970	7.1087	0.3200	12.5131	10.0591	6.1318
100	0.5227	9.3422	7.8884	4.3405	1.4257	7.3662	6.2925	3.7626
1000	0.0101	11.6411	6.0431	2.7872	0.0061	11.6411	5.8951	2.9689

5.4 Influence of adaptation gain using modified MRAC on interacting system

For the interacting system, we have also followed the same step as explained in section 5.3 for the tuning of the controller for a reliable response. After fine-tuning of controller, the gains in the PD controller have been found as $K_p = 5$, and $K_d = 2.5$. In this section, the interacting system's time response is analyzed with modified MRAC by varying the adaptation gain α . The results of a simulation with the gain values 0.1, 1, 10, 50, 100, and 1000 have been shown in Figure 14 and Figure 15 with the MIT rule and in Fig. 16 and Fig. 17 with the Lyapunov rule for the reference model (30). Figure 14 and Figure 16 show the step response and Figure 15 and Figure 17 shows the error between the system and reference model.

From Figure 14 to Figure 17 and Table 5 it may be observed that by designing a modified MRAC the interacting coupled system became stable; overshoot and oscillations are removed for a higher range of adaptation gain values with both the MIT and Lyapunov rule. As the adaptation gain increased, the system response became fast as shown in Table 5. The performance of the system w.r.t. rise time, settling time, peak time, and peak overshoot is shown in Table 5 and it can also be observed that the Lyapunov rule gives a better result than the MIT rule.

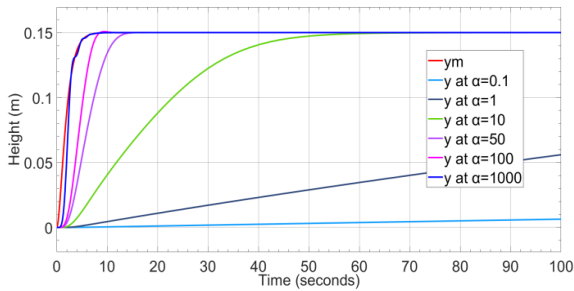


Figure 14: Simulation Results of Modified MRAC with MIT Rule for Various Values of Adaptation Gain (α)

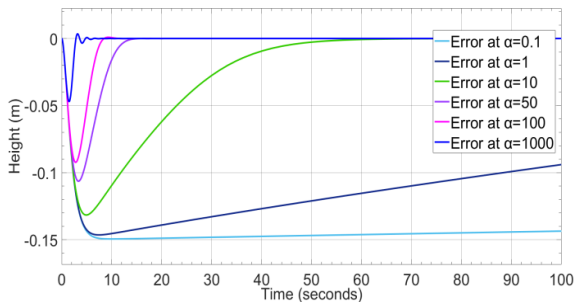


Figure 15: Error in Simulation Results of Modified MRAC with MIT Rule for Various Values of Adaptation Gain (α)

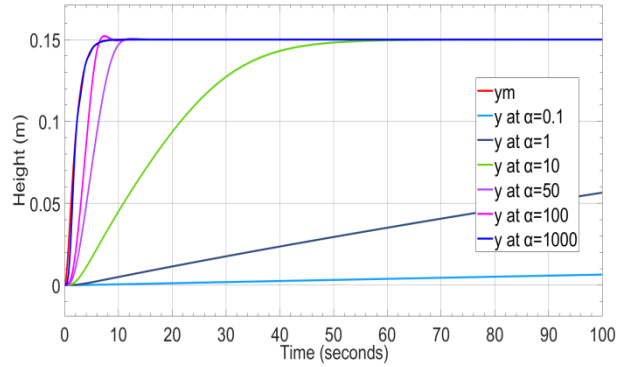


Figure 16: Simulation Results of Modified MRAC with Lyapunov Rule for Various Values of Adaptation Gain (α)

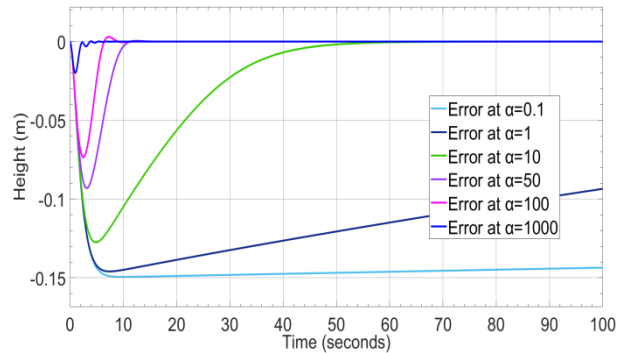


Figure 17: Error in Simulation Results of Modified MRAC with Lyapunov Rule for Various Values of Adaptation Gain (α)

6. CONCLUSIONS

In this paper, the traditional and Modified MRAC has been designed and simulated in MATLAB/Simulink for non-interacting and interacting two-tank coupled systems. The performance of normal MRAC systems has been compared to that of the modified systems, which is the MIT and Lyapunov technique superimposed with a PD controller in their design. The traditional and modified MRAC systems have demonstrated their adaptability and robustness in controlling non-interacting and interacting two-tank systems under various conditions. The adaptability gain parameter, a crucial component of these systems, has been shown to play a significant role in determining their performance. After carefully tuning this gain, we have achieved desirable control performance, with considerations for stability and convergence. Our investigation has highlighted the importance of a balanced adaptation gain. Too high gain can lead to overshooting and instability, while too low gain may result in sluggish responses and poor tracking performance. Achieving the right balance

is a critical aspect of the successful implementation of these adaptive control techniques. Through a rigorous examination of these adaptive control strategies, several key findings and insights have emerged. A significant contribution of this paper lies in the comparative stability and adaptation gain analysis of both controller designs. This comparative aspect is crucial for engineers and researchers seeking the most effective control strategy for similar dynamic systems.

The conventional MRAC is suitable only for a few lower adaptation gain values. It has been observed with the conventional MRAC that both the non-interacting and interacting two-tank liquid-level coupled systems have given very poor results with very high oscillations and reduced tracking performance. To overcome these problems the modified MRAC has been designed. The modified Model Reference Adaptive Controller has been designed for both the non-interacting and interacting two-tank coupled systems for a wide range of adaptation gain values. From the modified MRAC it has been observed that both the systems are stable and tracked the desired response for a wide range and very high gain values. With the increase in the values of adaptation gain, the system's performance is improving in terms of fast response, lesser settling time, a drop in overshoot, and well-tracking of desired response. There are no oscillations in the response for a large range and high gain values for both systems. It has also been observed that both systems have better responses with the Lyapunov rule as compared to the MIT rule.

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All the data generated or analyzed during this study are included in this manuscript and their citations and references are also provided. The first author conceptualized the proposed work and prepared the manuscript while the second and third authors supervised the work and reviewed the manuscript to improve the quality.

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All the authors declare that there is no conflict of interest regarding the publication of the paper.

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NOMENCLATURE

Q_{in}	inflow rate of liquid in tank-1
Q_1	outflow rate of liquid in tank-1
Q_2	outflow rate of liquid in tank-2
H_1	height of liquid in tank-1
H_2	height of liquid in tank-2
R_1	resistances to flow of liquid from tank-1
R_2	resistances to flow of liquid from tank-2
A_1	cross-sectional area of tank-1
A_2	cross-sectional area of tank-2
J	cost function
T_r	rise time
T_s	settling time
M_p	Peak overshoot
$a,$	constants

$a_m,$	
$b_1,$	
$b_2,$	
b_m	
K_p	proportional gain
K_d	derivative gain

Greek symbols

ρ	density of liquid
ζ	damping ratio
ω_n	natural frequency
$\theta_1,$	adaptation parameters
$\theta_2,$	
$\gamma,$	adaptation gains
α	

Superscripts

in	input
m	Reference model

Acronyms and Abbreviations

MIT	Massachusetts Institute of Technology
MRAC	Model Reference Adaptive Controller
PD	Proportional Derivative
NITT	Non-inverting Two-Tank
ITT	Inverting Two-Tank

ИСТРАЖИВАЊА О МОДИФИКОВАНОМ МРАЦ-У ЗАСНОВАНОМ НА ПРАВИЛИМА МИТ-А И ЉАПУНОВА ЗА СИСТЕМЕ СА ДВА РЕЗЕРВОАРА КОЈИ НИСУ У ИНТЕРАКЦИЈИ И МЕЋУСОБНО ДЕЛУЈУ

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Метода адаптивне контроле је техника која аутоматски и континуирано мери динамичке карактеристике постројења како би се направила поређење са његовом потребном производњом. Користи разлику између излаза постројења и референтног излаза за израчунавање прилагодљивих параметара система за одржавање оптималних перформанси без обзира на варијације система. На понашање правила прилагођавања значајно утиче вредност адаптационог добитка. Овај рад описује дизајн МИТ-а (Masachusetts Institute of Technology) и модификованог адаптивног контролера заснованог на Љапуновљевим правилима (МРАЦ) како би се стабилизовао процесни систем са два резервоара који није у интеракцији и који је у интеракцији. Такође је рађено истраживање варијације у појачању адаптације. У почетку, традиционални МРАЦ заснован на правилима МИТ-а и Љапунова је дизајниран да стабилизује систем за вучу-резервоар који није у интеракцији и који је у интеракцији и утврђено је да су оба система стабилна само за неколико вредности појачања прилагођавања. Да би се превазишао овај проблем, модификовани МРАЦ је планиран да стабилизује и побољша одзив система. Модификована МРАЦ шема је само ПД (пропор-

ционални дериват) контролер који је суперпонован на МРАЦ метод контроле. Сада са модификованим МРАЦ-ом, системи су стабилизовани и њихов одзив је побољшан за широк спектар побољшања прилагођавања. Такође је представљена компаративна анализа традиционалног и модификованог МРАЦ-а.

Анализа перформанси у смислу времена пораста, времена смиревања и прекорачења врха је спроведена упоређивањем резултата добијених за сва наведена правила са варијацијама у појачању адаптације, на МАТЛАБ/Симулинк платформи. Добијени резултати дају охрабрујуће резултате.