Analytical Determination of High-Feed Turning Procedures by the Application of Constructive Geometric Modeling

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The development of different machining procedures requires the exact mathematical description of those. This is especially important in finishing procedures, where the final geometry and surface characteristics of the parts are produced. In this paper, three high-feed turning procedures are analysed: skiving turning, tangential turning, and rotational turning. All three promise the creation is ground-like surfaces with low roughness characteristics, while reaching high values of productivity. The analytical determination of these procedures is carried out by the application of constructive tool geometric modeling. After the analysis of the kinematic and geometric relations of each procedure, the proper coordinate systems are defined. The transformational equations are determined, which describe the geometric boundary conditions and the movement of the workpiece and the tool. In the next step, the equation of motion is defined for the three studied procedure. Finally, the one variable equation of the cut surface section in the base plane is determined. Experiments were also carried out, which validated the achieved results.

Keywords: *analytical modelling, constructive tool geometry, cut surface, equation of motion, rotational turning, skiving turning, tangential turning.*

1. INTRODUCTION

The main machining procedure in the manufacturing of outer cylindrical surfaces is turning, where – in the traditional case – the tool performs an axial or radial feeding movement in addition to the rotating main movement of the workpiece [1]. The use of single-edged tools is an advantage because the size and shape of the surfaces are not limiting its application in the industry. However, the point-like contact of the tool nose and the adjacent section of the cutting edge is also a disadvantage because the tool wear is concentrated in this point thus limiting the tool life [2]. Furthermore, the heat load will be higher in this small section [3].

Due to the kinematic and geometric relations of the turning procedure, the surface-generating point of the cutting edge describe a helical curve on the workpiece machined surface, generating a periodically changing profile. Felhő and Varga showed, that increasing the feed rate leads to a more orderly generated surface [4]. There can be also problem with the roughness changes locally in surfaces with untraditional geometries, as shown by Matras et al. in their study of curvilinear sur–faces [5]. These can be limiting factors in the appli–cation of turning as finishing procedure, therefore there are many ongoing developments to lessen these effects. An important factor is the cutting time and tool wear rate, where both of these should be minimalised simul–taneously [6]. However, in turning of hard surfaces, higher heat is produced, which causes increased tool wear on the cutting tool [7]. In the application of hard tool materials different compositions should be used for different applications, which requires further optimi–zation [8]. Among lowering the cutting time there is the need for increasing the material removal rate in different machining procedures [9]. If the achievement of high material removal rate, low surface roughness and low tool wear rate is necessary, then the depth of cut is the most important factor, which is followed by the cutting speed and feed rate [10]. The search for reduction of the disadvantageous attributes of turning, while preserving its beneficial characteristics, can lead to that kind of procedure variants, which can increase the usability of the traditional lathe machine tools by the application of novel geometrical or kinematical rela–tions. The study of these procedures can result that kind of solutions which means small investment; however, higher productivity, better surface quality, and higher energy efficiency are achievable.

In the last decades, the main development directions of the machine tools were the following as shown in the work of Byrne et al. [11]: powerful high frequency work spindles, innovative drive systems, roller or ball type linear guideways, lightweight materials and const–ructions, innovative kinematic concepts, sensors and actuators providing process stability. These directions can be seen in the field of development of turning procedures. This study is focusing on those novel turning procedures, which can provide high productivity while also capable of producing surfaces with high quality. To achieve these goals, special geometric or kinematic relations should be applied between the workpiece and cutting tool. The former is manipulated in the simplest way by altering the cutting edge

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geometry. The edge design has a great impact on the machining process [12], since the shape, size and position of the cutting edge affect the micro- and microgeometry of the generated surface, the cross-sectional area of the chip and the cutting forces [1]. Therefore, the modification of the geometrical and kinematical relations in already existing procedures can lead to good results with small alterations [13]. In tra–ditional turning, better surface roughness can be obtained at high cutting speed and lower feed rate [14]. However, by increasing the inclination angle and choo–sing a proper cutting tool, the surface quality can be improved even in higher feed rates [15], while the tool life and the friction coefficient between the tool and workpiece will be also better [16]. The ratio of the cutting force components can be altered in an advan–tageous way [17].The change of the kinematic relations means the alteration of the usually applied longitudinal (axial) feeding motion. From the other two main feeding directions, the radial movement of the tool limits the machinable length of the workpiece and the cutting tool, while the tangential movement (with the possible combination with axial movement) has a wider appli–cation range.

From the several turning procedures, where higher inclination angle (λ_s> 10°) is usable and applied, three variants became the focus of my studiesdue to their similar characteristics and interesting attributes. In this paper the following high-feed turning procedures are the following procedures are presented and analysed: skiving turning [15], tangential turning [18], and rotational turning [19].

Skiving turning (Figure 1) is the most similar to traditional turning among the three procedures. In this case a longitudinal feed is applied, but the tool position is different due to the relatively high inclination angle. It can be applied on traditional turning machine tools; however, rather high run-in and run-out sections must be kept on the surface.

The applied cutting tool is very similar in tangential turning to the previously described skiving turning, as it can be seen in Figure 2. However, as the name of the procedure suggest, in this case the feed rate is tangential directional to the workpiece outer. To achieve this kind of feeding movement, a particular lathe is needed, where tool movements can be programmed in all three directions (since on the traditional lathes the feeding movement can only be programmed in one plane). Still, this kinematic relation makes it possible to machine such outer cylindrical surfaces, where there is no space for the tool run-in and run-out.

The third studied high-feed turning variant is rotational turning. The basic layout can be seen in Figure 3. The special characteristic of this procedure is its complex turning tooland unique feeding movement. The cutting edge describes a helical curve, which pitch angle represents the inclination angle. The material removal is done by the rotational movement of the tool. It is important to mention, that the revolution speed of the workpiece must be significantly higher than the revolutions of the tool, otherwise discontinuous chip-removal will occur, and the process will be closer to milling, than turning.

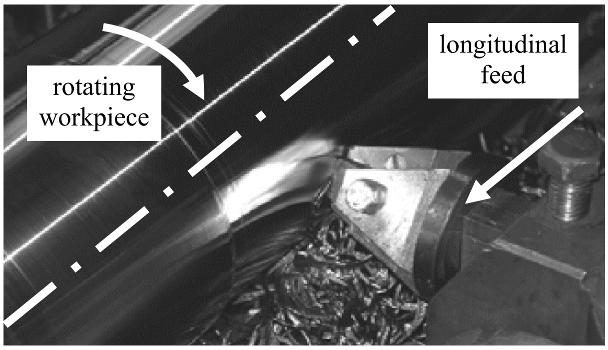

Figure 1. Skiving turning [15]

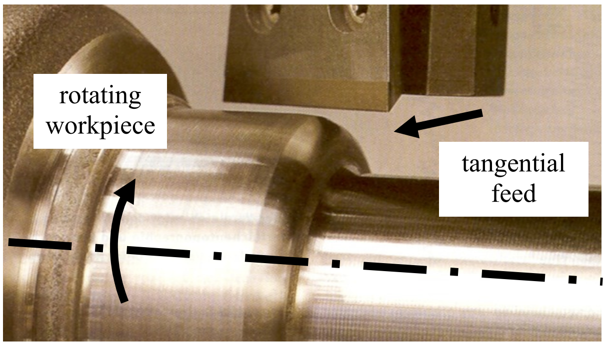

Figure 2. Tangential turning [20]

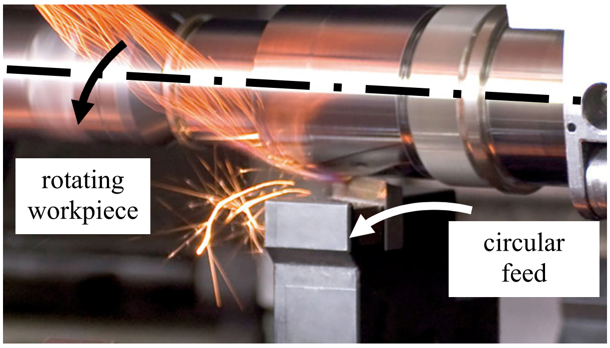

Figure 3. Rotational turning [21]

The common characteristic of these procedure variants is the applied relatively high value of the inclination angle. The projection of the cutting edge in the base plane becomes different from the usually common shape in traditional turning, which results in a wider and shallower shape of the roughness profile valleys. In theory, this should lead lower surface rough–ness at the same feed rate compared with traditional turning [16]. The special kinematic relations in tan–gential and rotational turning (the tangential and the circular feeding movement) enable the application of high feeds. To thoroughly study these procedures, a proper mathematical description of the material removal process is necessary. In this paper kinematical model of these high-feed turning procedures are determined and the analytical description of the cut surface is also worked out. The analysis of the theoretical values of the machined surface roughness, the uncut chip cross-section, machining time, and efficiency becomes pos–sible by the determination of the kinematical model and the formulation of the equations of motion of these procedures. The results of this paper are the basis of the deeper analysis of skiving turning, tangential turning and rotational turning.

2. CONSTRUCTIVE GEOMETRIC MODELING

A machining procedure can be studied experimentally and analytically in many ways; therefore, the first step is the picking of the right method to establish the basis for the further theoretical studies. In the following chapter I present the constructive geometric modeling, which is widely used by the constructive design and techno–logical planning as well.

The geometrical and analytical apparatus of mat–hematics are applied in a long time for the description and determination of complex connection cases both in the designing of machine parts and the planning of manufacturing processes. The work of Litvin should be considered a foundational work in the international literature for the determination of workpiece surfaces [22], by which complex surfaces can be easily deter–mined for the practiceby the application of differential-geometry and kinematical method.

The kinematic method deepened further also by Perepelitsa enhancing the mathematical apparatus of multi parameter mappings of affine space [23], which become capable of the mathematic-analytical determi–nation of shaped cutting tools [24] and the analysis of the geometric relations during cutting [25]. The combi–ned description of the cutting edge movement and the cutting speed vector makes it possible to study the angles [26] and other edge geometrical elements [27].

Hsieh showed that by the application of constructive geometric modeling, the cutting capability of the tools can be analysed [28]. By the application of kinematic modeling, the calculation and correction of the geo–metrical errors of 5D gear profile grinders [29] and machining centres [30] are possible. Dudas created and improved a mathematical model for the geometric description of worm gear drives [31-32]. Bodzás applied the presented kinematical method to create a mathe–matical model for the technological analysis, researches and designing milling technology for the spline shaft [33], and to modify the position of the tool in comparison with the workpiece in face milling [34]. Balajti et al. developed a function description of the ad–justment parameters of the post-sharpened hob to support production accuracy of the mated gear [35]. The study of undercuts [36] is also possible with the constructive geometric modeling, which undercuts can be mapped for example in the case of a helical drive with a circular profile in the axial section by taking into account the mutual influence of the relationships of the interacting mathematical parameters [37]. Balajti also developed a new mathematical procedure for positi–oning the CCD cameras with mathematical precision to ensure that the cutting-edge curve can be reconstructed from digitized images for wear measurement [38].

It can also be seen from this brief review of the international literature, that the constructive geometric modeling completed with the kinematic method is capable in the modeling and determination of complex geometric relations. Its main characteristic is the specification of the surfaces with vector equations. The essence of the method is the determination of appropriate coordinate system, in which the different geometric objects can be given by these vector equ–ations. Transformational equations must be written between these coordinate systems, which equations contains the kinematic motion of the described geo–metric objects. The solving of the resulting equations under different boundary conditions will lead to the exact mathematical description of the studied subject. Therefore, in the analysis of machining procedures, the following steps must be followed:

1) geometrical and kinematical analysis,
2) definition of the needed coordinate systems,
3) characterization of the transformational equations,
4) specifying the vector equation of the cutting edge,
5) determination of the equation of motion.

In this paper, these tasks are performed to deter–minate the machined surface analytically for skiving turning, tangential turning, and rotational turning. The mathematical description of the produced workpiece is the basis of the future studies of these high-feed turning procedures.

3. GEOMETRIC AND KINEMATIC ANALYSIS OF THE STUDIED PROCEDURES

The basis of the proper application of constructive geometric modeling in the manufacturing science is the throughout analysis of the procedure to be described to reveal the geometric and kinematic relations between the cutting tool and the workpiece. Therefore, the first step in this study is the examination of the procedures and the determination of the markings of the later to be used geometric and kinematic parameters. In this chapter the skiving turning, tangential turning and rotational turning is analysed.

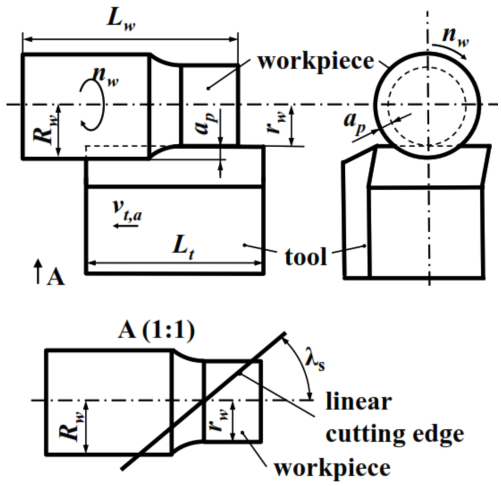

Figure 4. Schematic figure of skiving turning

Skiving turning is described according to Figure 4. Here it can be seen, that the kinematic relations are similar to traditional longitudinal turning: the workpiece does an axisymmetric rotary motion (n_w), which results the main cutting movement; while the cutting tool moves on a linear path parallel to the axis of the work–piece ($v_{t,a}$), which results the secondary feeding move–ment. The cutting tool consist of a working part, which is usually a turning insert, and a tool holder. The geometry of the cutting edge is a straight line which forms a pair of skew lines with the symmetry axis of the workpiece.

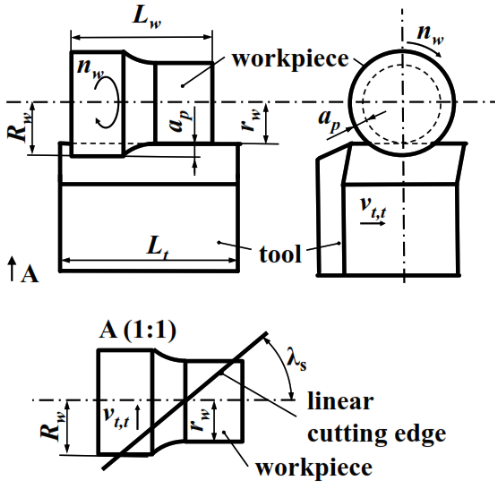

Figure 5. Schematic figure of tangential turning

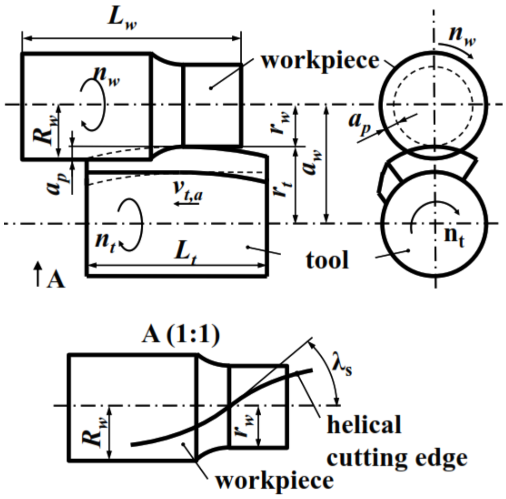

Figure 6. Schematic figure of rotational turning

The angle between the cutting edge and the base plane is the inclination angle (λ_s). The extent of this angle is needed to achieve proper chip removal and to lessen harmful effects like vibration, which can be expected with machining with parallel edged cutting tool. The length of the workpiece (L_w) can be lower or higher than the length of the tool projection on the base plane (L_t), however enough run-out length is needed following the end of the workpiece to machine the complete surface. The figure also shows the radius of the to be machined surface (R_w), the radius of the machined surface (r_w), and the depth of cut (a_p), where the following formula can be written: $a_p = R_w - r_w$.

Figure 5 shows the kinematic and geometric relations of tangential turning. The most important difference from skiving turning is the direction of the feeding movement. Here the cutting edge moves on a path which is tangential to the cylinder defined by the machined surface radius (r_w), which results in the feeding movement ($v_{t,t}$). The cutting movement is also resulted from the rotary movement of the workpiece (n_w). A significant limit in the application of turning with solely tangential feeding movement is the length of the projection of the cutting edge in the base plane (L_t), which must be higher than the length of the workpiece (L_w). Small runouts are also necessary on both ends of the workpiece. The cutting edge is not parallel to the symmetry axis of the workpiece due to similar reasons described before.

The kinematic and geometric relations of rotational turning is described based on Figure 6.The main cutting movement is done by the rotations of the workpiece (n_w) in this procedure as well. The circular feed is resulted from the rotation of the helical cutting edge (n_t), which axis is parallel to the symmetry axis of the workpiece. The cutting edge moves into the workpiece material during its slow, rotary movement, which causes the material removal. The cross-sectional area of the chip becomes constant when the cutting edge reaches the plane defined by the axis of the helix and the symmetry axis of the workpiece. The continuous chip removal starts at this point which is characteristic for turning procedures. If the length of the workpiece (L_w) is smaller than the length of the projection of the cutting edge onto the base plane (L_t), then the previously described circular feed is enough to machine the complete surface of the workpiece. However, in case L_t is smaller than L_w, this circular feed is not enough. The solution is the application of additional axial feed rate ($v_{t,a}$), which must be precisely synchronised with the rotary feeding movement in order to machine the chosen outer cylindrical surface. The depth of cut (a_p) can be adjusted by changing the axis distance (a_w) between the workpiece and the cutting tool.

Although these three high-feed procedures apply a different kind of cutting tool or feeding movement than what is usual in longitudinal turning in the machining of outer cylindrical surfaces, it can be concluded from the above analysis, that this variants also produces a constant cross-sectional chip during the continuous material removal in the most defining part of the machining process (only exemptions are the run-in and run-out phases, which can be neglected if the length of the workpiece is high enough). However, due to the special kinematic and geometric relationships between the workpiece and the cutting tool, surfaces with better quality can be expected than in traditional turning.

4. DEFINITION OF COORDINATE SYSTEMS

The next step – after the analysis of the procedures – in the constructive geometric modeling is the definition of the coordinate systems. In this mathematical description the vector equation of the cutting edge is needed to be determined, which will be transformed into the vector equation of the cut surface. These transformations are done by the determined transformation-equations between the defined coordinate systems.

To start with, the number of coordinate systems should be decided. The reviewing of the relevant literature pointed out that it is practical to define 4 coordinate systems (CS). The first will be the initial CS, in which the vector equation of the cutting edge should be given. This will be linked to the tool, moving simultaneously during the machining, therefore it is called "Tool Moving CS" ($K_{t,m}$). The second CS is linked to the machine and the transformation of the Tool Moving CS into the current the previous and the current will represent the feeding movement of the tool. This is called "Tool Standing CS" ($K_{t,s}$).

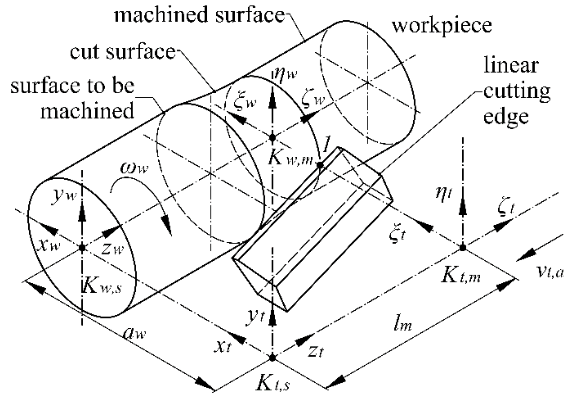

Figure 7. Coordinate systems and kinematic relations in skiving turning

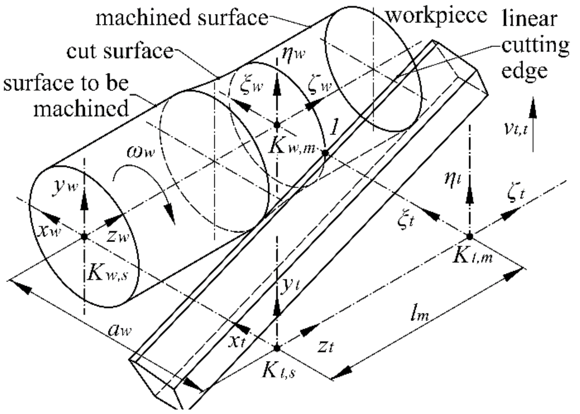

Figure 8. Coordinate systems and kinematic relations in tangential turning

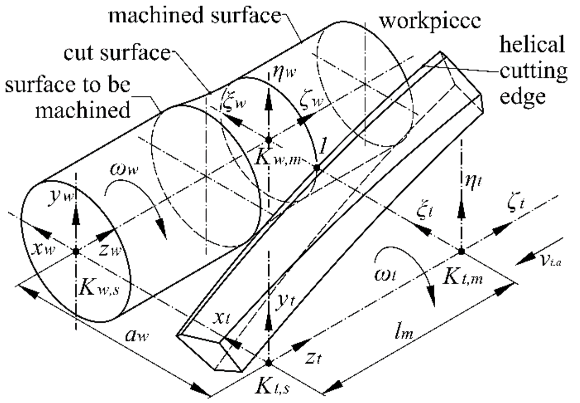

Figure 9. Coordinate systems and kinematic relations in rotational turning

The third coordinate system will be linked to the machine again, however it is connected to the work–piece null point, therefore it is called "Workpiece Standing CS" ($K_{w,s}$). The transformation between the two standing coordinate systems will represent the radial distance between the axes of the workpiece and the tool (a_w). The fourth coordinate system is linked to the workpiece, however it is rotating with it, therefore it is called "Workpiece Moving CS" ($K_{w,m}$). The trans–formation between the standing and moving coordinate systems of the workpiece will represent the rotary motion of the machined part. In summary, two CS are linked to the workpiece and two CS are linked to the tool, from which 1-1 will be moving with its linked geometric object, while 1-1 will be connected to the machine. The resulted coordinate systems can be seen in Figure 7-9 for skiving turning, tangential turning, and rotational turning respectively. These figures also present the movements of the workpiece and the tool among the defined CS. The rotation of the workpiece is given by the angular speed (ω_w) in all three cases, due to its better usability in the mathematical deduction. The circular feed is also given by its angular speed value (ω_t) in rotational turning.

After the correct starting points of the coordinate systems are chosen, further considerations should be taken, which are either needed for the analytical determination or helps the mathematical deduction.

ξ_i, η_i, ζ_i Greek letters represent the axes of a moving CS, while x_i, y_i, z_i Latin letters symbolize the axes of a standing CS, where $i = w$– "workpiece" and t– "tool". The ζ_i and z_i axes are parallel with the symmetry axis of the workpiece. The base plane will be described by the $[x_i; z_i]$ and $[\xi_i; \zeta_i]$ planes. The axis in a given CS should form a right-wing coordinate system (which defines the position of η_i and y_i). The ξ_i coordinate axis of the tool and workpiece moving CS go through the surface generating point (marked by "*1*").

5. TRANSFORMATIONAL EQUATIONS

Afterwards the coordinate systems are established and well defined for the studied high-feeding turning proce–dures, the mathematical analysis continue with the next step. In constructive geometric modeling, the transfor–mation between the coordinate systems must be defined in order to reach our goal. This transformation method is explained based on Figure 10, which present a general transformation between two coordinate systems.

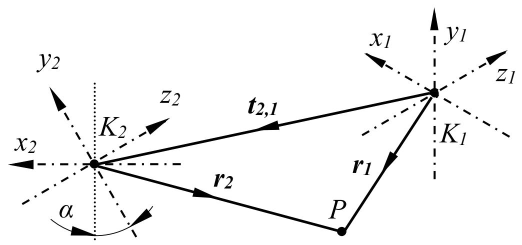

Figure 10. Meaning of the transformational equation

Let $\boldsymbol{r_1}$ vector – which refers to the P point – be given in K_1 coordinate system, and its transformed $\boldsymbol{r_2}$ vector is sought in the K_2 coordinate system. In this case the lattervector can be written as:

$$r_2 = R_{2,1} r_1 + t_{2,1} \tag{1}$$

where $\boldsymbol{R_{2,1}}$ is a matrix representing the rotation between the two coordinate system, and $\boldsymbol{t_{2,1}}$ is a vector indicating the translational movement between the origin points of K_1 and K_2. There could be three possible arrangement of variables in the rotational matrix according to the axis of rotation. For example, the α degree rotation around the z_1 axis will be written as(where $\boldsymbol{R_{2,1}}^z$ is the marking of the matrix of rotation around the z axis):

$$R_{2,1}^z = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The translation vector takes the following for, in case of a value movement in x direction, b value movement in y direction and c value movement in z direction:

$$t_{2,1} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{3}$$

After the basics have been covered, the study gets to the mathematical description of the transformational equation. My goal is to give combined equations for skiving turning, tangential turning, and rotational tur–ning. The mathematical description of each procedure can be deducted by the proper changing of the variables.

Firstly, the transformation between the moving and standing coordinate systems of the tool is needed to be defined. As it can be sawn in the previously done kinematical analysis, 3 kinds of movement can occur in these cases. The tool makes an axial movement in skiving turning, tangential motion is presented in tangential turning, and rotational and axial movement occur in rotational turning. In order to describe these movement mathematically, a rotational matrix and a translational vector is needed:

$$r_{ts} = R_{ts,tm} r_{tm} + t_{ts,tm} \tag{4}$$

where $\boldsymbol{r_{tm}}$ is the vector equation of the cutting edge in the moving CS of the tool, $\boldsymbol{r_{ts}}$ is the resulted vector equation in the standing CS of the tool; $\boldsymbol{R_{ts,tm}}$ is the rotational matrix, and $\boldsymbol{t_{ts,tm}}$ is the translational vector between the moving and standing CS of the tool. The rotation around the z_t axis will be given by the multiplication of the angular speed of the tool (ω_t) and the time parameter ($t \in \mathbb{R}$), which results in the angle of rotation. Two kind of linear motion will be given in the translation vector: the y_t directional movement is given by the multiplication of the tangential feed rate ($v_{t,t}$) and the time parameter, while the z_t directional movement is given by the multiplication of the axial feed rate ($v_{t,a}$) and the time parameter. Due to the initial l_m distance between the two coordinate systems, this value is also included in the latter movement. To sum up the above, (4) takes the following form:

$$r_{ts} = \begin{bmatrix} \cos\omega_w t & -\sin\omega_w t & 0 \\ \sin\omega_w t & \cos\omega_w t & 0 \\ 0 & 0 & 1 \end{bmatrix} r_{tm} + \begin{bmatrix} 0 \\ v_{t,t} t \\ l_m - v_{t,a} t \end{bmatrix} \tag{5}$$

The second transformation happens between the sta–nding coordinate systems of the tool and the workpiece. There is no kinematical movement involved between these two CS in the mathematical description, however, there is a geometrical translation which represents the radial distance between the z_i axis of the workpiece and the tool in the x_i direction (where $i = w$ and t). This transformation can be written in the form of the following equation:

$$r_{ws} = R_{ws,ts} r_{ts} + t_{ws,ts} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} r_{ts} + \begin{bmatrix} -a_w \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

where $\boldsymbol{r_{ws}}$ is the resulted vector equation in the standing CS of the workpiece; $\boldsymbol{R_{ws,ts}}$ is the rotational matrix, and $\boldsymbol{t_{ws,ts}}$ is the translational vector between the standing CS of the tool and the workpiece.

The third and final transformation between the stan–ding and moving coordinate systems of the workpiece represents the rotational movement of the machined part. This contains two mathematical operators. One of them is a rotational matrix, where the rotary motion around the z_w axis will be given by the multiplication of the angular speed of the workpiece (ω_w) and the time parameter (t), which results in the angle of rotation.

The other is a translational vector, where the initial l_m distance is given in the opposite direction than before. It must be added to the deduction, that this initial distance is not needed in the mathematical description, it also works with the same origin points of the moving and standing CS. However, the involvement of l_m value helps the understanding of the explanation, and it also helps in the generalization of the constructive modeling method.

In summary, the transformation between the moving and standing coordinate systems of the workpiece are sought in the following form:

$$r_{wm} = R_{wm,ws} r_{ws} + t_{wm,ws} \tag{7}$$

where $\boldsymbol{r_{wm}}$ is the resulted vector equation in the moving CS of the workpiece; $\boldsymbol{R_{wm,ws}}$ is the rotational matrix, and $\boldsymbol{t_{wm,ws}}$ is the translational vector between the standing and moving CS of the workpiece. This equation takes the following form with the substitution of the above-described variable parameters:

$$r_{wm} = \begin{bmatrix} \cos\omega_t t & \sin\omega_t t & 0 \\ -\sin\omega_t t & \cos\omega_t & 0 \\ 0 & 0 & 1 \end{bmatrix} r_{ws} + \begin{bmatrix} 0 \\ 0 \\ -l_m \end{bmatrix} \tag{8}$$

The defined three transformation can be summa–rized in one equation, which will represent the trans–formation between the moving coordinate systems of the tool and the workpiece. This can be calculated by solving the following equation:

$$r_{wm} = R_{wm,ws}\left[R_{ws,ts}\left(R_{ts,tm} r_{tm} + t_{ts,tm}\right) + t_{ws,ts}\right] + \\ + t_{wm,ws} \tag{9}$$

The following form is resulted by settling (9):

$$r_{wm} = \left[R_{wm,ws} R_{ws,ts} R_{ts,tm}\right] r_{sm} + \\ + \left[R_{wm,ws} R_{ws,ts} R_{ts,tm} + R_{wm,ws} R_{ws,ts} + t_{wm,ws}\right] \tag{10}$$

The final resultant transformational equation can be also written in the following form, where one combined rotational matrix and one combined translational vector is presented:

$$r_{wm} = R_{wm,tm} r_{tm} + t_{wm,tm} \tag{11}$$

where $\boldsymbol{R_{wm,tm}}$ is the resultant rotational matrix, and $\boldsymbol{t_{wm,tm}}$ is the resultant translational vector between the moving CS of the tool and the workpiece.

The resultant rotational vector will take the follo–wing form after the mathematical conversions:

$$r_{wm}=\begin{bmatrix}\cos(\omega_t t-w_w t) & \sin(\omega_t t-w_w t) & 0\\ \sin(\omega_t t-w_w t) & \cos(\omega_t t-w_w t) & 0\\ 0 & 0 & 1\end{bmatrix} r_{tm}+ \\ +\begin{bmatrix} v_{t,t}t\sin\omega_w t-a_w\cos\omega_w t\\ v_{t,t}t\cos\omega_w t+a_w\sin\omega_w t\\ -v_{t,a}t\end{bmatrix} \tag{12}$$

The transformational equation for the three studied high-feed turning procedure can be derived from (12) by the substitution of the following:

a) skiving turning: $a_w = r_w$, $\omega_t = 0$, and $v_{t,t} = 0$,
b) tangential turning: $a_w = r_w$, $\omega_t = 0$, and $v_{t,a} = 0$,
c) rotational turning: $a_w = r_t + r_w$ and $v_{t,t} = 0$.

6. VECTOR EQUATIONS OF THE CUTTING EDGES

To finalise the mathematical description of the studied high-feed turning procedures, the next step is the specification of the vector equations of the cutting edges. Two types of cutting edge geometry is presented in this study: linear edge in case of skiving and tangential turning, and helical cutting edge in rotational turning. The deduction will be presented based on Figure 11, which shows the two types of curves which is considered in this study. The linear cutting edge represented by curve (a), which intersects with the workpiece in 1^a point; and the helical cutting edge is represented by curve (b) and 1^b intersection point.

At first, the vector equation of the linear cutting edge will be defined, which shown as curve (a) in Figure 11. Let $\boldsymbol{a}$ is the vector to a point through which the line passing and $\boldsymbol{b}$ is the direction vector of the line. The general parametric equation of a line ($\boldsymbol{r}_{line}(p)$) can be written as follows where each coordinate of a point on the line is given by a function of $p\in\mathbb{R}$:

$$r_{line}(p)=a+pb \tag{13}$$

As it can be seen in Figure 11, the line of the cutting edge goes through point 1^a, which is the origin of the $K_{t,m}$ coordinate system. The direction vector can be written in function of the inclination angle. Considering the above written, the two vectors can be defined as:

$$a=\begin{bmatrix}0\\0\\0\end{bmatrix}, b=\begin{bmatrix}0\\ ctg(\lambda_t)\\ 1\end{bmatrix} \tag{14}$$

The substitution of (14) into (13)leads to the p parametric vector equation of the linear cutting edge in the moving coordinate system of the tool ($\boldsymbol{r}_{tm,linear}(p)$), which result can be seen in (15).

$$r_{tm,linear}(p)=\begin{bmatrix}0\\ p\,ctg(\lambda_t)\\ p\end{bmatrix} \tag{15}$$

The second deduction is the definition of the para–metric vector equation of the cutting edge in rotational turning. The to be defined helical curve (b) can be seen in Figure 11. The sketch shows a helix with ζ_t axis which passes the ξ_t coordinate axis through point 1^bin which point the dashed line present the tangent to the curve at this point. This ζ_t directional right-handed helix has a radius of r_t and a pitch of m_t; additionally, the angle between the before mentioned tangent line and the ζ_t axis is λ_s, which is the inclination angle. The general parametric equation of this particular helix with p parameter is written as the following:

$$\begin{cases}\xi_t = r_t\cos(p)\\ \eta_t = r_t\sin(p)\ ,\ p\in\mathbb{R}\\ \zeta_t=\dfrac{1}{2\pi}m_t(p)\end{cases} \tag{16}$$

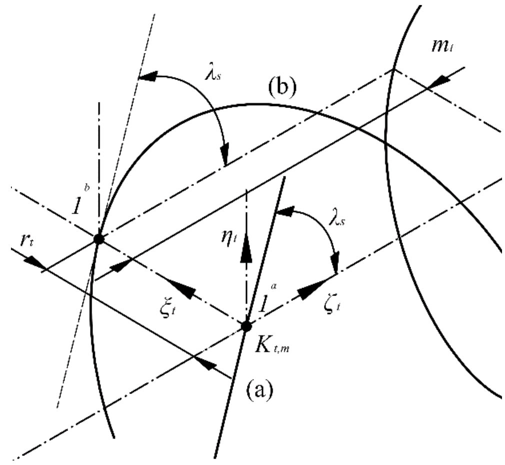

Figure 11. Geometrical representation of the cutting edges: (a) linear, (b) helical curve

The following expression can be written for the pitch, tool radius, and inclinational angle by the unfol–ding of the helix onto a plane:

$$tg(\lambda_t)=\frac{2\pi r_t}{m_t}\rightarrow\frac{m_t}{2\pi}=r_t ctg(\lambda_t) \tag{17}$$

In conclusion, the vector equation of the helical cutting edge in the moving coordinate system of the tool ($\boldsymbol{r}_{tm,helix}(p)$) can be written as the following in function of the tool radius, inclinational angle, and p parameter:

$$r_{tm,helix}(p)=\begin{bmatrix}r_t\cos(p)\\ r_t\sin(p)\\ p\,r_t ctg(\lambda_t)\end{bmatrix} \tag{18}$$

7. EQUATION OF MOTION OF THE CUTTING EDGE

The next step in this study is the determination of the two-parametric vector equations of motion of the cutting edge, which describe the machined surface. The three different high-feed turning procedures will have different equations due to the non-identical kinematic relations and geometries of the cutting edge.

The equation, describing the movement of the cutting edge in skiving turning, is determined, by the substitution of (15) into (12). During the solution, the following statements are taken into account: $a_w = r_w$, $\omega_t = 0$, and $v_{t,t} = 0$. The resulted two-parametric vector equation of the cut surface can be seen in (19).

$$r_{wm,skiving}(p,t)=\begin{bmatrix} pctg(\lambda_s)\sin(\omega_w t)-r_w\cos(\omega_w t) \\ pctg(\lambda_s)\cos(\omega_w t)+r_w\sin(\omega_w t) \\ p-v_{t,a}t \end{bmatrix} \tag{19}$$

$$r_{wm,tangential}(p,t)=\begin{bmatrix} pctg(\lambda_s)\sin(\omega_w t)-r_w\cos(\omega_w t)+v_{t,t}t\sin(\omega_w t) \\ pctg(\lambda_s)\cos(\omega_w t)+r_w\sin(\omega_w t)+v_{t,t}t\cos(\omega_w t) \\ p \end{bmatrix} \tag{20}$$

$$r_{wm,rotational}(p,t)=\begin{bmatrix} r_t\left[\cos(p+\omega_t t-\omega_w t)-\cos(\omega_w t)\right]-r_w\cos(\omega_w t) \\ r_t\left[\sin(p+\omega_t t-\omega_w t)+\sin(\omega_w t)\right]+r_w\sin(\omega_w t) \\ r_t pctg(\lambda_s)-v_{t,a}t \end{bmatrix} \tag{21}$$

The vector equation for tangential turning can be determined by the substitution of (15) into (12) as before. However, the specified values of given constants are different from skiving turning. In this case, the following declarations must be fulfilled during the determination: $a_w = r_w$, $\omega_t = 0$, and $v_{t,a} = 0$. As a result, the two-parametric vector equation of the machined surface in tangential turning is shown in (20).

The machined surface can be determined for rota–tional turning by the substitution of (18) into (12), taking into account the 0 value of the additional tan–gential feed rate ($v_{t,t} = 0$). The resulted two-parametric vector equation is presented in (21).

8. DETERMINATION OF THE MACHINED SURFACE

Following the analytical determination of the machined surfaces, it is practical to define the sectional view of them in the base plane, which is the [ξ_w; ζ_w] plane. It is either necessary or practical to among other things, determine the geometric characteristics of the chip cross-section and the theoretical values of the roughness parameters. The sectional view of the cut surfaces is derived from the two-parameter equation of motion of the cutting edge–(19-21). The base plane is passing through the inspected point of the tool (point 1), at which point the base plane is perpendicular to the cutting speed (the peripheral speed resulting from the rotating movement of the workpiece).

Firstly, the equation is deducted for skiving turning. In the exact cases of this study, the equation of the section of the machined surface is needed to be determined in the plane stretched by the ξ_w and ζ_w axes in the $K_{w,m}$ coordinate system. This is true, when the η_w component of the vector equation in (19)is equal to zero, which can be written as the following:

$$\eta_w = pctg(\lambda_s)\cos(\omega_w t)+r_w\sin(\omega_w t)=0 \tag{22}$$

The p parameter in function of the t parameter can be defined as:

$$p(t)=-\frac{\sin(\omega_w t)r_w}{\cos(\omega_w t)ctg(\lambda_s)} \tag{23}$$

Equation 23 means that how much the value of p should be for any given value of t so that the defined point will be in the base plane. The next step is the substitution of (23) into (19), which results in the one parametric system of equations of the cut surface in the base plane, as it can be seen in (24) and (25).

$$\xi_w(t)=-\frac{r_w}{\cos(\omega_w t)} \tag{24}$$

$$\zeta_w(t)=-\frac{\sin(\omega_w t)r_w}{\cos(\omega_w t)ctg(\lambda_s)}-v_{t,a}t \tag{25}$$

From the system of equations of the machined surface, the machined surface can be derived, therefore, for example, the chip cross-section and the theoretical roughness can also be characterized.

Still, it is more appropriate to use the one-variable function form $\xi_w(\zeta_w)$ instead of the one-parametric system of equations form of $\xi_w(t)$, $\zeta_w(t)$, for example, for the mathematical-analytical determination of the chip width (which needs an integration when calculating the arc length).

The expression of the parameter t from (24) and (25) does not give a well-handled relation even in technical practice since the parameter t is also contained in mul–tiple complex functions. Therefore, an approxima–tion is necessary.

For the approximation of the Equations, Taylor series was chosen among the available options, since it can approximate trigonometric and hyperbolic functions with negligible error. By increasing the order, the degree of error decreases, but the complexity of the approximation increases. In this study the second order expansion was chosen, which can be increased, if it is needed in the practical use. The result of expanding of (25) into a second-order Taylor series in the point of t = 0 is shown in (26).

$$-\frac{\sin(\omega_w t)r_w}{\cos(\omega_w t)ctg(\lambda_s)}-v_{t,a}t \to -t\left(\frac{\omega_w r_w}{ctg(\lambda_s)}+v_{t,a}\right) \tag{26}$$

As a result, the deduction can be continued by exp–ressing the t parameter in function of ζ_w independent variable, which can be seen in (26):

$$t(\zeta_w)=-\frac{\zeta_w}{\omega_w r_w\tan(\lambda_s)+v_{t,a}} \tag{27}$$

The sought one-variable function form of the cut surface sectional view in the base plane can be expressed, if (27) is substituted into (24):

$$\xi_w(\zeta_w)_{skiving} = -\frac{r_w}{\cos\left(\frac{\omega_w\zeta_w}{\omega_w r_w \tan(\lambda_s)+v_{t,a}}\right)} \quad (28)$$

The deduction follows the same steps with different results for the determination of the cut surface in tange–ntial turning. The initial condition for η_w can be written as:

$$\eta_w = pctg(\lambda_s)\cos(\omega_w t)+r_w\sin(\omega_w t)+ \\ +v_{t,t}t\cos(\omega_w t)=0 \quad (29)$$

From (29), the formula for p parameter is expressed in function of t parameter. This results in that value for p parameter, which results in an intersecting point of the cutting edge with the base plane for a given t parameter:

$$p(t) = -\frac{r_w\sin(\omega_w t)+v_{t,t}t\cos(\omega_w t)}{\cos(\omega_w t)ctg(\lambda_s)} \quad (30)$$

The one-parametric system of equations is resulted for tangential turning by the substitution of(30) into (20):

$$\xi_w(t) = -\frac{r_w}{\cos(\omega_w t)} \quad (31)$$

$$\zeta_w(t) = -\frac{\sin(\omega_w t)r_w+v_{t,t}t\cos(\omega_w t)}{\cos(\omega_w t)ctg(\lambda_s)} \quad (32)$$

The solution here is also too complex to use it prac–tically, therefore the same Taylor approximation is used, as it can be seen in (33).

$$-\frac{\sin(\omega_w t)r_w+v_{t,t}t\cos(\omega_w t)}{\cos(w_w t)ctg(\lambda_s)} \rightarrow -t\frac{\omega_w r_w+v_{t,t}}{ctg(\lambda_s)} \quad (33)$$

Based on (32) and (33), the t parameter in function of ζ_w independent variable is expressed as the following:

$$t(\zeta_w) = -\frac{\zeta_w ctg(\lambda_s)}{w_w r_w+v_{t,t}} \quad (34)$$

To conclude the deduction above the one-variable function form of the cut surface in tangential turning is expressed by the substitution of (34) into (31).

$$\xi_w(\zeta_w)_{tangential} = -\frac{r_w}{\cos\left(\frac{\omega_w\zeta_w ctg(\lambda_s)}{\omega_w r_w+v_{t,t}}\right)} \quad (35)$$

In this study, the final deduction is the determination of the cut surface in rotational turning. The basis step is the definition of the boundary condition in this case as well. The following formula is resulted from (21)to fulfil the base plane intersecting condition:

$$\eta_w = r_t\left[\sin(p+\omega_w t)+\sin(\omega_w t)\right]+ \\ +\sin(\omega_w t)r_t = 0 \quad (36)$$

The constrain for p parameter in function of t pa–rameter is determined by reordering (36), which result is the following:

$$p(t) = -\arcsin\left[\frac{\sin(\omega_w t)(r_t+r_w)}{r_t}\right]+(\omega_w-\omega_t)t \quad (37)$$

The one-parametric system of equations of the intersecting curve of the cut surface and the base plane is deducted by the substitution of(37) into (21), which operation leads to the following:

$$\xi_w(t) = r_t\sqrt{1-\left(\frac{\sin(\omega_w t)(r_t+r_w)}{r_t}\right)^2} - r_t\cos(\omega_t t) - \\ -r_w\cos(\omega_w t) \quad (38)$$

$$\zeta_w(t) r_t ctg(\lambda_s)\left(\omega_w t-\omega_t t-\arcsin\left[\frac{\sin(\omega_w t)(r_t+r_w)}{r_t}\right]\right) \\ -v_{t,a}t \quad (39)$$

For further deduction, an approximation is need in this case as well, since the independent variable (t) is presented in multiple complex functions. In order to minimize the error, only the inverse sine function is approximated. The second order Taylor expansion of the described function takes the form of the following:

$$\arcsin\left[\frac{\sin(\omega_w t)(r_t+r_w)}{r_t}\right] \rightarrow \frac{\omega_w t(r_t+r_w)}{r_t} \quad (40)$$

The t parameter is determined in function of ζw by the substitution of (40) into (39), and the rearrangement of the result into the following formula:

$$t(\zeta_w) = -\frac{\zeta_w}{ctg(\lambda_s)\omega_w r_w+ctg(\lambda_s)\omega_t r_t+v_{t,a}} \quad (41)$$

Finally, by the substitution of (41) into (38), the one-variable function of the cut surface sectional view in the base plane is determined:

$$\xi_w(\zeta_w) = \\ \sqrt{r_t^2-\left((r_t+r_w)\sin\frac{\zeta_w\omega_w\tan(\lambda_s)}{\omega_w r_w+\omega_t r_t+v_{t,a}\tan(\lambda_s)}\right)^2} - \\ -(r_t+r_w)\cos\frac{\zeta_w\omega_w\tan(\lambda_s)}{\omega_w r_w+\omega_t r_t+v_{t,a}\tan(\lambda_s)} \quad (42)$$

9. VALIDATION

By the application of different mathematical methods, the equations of the cut surface in the sectional view of the base plane are determined for the studied high-feed turning procedures. However, in order to prove the applicability of these formulas, validation is necessary. The conformation is done by experimentally producing these surfaces, and comparing the theoretical and real surfaces to each other, determining the error between these curves.

An EMAG VSC 400 DS hard machining centre was used for the study. This machine tool can provide the necessary tangential and circular feed, moreover it has especial rigidity to do the precise cutting experiments. In tangential turning, the chosen material was 42CrMo4 grade alloyed steel, which processed by hardening heat treatment to 60 HRC hardness before the experiments. In the experiments, a cylindrical workpiece is machined, which outer diameter was 70 mm. A tool with 45° inclination angle was used. The indexable turning tool is made by HORN Cutting Tools Ltd. and consisted of two parts: S117.0032.00 insert and H117.2530.4132 holder. The working part of the tool was an uncoated carbide insert (MG12 grade). The experimental setup for tangential turning is presented in Figure 12.

The rotational turning experiments are carried out on heat-treated C45 cylindrical steel workpieces with Ø40 mm diameter. The material removal is done by a Fraisa P5300682 cutting tool with 30° inclination angle. The tool clamped in the driven tool holder of the machine tool and the workpiece clamped in the spindle can be seen in Figure 13.

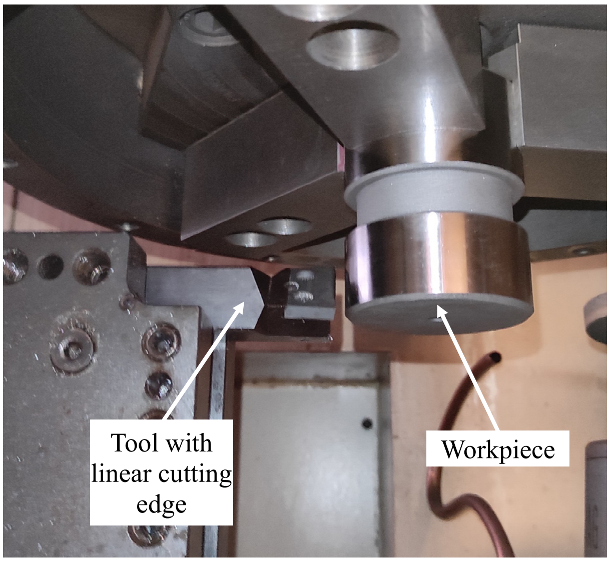

Figure 12. Experimental setup in tangential turning

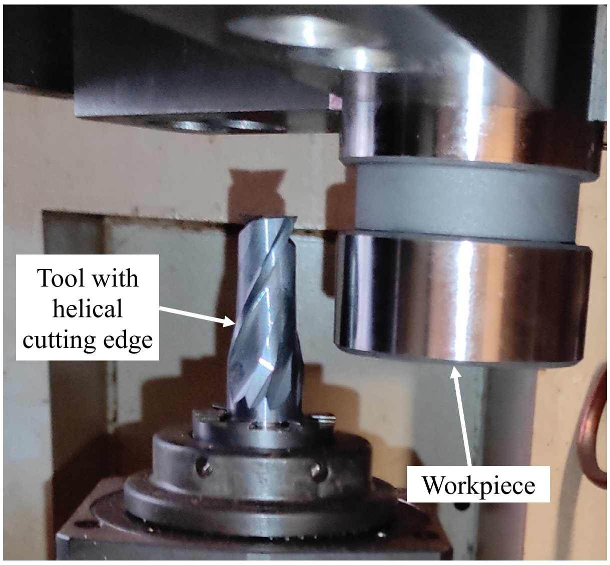

Figure 13. Experimental setup in rotational turning

The main idea in the technological data selection was to analyse the results in a wider range of para–meters. Therefore, 2-2 kinds of inclination angle (λs), cutting speed (vc), feed (f), and depth of cut (a) were chosen for the experiments. The resulted setups are shown in Table 1.

An AltiSurf 520 three-dimensional topography mea–suring instrument was applied for the measurements, which were carried out after the experiments using a confocal chromatic probe. The setup parameters for the measurements and evaluations were chosen according to the ISO 21920:2021standard.

Table 1. Experimental setup parameters

Setup	Proc.	λ_s [°]	v_c [m/min]	f [mm/r.]	a [mm]
A	tan.	45	200	1.0	0.2
B	tan.	45	250	0.8	0.1
C	rot.	30	200	1.0	0.1
D	rot.	30	200	0.8	0.1

After the measurements are completed, the rough–ness profiles of machined surfaces in each setup were drawn. Furthermore, the theoretical curves of the cut surface were also determined based on (35) in tangen–tial turning and (42) in rotational turning. The results can be seen in Figure 11-14.

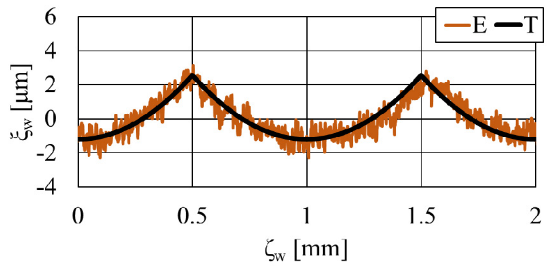

Figure 11. Comparison of the experimental (E) and theoretical (T) curves in tangential turning (Setup A)

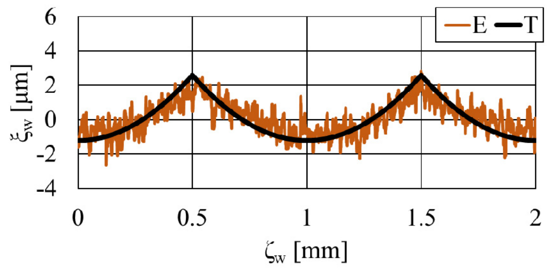

Figure 12. Comparison of the experimental (E) and theoretical (T) curves in tangential turning (Setup B)

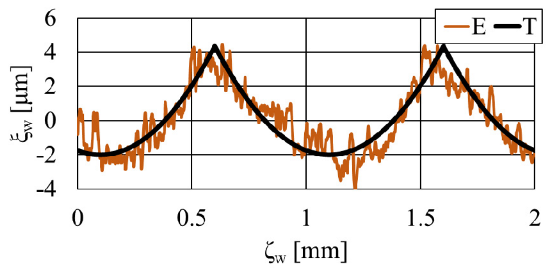

Figure 13. Comparison of the experimental (E) and theoretical (T) curves in rotational turning (Setup C)

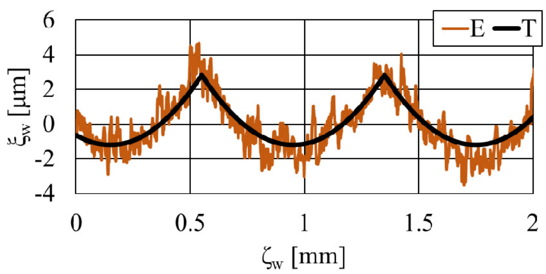

Figure 14. Comparison of the experimental (E) and theoretical (T) curves in rotational turning (Setup D)

The calculated correlations in each Setup are the following: A = 0.91, B = 0.84, C = 0.91, D = 0.89. The coefficients of determination in each case are deter–mined as follows: A = 0.82, B = 0.71, C = 0.82, D = 0.80.

It can be concluded based on the Figures and these values, that the determined curves describe the cut surface well. Even in a very small scale, the accuracy is acceptable.

10. CONCLUSION

The exact analytical determination of a given procedure is a complex task, which requires careful planning and the proper use of the methodical apparatus. Skiving turning, tangential turning, and rotational turning are determined mathematically in this paper. The cons–tructive geometric modelingis applied to achieve the proposed goal. After the throughout kinematical and geometrical analysis, the equations of motion of these high-feed turning procedures are determined through the transformation between the defined coordinate systems. The one variable equations of the cut surface are also defined. The carried-out experiments shown that the resulted equations describe the motion of the cutting edge well. The study will continue to determine mathe–matically among others: the geometric attributes of the uncut chip, the generated surface topography in 2D and 3D, and the characteristic parameters of the produc–tivity.

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АНАЛИТИЧКО ОДРЕЂИВАЊЕ ПОСТУПАКА СТРУГАЊА СА ВЕЛИКИМ ПОМАКОМ ПРИМЕНОМ КОНСТРУКТИВНОГ ГЕОМЕТРИЈСКОГ МОДЕЛИРАЊА

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Развој различитих поступака обраде захтева њихов тачан математички опис. Ово је посебно важно у поступцима завршне обраде, где се производе коначна геометрија и површинске карактеристике делова. У овом раду су анализирана три поступка стругања са великим помаком: стругање с љуштењем, тангенцијално стругање и ротационо стругање. Сва три обећавају стварање површина налик на тло са ниским карактеристикама храпавости, уз постизање високих вредности продуктивности. Аналитичко утврђивање ових поступака врши се применом конструктивног алатног геометријског моделирања. Након анализе кинематичких и геометријских односа сваког

поступка, дефинишу се одговарајући координатни системи. Одређене су једначине трансформације које описују геометријске граничне услове и кретање радног предмета и алата. У следећем кораку дефинише се једначина кретања за три проучавана поступка. Коначно се одређује једна промењива једначина пресека површине реза у основној равни. Спроведени су и експерименти који су потврдили постигнуте резултате.