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Implementation of Stability Analysis Approach for In-pipe Inspection Robot Movement through Pipeline Fittings

The paper presents the implementation of the system stability testing approach based on the direct Lyapunov method. For this purpose, a pipeline inspection robot consisting of a driving and a driven part was used as an example. A kinematic model of the robot is shown, which describes its movement through the pipe fittings, the elbow, and the T-piece. In addition, a dynamic model based on the Lagrange function is given. The data set of this model is linearised and given in the so-called state space form, which is suitable for the application of the direct Lyapunov method. The approach is illustrated in a flow chart and is iterative in nature. The simulation was carried out with the computer program ©MATLAB.

Keywords: *stability analysis, in-pipe inspection robot, movement, pipeline, fittings, simulation, Lyapunov method*

1. INTRODUCTION

In today's industry, pipelines are important system components that are used to transport various media in different branches of industry. Precisely because they are so important, they must be carefully maintained [1]. They must be inspected regularly to obtain information about the condition of the pipeline and any damage it may have sustained [2]. Of particular importance is damage inside the pipeline, which can be caused either by wear and corrosion of the material or by the accu–mulation of solid deposits of other materials along the pipeline wall. Their inspections are sometimes dangerous due to their purpose and the media they transport, as well as their location in space. In addition, these inspection procedures can be time-consuming, especially when performed manually. Furthermore, this type of inspection is inaccessible to humans and difficult to perform for smaller pipe diameters and complex geometries. Manual methods of detecting leaks or defects in pipelines are expensive and time-consuming. There is also a high probability that some of the leaks will not be detected, which reduces the work efficiency of such a method.

For this purpose, different variants of robotic models have been developed for the inspection of pipelines. Depending on the type of movement and the architecture of the robot [3], the extraordinary mobility of these robots through the inside of the inspected pipeline [4]. This type of robot should be designed as a replacement for humans due to the dangerous and inaccessible working environment. The conceptual design model of such a robotic system is a particularly complex task, as the designer has to take many influential parameters into account. In addition, engineers are faced with complex working conditions when performing tests and collecting data on pipeline damage. Many factors can influence the reading errors of defects in pipelines. The environment has a major influence, as does the noise generated. A robotic system that moves through a given surface structure with its legs must be designed to adapt to possible predicted disturbances. Conceptually, it is assumed that it must be designed so that it has a sufficiently robust architecture and can overcome all possible estimated obstacles during its movement. Noise and damage to the leg elements are caused by dynamic vibrations [5] during robot movement. To reduce these to an acceptable level, elastic elements are used that are built into the robot legs.

Regardless of the fact that different architectures of these robots as well as the corresponding control algorithms have been developed, the question arises whether the robot achieves a stable state of motion during its movement in order to overcome all obstacles it may encounter. A major problem during its motion is the piping components it may encounter, such as constrictions and widenings, elbows, T-pieces, and the rise and fall of the piping following the configuration of the terrain. During its complex movement through the pipeline, the robot performs a combination of translational and rotational movement [6], so-called spinal (screw) movement. Stiffness also occurs in the joints of the robot legs [7], which is partially solved by the installation of torsion springs, while the robot legs are considered as rigid bodies.

This paper presents an approach to solve the stability problem of a robot for inspection inside a pipeline in the most critical cases of traversing a pipeline (bends and T-pieces). It is also suitable for pipelines that change their diameter and orientation in individual sections. The approach is based on the so-called iterative character and makes it possible to propose some improvements and changes to the robot architecture model and the control algorithm model based on the analysis and the feedback loop. A reorganization of the system is carried

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out, i.e., the inclusion of new or the removal of existing subsystems or modules via a feedback loop. In addition, changes must first be made to the customer requirements and the technical requirements created on their basis. The direct Lyapunov method is used for this.

2. LITERATURE REVIEW

There are many papers in the literature that present the development of control models and algorithms as well as stability studies of pipeline inspection robots. Only those that are relevant and show similarities with the robot architecture proposed in this thesis are listed below. Pipeline robots are generally categorized into actively traveling and passively traveling robots - the cate–gorization is based on the differences in the energy so–urce and controllability of the inspection mechanism [3].

Li et al. [8] describe a pipeline inspection robot whose motion is based on the screw principle and has a modular design. In addition, the robot's drive legs are attached to its body together with the wheels and rotate together with them. It is designed to move through pi–pelines filled with liquid and gas. It includes a CCD camera attached to the front body of the robot to inspect the inside surface of the pipeline. Its architecture also includes an adaptive mechanism containing wheels with legs, springs, actuators, joints, etc.

Moghaddam and Arbabtafti [9] have developed a device that is able to move through a pipe with a span of 250-350 mm, with the possibility of diameter adjustment. It can move through the pipe regardless of its geometry and position. In addition, it can avoid obstacles due to the design of the legs. The pressure force on the pipeline wall is regulated by the operator via a built-in sensor using a computer program.

Nishimura et al. [10] show a robot that also moves according to the screw principle. Its architecture consists of three parts – front, center, and rear. Its drive mechanism is located in the rear part and has two DC motors. One motor is used to drive the robot, and the other is used to slow it down. The movement of the front part is based on the principle of rotation around an axis due to its articulated connection, which enables it to negotiate bends and T-pieces in the pipeline. It can also move through conical pipes and pipelines with long bends.

The authors Nayak and Pradhan [11] also show the development of a wheeled robotic system based on helical motion through a pipe. The robot is designed to move through all parts of the pipeline. It consists of three modules, the stator, the rotor, and the control part.

Tang et al. [12] have developed a robot that also moves according to the screw principle. It consists of 4 parts and a mechanism that ensures the pressure of the wheels on the tube wall. It is also designed to avoid obstacles through rotation and screw movement. It can also move through pipes with round and rectangular cross-sections.

The fifth group that should be mentioned here is the MRINSPECT robot series, which is an abbreviation for Multifunctional Robotic Crawler for In-pipe Inspection [13 - 19]. MRINSPECT (II-VII) is a product family of robotic systems for the inspection of pipelines. Several versions of these robots have been developed, and their operating principle is based on the pressure force with which they can be controlled. They can move through pipelines with different diameters. The so-called spring mechanism is used for this purpose.

Torajizadeh et al [20] developed a robot that is adaptable and can move through a larger range of pipe diameters according to the screw principle. It consists of two parts, a stator and a rotor. The pressure of the leg on the pipe wall is provided by a passive element with a spring.

Finally, two works by Osman and Kovačić should be mentioned. In the first paper [21], the architecture and behavior model of the robot, which is intended for inspecting the inside of pipelines, were described and presented. The architecture of the robot consists of a driven part and a driving part, each of which has three legs with two joints. A model for the behavior of the robot was also developed, including its kinematic and dynamic models. In another paper [22], a kinematic and dynamic analysis for the case of pipe sections (bends, T-pieces, constriction, and expansion points) is presented. In addition, a robot control algorithm was developed using the so-called co-simulation approach, a combi–nation of several computer programs: ©SolidWorks, ©MSC ADAMS, and ©MATLAB Simulink.

The contribution of this work is reflected in the notation of the dynamic model by the Lagrange function, i.e., its kinetic and potential energies. By adding individual components to the architecture and including them in the mathematical model, it is possible to influence the change in the values of individual influential parameters in the system matrix. The stability test with the proposed method is a quick way to appro–ximately assess and evaluate the robot architecture. In addition, the mathematical model makes it possible to perform an analytical test by changing the numerical values. Moreover, the given description and flowchart show all the necessary steps in the creation of the program sequence to perform this test. To date, there is no published work that has tested the system stability of this type of robot. The authors believe that this work will contribute to new knowledge in this regard.

The authors believe that such a structured approach to calculating approximate system stability will be of great help to engineers working in this field. The state-space representation of the differential equations of the system facilitates the application of Lyapunov's direct method. The use of 3D software systems after the creation of the system architecture enables the selection of materials and the calculation of the dynamic moments of inertia of the individual components. The approach is also applicable to other types of mobile robotic systems.

3. APPROACH TO SYSTEM STABILITY TESTING

The observed approach consists of several steps, which are described in more detail below using the steps shown in the flowchart in Figure 1:

1. *Introduction of assumptions and constraints* - assumptions are introduced to simplify the problem, but without neglecting the essential processes and

parameters in the system. There are also some constraints that the mechanical designer should define, e.g., the speed of the robot in the pipeline, the diameter of the pipeline, etc.

When defining the technical requirements, the auto–mation engineer works together with the mechanical engineer. Both are created based on the given customer requirements and engineering requirements.

2. *Creation of an initial conceptual model of the robot architecture and control algorithm* with known system components and selected materials as well as the selected control algorithm with its parameters. The mechanical designer carries out a preliminary calcu–lation to select the equipment, together with a concep–tual solution for the components to be manufactured. Based on the given parameters that the designer knows, the automation engineer makes the selection of his equipment.

3. *Kinematic analysis of the robot motion* - based on the previously created conceptual robot architecture, a kinematic motion model is created and a kinematic analysis is performed with the aim of analyzing the observed parameters over time. In this case, the simulation analyses the change in distance, speed, and acceleration as the robot moves through the pipeline sections.

4. *Dynamic analysis of the robot motion* - dynamic equations of motion are established, i.e., a dynamic model of the robot is created. The aim of the analysis is to observe the changes in the selected parameters over time. In this analysis, the contact forces, frictional forces, and torques in the joints are analyzed. A new choice of materials, especially for the leg wheels, or a change in the design of the leg joints is possible. In addition, the mass should be reduced through lower torques.

5. *Display* the previously created dynamic model *in the state space form*. Generalized forces are generated by applying the Lagrange equations of the second kind. On their basis, the differential equations of motion are obtained, with which the input states of the state vector are defined. The purpose of this is to obtain the System dynamics matrix **A** for later stability analysis.

6. *Stability analysis of the robot system* using the direct Lyapunov method – explained in more detail in the next chapter. The analysis is carried out through a created program procedure in the ©MATLAB software tool.

7. *Display the simulation results via diagrams*. The input state responses to the state space model over time are displayed.

8. *Restructuring phase* – possible step - depending on the stability check of the system, changes to the customer requirements, the engineering requirements, and the created system architecture as well as changes to the parameters in the control algorithm are possible.

4. THE STABILITY ANALYSIS OF A ROBOTIC SYSTEM USING THE DIRECT LYAPUNOV METHOD

The algorithm for testing system stability is shown in the flow chart in Figure 2. The stability of the system is tested on the basis of the created dynamic model of the system and its representation in the state space. The presented algorithm is applicable for systems that are extremely linear as well as for non-linear systems that can be linearized. It is also only applicable to the group of time-invariant (LTI) systems [23].

A linear unexcited system, time-invariant, is obser–ved, which can be described by the following equation:

$$\dot{x}(t) = Ax(t) \quad x \in R^n \tag{1}$$

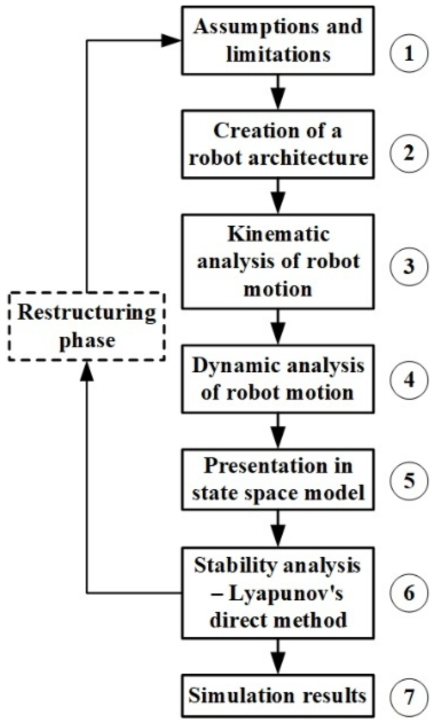


Figure 1. Diagrammatic representation of the approach to system stability testing

The system matrix of system **A** was read from the dynamic model record in the state space form.

For such a system, we can look for the Lyapunov function [16] in the form of a quadratic function:

$$V = \mathbf{x}^T \mathbf{P} \mathbf{x} \tag{2}$$

where is:
P – symmetric positive definite matrix.

For such a system to be stable, the derivative of the Lyapunov function must be negative definite, i.e.s

$$\dot{V} = -\mathbf{x}^T \mathbf{Q} \mathbf{x} \tag{3}$$

where is:
Q – some symmetric positive definite matrix.

If we differentiate equation (2) with respect to time, we get:

$$\dot{V} = \dot{\mathbf{x}}^T \mathbf{P} \mathbf{x} + \mathbf{x}^T \mathbf{P} \dot{\mathbf{x}} \tag{4}$$

By equating equations (3) and (4), we can write:

$$\dot{V} = \dot{\mathbf{x}}^T \mathbf{P} \mathbf{x} + \mathbf{x}^T \mathbf{P} \mathbf{x} = -\mathbf{x}^T \mathbf{Q} \mathbf{x} \tag{5}$$

If we insert equation (1) into (5), and write it out, we get:

$$\mathbf{x}^T \mathbf{A}^T \mathbf{P}\mathbf{x} + \mathbf{x}^T \mathbf{P}\mathbf{A}\mathbf{x} = \mathbf{x}^T \left(\mathbf{A}^T \mathbf{P} + \mathbf{P}\mathbf{A} \right) \mathbf{x} = -\mathbf{x}^T \mathbf{Q}\mathbf{x} \quad (6)$$

From this, we get:

$$\mathbf{A}^T \mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q} \quad (7)$$

which is known as the Lyapunov matrix equation, whe–re: **P** and **Q** are positive definite symmetric matrices.

The stability test is performed by first choosing a symmetric matrix **Q**, which must be positive definite (usually it is chosen as a positive matrix).

Then the symmetric matrix **P** is calculated from equation (7). If it is positive, then the system is globally asymptotically stable in the sense of Lyapunov.

Now we can apply the conditions for local and global asymptotic stability to the unexcited system and perform the stability test.

For the sake of simplicity, we can take T = 1. We describe the Lyapunov function by a positive definite quadratic function:

$$V\left[x(k)\right] = x^T(k) \cdot \mathbf{P} \cdot x(k) \quad (8)$$

The function $V[x(k)]$ will be positive definite if the symmetric matrix **P** is positive definite (this can be checked, for example, by Sylvester's criterion).

Also, the difference of the function $\Delta V[x(k)]$ should be negatively defined, i.e.:

$$\Delta V\left[x(k)\right] = -x^T(k) \cdot \mathbf{Q} \cdot x(k) < 0 \quad (9)$$

It also follows that the matrix **Q** must also be posi–tive definite:

$$-\mathbf{Q} = \Phi^T \cdot \mathbf{P} \cdot \Phi - \mathbf{P} \quad (10)$$

When analyzing the asymptotic stability of discrete systems, it is necessary to choose the matrix **Q** so that it is positive definite and then to determine the matrix **P**.

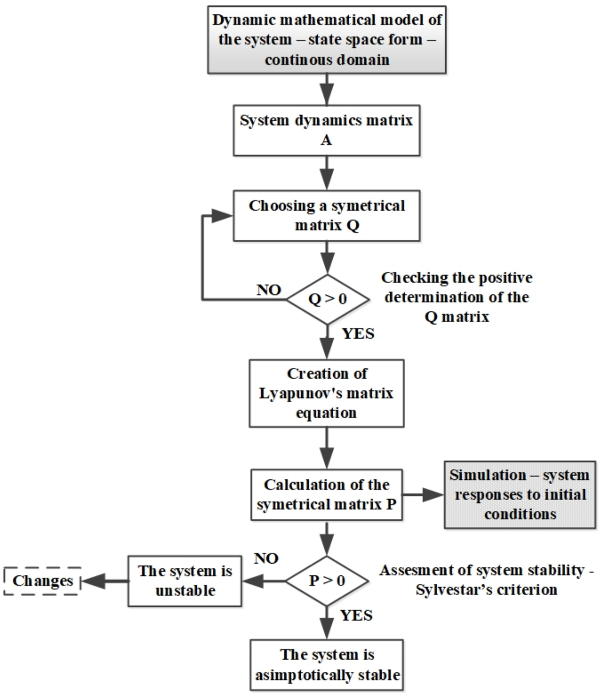


Figure 2. Diagrammatic representation of the stability analysis

If it is positive definite, the system is asymptotically stable.

If the system becomes unstable, you need to make changes in the structural part of the system (reorga–nization - by adding new or removing existing subsys–tems or modules) through feedback. The customer re–quirements and the engineering requirements must be changed beforehand.

5. DESCRIPTION OF THE CONCEPTUAL MODEL OF THE ARCHITECTURE OF THE ROBOT SYSTEM

A conceptual model of the architecture of the robot system is shown in Figure 3. The robot consists of two parts, the driving part and the driven part, which are connected by a shaft via a spherical joint. Each of these two parts contains three legs with wheels that are 120° apart. When defining the concept for the robotic system, the first task was to define the minimum number of legs that the robot needs to grip the walls of the pipeline well enough and also to move satisfactorily within the pipeline without tipping over. The system concept also provides for two parts of the robot, the driving part and the driven part. This is for easier handling of curved surfaces in the pipeline, such as elbows and T-pieces.

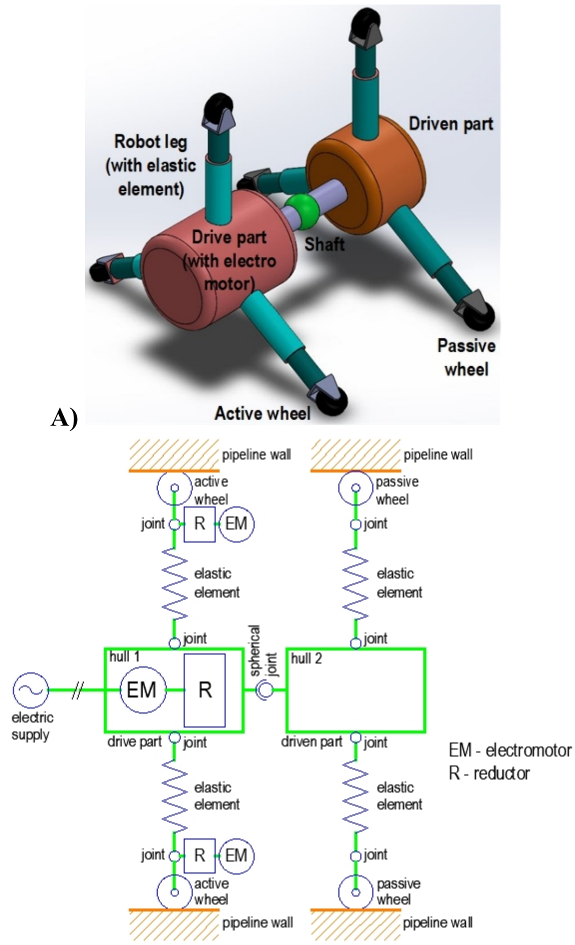


Figure 3. a) 3D Representation of the conceptual model of the architecture of the robotic system, b) Kinematical schema of the robotic system

The pressure of the wheels on the pipeline wall is ensured by an elastic element located on each leg of the robot. The driving part of the robot contains an electric motor and a power transmission that enables its move–ment. The driving part of the robot has active wheels. The driven part contains passive wheels. Active wheels are those that are controlled, i.e., driven by an actuator at the joint. Passive wheels, on the other hand, rotate freely, i.e., they adapt their position to the current position in the pipeline. The model was developed using the computer program ©SolidWorks.

6. KINEMATIC MODEL OF THE ROBOT SYSTEM

This chapter presents the kinematic model of the robot. When the robot moves through a straight pipe, it is necessary to define the cylindrical coordinate system R, Θ, and z, which is shown in Figure 4. It shows a spiral curve (helix) along which the robot moves to realize a helical movement (spiral) [24].

According to Figure 4, the variable R is defined, which represents the vector of the radius on the circular plane, i.e., the inner radius of the tube. Its components in the cylindrical coordinate system are represented by the following matrix:

$$R=\begin{bmatrix} R\cos\Theta \\ R\sin\Theta \\ 0 \end{bmatrix} \tag{11}$$

where is:

Θ – angle of rotation around the z-axis,

α – inclination angle of the active wheel against the circular plane.

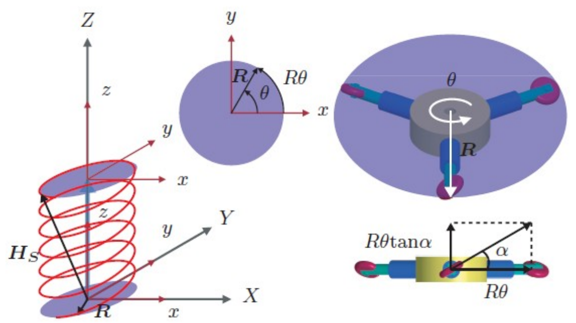


Figure 4. Display of parameters in a cylindrical coordinate system

The observed point on the helix curve can be rep–resented by the following equation:

$$\begin{bmatrix} H_s \\ 1 \end{bmatrix} = T_z \cdot \begin{bmatrix} R \\ 1 \end{bmatrix} = \begin{bmatrix} R\cos\Theta \\ R\sin\Theta \\ R\Theta tg\Theta \\ 1 \end{bmatrix} \tag{12}$$

Movement in pipeline fittings, e.g., in an elbow, is represented by the following equation:

$$\begin{bmatrix} H_{cp} \\ 1 \end{bmatrix} = T_{\Phi x} \cdot \begin{bmatrix} R \\ 1 \end{bmatrix} = \begin{bmatrix} (R_0 + R\cos\Theta)\cos\Theta \\ R\sin\Theta \\ (R_0 + R\cos\Theta)\sin\Theta \\ 1 \end{bmatrix} \tag{13}$$

where is:

R_0 – position vector

Φ – rotating angle around the Y-axis

According to Figure 5, it can be written:

$$R d\Theta tg\alpha = R_0 d\Phi \tag{14}$$

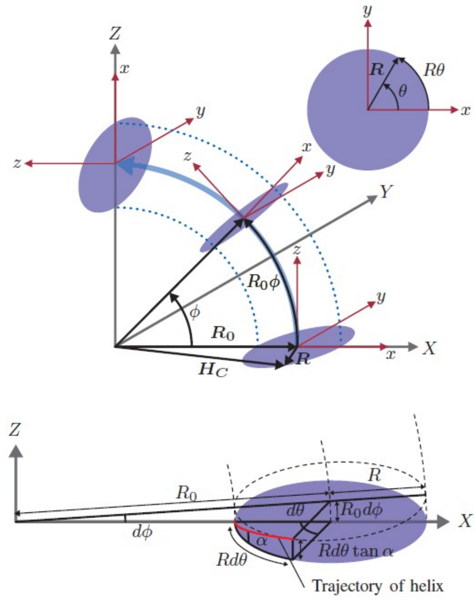


Figure 5. a) Display of parameters in the elbow, b) Elbow trajectory display

Equation (13) can be written more briefly as:

$$\begin{bmatrix} H_{cp} \\ 1 \end{bmatrix} = \begin{bmatrix} (R_0 + RC_\Theta)C_{K\Theta} \\ RS_0 \\ (R_0 + RC_\Theta)S_{K\Theta} \\ 1 \end{bmatrix} \tag{15}$$

For the case of the motion of a wheeled robot through a knee (Figure 6), the following equations (16) – (21) can be written:

Geometry parameters A and B are calculated by the following expressions (16-17):

$$2A = \sqrt{(R_0 + R)^2 - L^2} - \sqrt{(R_0 - R)^2 - L^2} \tag{16}$$

$$B = \sqrt{(R_0 - R)^2 - L^2} \tag{17}$$

The radius of the pipeline R_p is represented with geometry parameter A and angle Θ (18):

$$R_p = \begin{bmatrix} A\cos\Theta \\ A\sin\Theta \\ 0 \end{bmatrix} \tag{18}$$

The observed point on the helix curve is represented in the form of dependence on geometric parameters A, B, and L (19):

$$\begin{bmatrix} H_s \\ 1 \end{bmatrix} = T_p \cdot \begin{bmatrix} R \\ 1 \end{bmatrix} = \begin{bmatrix} \{A(1+C_\Theta)+B\}C_{K\Theta} - LS_{K\Theta} \\ AS_\Theta \\ \{A(1+C_\Theta)+B\}S_{K\Theta} + LC_{K\Theta} \\ 1 \end{bmatrix} \tag{19}$$

$$v_{pwc} = \frac{dH_p}{dt} = \begin{bmatrix} \left(-AS_\Theta - LK\right)C_{K\Theta} - KbS_{K\Theta} \\ AC_\Theta \\ \left(-AS_\Theta - LK\right)S_{K\Theta} + KbC_{K\Theta} \end{bmatrix} \quad (20)$$

$$b = A\left(1 + C_\Theta\right) + B \quad (21)$$

where is:
v_{pwc} – velocity vector of the passive wheel.

Equations (16), (17), and (21) show the calculation of geometric parameters according to Figure 6.

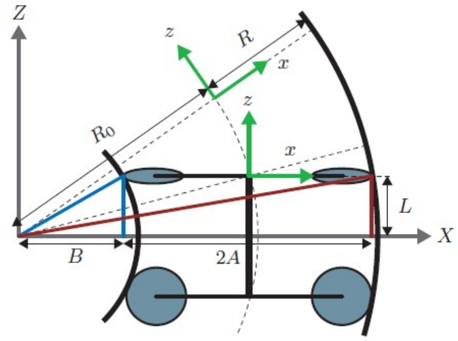


Figure 6. Display of robot motion through the elbow

7. DYNAMIC MODEL OF THE ROBOT SYSTEM

Definition of the Lagrange function L for dynamic model creation [25]:

$$L = E_k - E_p \quad (22)$$

where is:
E_K – kinetic energy,
E_p – potential energy.

The potential energy E_p is assumed to have a (-) sign because the robot moves through a pipe that goes upwards. For this purpose the most difficult case of movement through a pipeline is chosen.

The differential of potential energy dE_p, was calculated and set up with the following equation:

$$dE_p = \left(M_{mot} + M_{hull} + nm\right)\left(b + r\right) gd\varnothing tg\alpha \quad (23)$$

The total kinetic energy $E_{k,tot}$, was calculated and set up the following equations:

$$E_{k,tot} = E_{k,mot} + E_{k,hull} + n\left(E_{k,\omega 1} + E_{k,\omega 2}\right) \quad (24)$$

The kinetic energy of the electromotor was calcu–lated as:

$$E_{k,mot} = \frac{1}{2} m_{mot} z^2 \quad (25)$$

In addition, the kinetic energy of the hull was cal–culated as:

$$E_{k,hull} = \frac{1}{2} m_{hull} z^2 + I_B \varnothing^2 \quad (26)$$

The kinetic energies of the wheels were calculated as:

$$E_{k,\omega 1} = \frac{1}{2}\left\{m_w r^2 + I_{wz}\right\}\left(\frac{b\cos\alpha}{b+r}\right)^2 + \\ +\left(m_w r^2 + l_{wx}\right)\left(\sin\alpha\right)^2 \dot{\varnothing}^2 \quad (27)$$

$$E_{k,\omega 2} = \frac{1}{2} n I_w \dot{\alpha}^2 \quad (28)$$

$$E_{k,tot} = \frac{1}{2}\left\{\left(\left(b+r\right)\frac{S_\alpha}{C_\alpha}\right)^2 m_{DM} + nb^2 m_{DM} + I_B\right\}\dot{\varnothing}^2 + \\ + \frac{1}{2} n I_w \dot{\alpha}^2 \quad (29)$$

The mass of the robot drive and driven part was calculated as:

$$m_{DM} = m_{mot} + m_{hull} + nm_w + n\frac{I_{wx}}{r^2} \quad (30)$$

$$m_{DM} = m_w + \frac{I_{wz}}{r^2} \quad (31)$$

where is:
n – number of steering wheels

It is necessary to determine the generalized forces Q_i for the Lagrange equation setup. The Lagrange equation of the second kind is used for this purpose:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \quad (32)$$

The first generalized force Q_1 could ne presented by the equation (33):

$$Q_1 = T_{mot} - T_{fr} \quad (33)$$

The second generalized force Q_2 represents the tor–sion moment of steering (34):

$$Q_2 = T_{steer} \quad (34)$$

The friction torque T_{fr} can be expressed as:

$$T_{fr} = nb\mu F_N \sin\alpha \quad (35)$$

The generalized force Q_1 was written as:

$$Q_1 = T_{mot} - nb\mu F_N \sin\alpha \quad (36)$$

Using Lagrange's equation of the second kind (32), dynamic differential equations were created. Firstly, equation (37) which describes the rate of change of the rotation angle of the motor:

$$\ddot{\varnothing} = \frac{m_{mot} Rg\left(1+\left(tg\alpha\right)^2\right)a - 2R^2 \dot{m}_{Dm}\frac{S_\alpha}{C_\alpha^3}\dot{\alpha}\dot{\varnothing} - n\mu bF_N S_\alpha}{nm_{Dm}b^2 + I_B + R^2 m_{Dm}\left(tg\alpha\right)^2} + \\ + \frac{T_{mot}}{nm_{Dm}b^2 + I_B + R^2 m_{Dm}\left(tg\alpha\right)^2} \quad (37)$$

The second differential equation (38) describes the change in the ascent angle of the helix:

$$\ddot{\alpha} = \left(\frac{R^2 m_{Dm}\left(tg\alpha + \left(tg\alpha\right)^3\right)}{nI_w}\right)\dot{\phi}^2 - \left(\frac{m_{tot} Rg\left(1+\left(tg\alpha\right)^2\right)}{nI_w}\right)\dot{\phi} + \\ + \frac{T_{steer}}{nI_w} \quad (38)$$

where is:
Φ – rotation angle of the motor,
α – ascent angle of the helix.

The input states to the state vector x(t), i.e. the input states in the state space model, are variables $\phi, \dot{\phi}, \alpha, \dot{\alpha}$.

The model in state space form was written with the following equations (39) – (42):

$$\dot{x}_1 = x_2 \tag{39}$$

$$\dot{x}_2 = \frac{m_{tot} Rg\left(1+\left(tgx_3\right)^2\right) x_4 - 2R^2 m_{Dm} \frac{S_{x3}}{C_{x3}} x_4 x_2 - n\mu b F_N S_{x3}}{n m_{Dm} b^2 + I_B + R^2 m_{DM}\left(tgx_3\right)^2} + \frac{T_{mot}}{n m_{Dm} b^2 + I_B + R^2 m_{DM}\left(tgx_3\right)^2} \tag{40}$$

$$\dot{x}_3 = x_4 \tag{41}$$

$$\dot{x}_4 = \left(\frac{R^2 m_{Dm}\left(tgx_3 + \left(tgx_3\right)^3\right)}{nI_w}\right) x_2^2 - \left(\frac{m_{tot} Rg\left(1+\left(tgx_3\right)^2\right)}{nI_w}\right) x_2 + \frac{T_{steer}}{nI_w} \tag{42}$$

The above equations are translated into linearized form about operating point $\left[\dot{\phi}, \alpha, \dot{\alpha}\right] = \left[x_2, x_3, x_4\right]$.

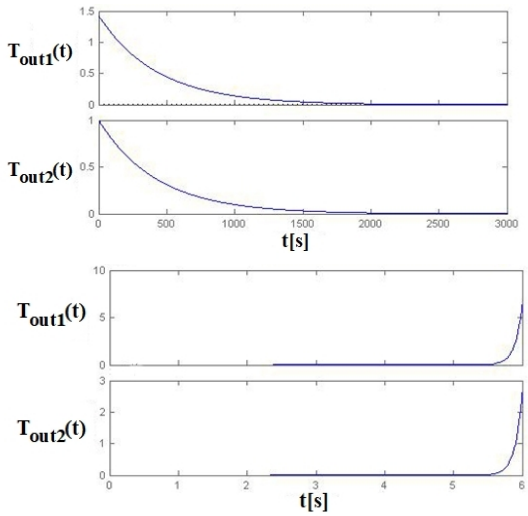
From the linearized state space model, System matrix **A** was written (43):

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{n\mu b F_N\left(0.016R^2 - 0.9\right)}{0.005R^2 - m_{DM} + n m_{Dm} b^2 + I_B} & \frac{1.015 Rg m_{tot}}{0.005R^2 - m_{DM} + n m_{Dm} b} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1.015 Rg m_{tot}}{nI_w} & 0 & 0 \end{bmatrix} \tag{43}$$

8. SIMULATION AND RESULTS

Input parameters for the simulation of stability analysis are:

- inner diameter of inspection pipeline: 500 – 750 mm,
- velocity of passive wheels (amplitude): 8,5 – 8,9 mm/s,
- mass of robot: 16,6 kg,
- power supply / number of phases / frequency: 400 V / 3 ph / 50 Hz,
- input power of the AC electromotor: 3,7 kW,
- length of the robot (drive + driven part): 950 mm,
- maximum length of robot: 200 mm,
- number of wheels: 6 (3 on drive and 3 on driven part).



a) b)

Figure 7. Simulation results for stability analysis: a) 2 wheels in contact inside the pipeline (on the drive part), b) 1 wheel in contact inside the pipeline (on the drive part)

Simulation results for stability analysis are given for testing the stability of the robot during its passage through a T-piece for different numbers of legs n. The number of legs was varied from n=1 to n=2 (Figure 7 a and b).

The simulation was carried out using a procedure developed in the ©MATLAB software according to the flow chart shown in Fig. 2.

9. CONCLUSION AND POSSIBLE DIRECTIONS FOR FUTURE RESEARCH

The paper presents a structured approach to test the stability of the system using Lyapunov's direct method. The approach is demonstrated using the case study of an in-line pipeline inspection robot. For the given example of a robot, which is structurally designed from a driving and a driven part, the following is shown:

- A kinematic and a dynamic system model. The kinematic model shows the complex movement of the robot through the pipeline wall, whereby it performs the so-called screw movement. In the dynamic model of the robot, the Lagrange function was used, which contains the kinetic and potential energy of the system. In addition, generalised forces are represented and dynamic differential equations of the robot are created. The dynamic model of the robot is represented in the state model.
- The notation of the dynamic robot model, which is written in the state space model, is suitable for the application of the direct Lyapunov method.
- The stability of the observed system was investigated in two cases of critical changes in the pipeline fittings: elbow and T-piece, by varying the number of robot wheels in direct contact with the pipeline wall.
- The approach is applied to a pipeline that does not contain fluid. Furthermore, it is intended that the

robot moves along an ideal pipeline wall without unevenness.

- The stability analysis shows that the robot must have at least 2 legs in contact with the pipeline wall in order to reach a state of asymptotic stability.

Possible directions for future research are:

- An attempt will be made to develop a model of the architecture and control algorithm of the observed robot using the so-called hybrid compliance control system, including its passive and active parts.
- To investigate the stability of the robot during movement through the pipeline segments in more detail, vary the movement speeds and the materials of the wheels and the pipeline walls.
- To investigate which architecture of the observed robot system is the most optimal for travelling through the pipeline segments and which best ensures the stability of the robot, its speed, and the accuracy of error detection in the pipeline.
- A prototype of the observed robot will be developed and an experimental test of the robot's behaviour during movement through the pipeline segments will be carried out.

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NOMENCLATURE

A	geometry parameter
$\mathbf{A}$	system dynamics matrix
B	geometry parameter
b	geometry parameter
C_α	geometry parameter
dE_p	differential of potential energy
$E_{k,hull}$	kinetic energy of the hull
$E_{k,mot}$	kinetic energy of the electromotor
$E_{k,tot}$	total kinetic energy
$E_{k,\omega 1}$	kinetic energy of the wheels
$E_{k,\omega 2}$	kinetic energy of the wheels
E_p	potential energy
F_N	normal force
H_{cp}	point during the movement in pipeline fittings
H_s	point on the helix curve
I_B	dynamic inertia moment of hull
I_w	dynamic inertia moment of wheel
K	geometry parameter
L	geometry parameter
m	mass
m_{hull}	mass of the hull
mDM	mass of the robot drive part
mDm	mass of the robot driven part
m_{mot}	mass of the electromotor
m_{tot}	total mass
m_w	mass of the wheel
L	Lagrange function
n	number of steering wheels
$\mathbf{P}$	symmetric positive definite matrix
Q_i	generalised force
Q_1	generalised force
Q_2	generalised force
$\mathbf{Q}$	some symmetric positive definite matrix
R	inner radius of the pipeline
r	r coordinate
R_0	position vector
R_{oi}	initial inner radius of the pipeline
R_p	pipeline radius
S_α	geometry parameter
T_{fr}	friction torsion moment
T_{mot}	torsion moment of electromotor
T_{steer}	torsion moment of steering
T_{out1}	output variable for stability analysis
T_{out2}	output variable for stability analysis
t	time
V	Lyapunov function
v_{pwc}	velocity vector of the passive wheel
ΔV	difference of the Lyapunov function
z	z coordinate

Greek symbols

α	variable
α	inclination angle of the active wheel against the circular plane
$\dot{\alpha}$	variable
Φ	rotating angle around the Y-axis
$\dot{\Phi}$	variable
μ	coefficient of friction
Θ	angle of rotation around the z-axis

Superscripts

cp	pipeline fittings
fr	friction
hull	hull
mot	motor
out	output
s	curve

steer	steering
tot	total

Abbreviations

MRINSPECT	Multifunctional Robotic Crawler for In-pipe Inspection
CCD	Charge-Coupled Device
DC	direct current
LTI	linear time invariant

ИМПЛЕМЕНТАЦИЈА ПРИСТУПА АНАЛИЗЕ СТАБИЛНОСТИ ЗА КРЕТАЊЕ РОБОТА ЗА ИНСПЕКЦИЈУ У ЦЕВИ КРОЗ ФИТИНГ ЦЕВОВОДА

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У раду је приказана имплементација приступа тестирању стабилности система заснованог на директној методи Љапунова. У ту сврху је као пример коришћен робот за преглед цевовода који се састоји од погонског и погонског дела. Приказан је кинематички модел робота који описује његово кретање кроз цевне спојнице, колено и Т-комад. Поред тога, дат је и динамички модел заснован на Лагранжовој функцији. Скуп података овог модела је линеаризован и дат у такозваном облику простора стања, који је погодан за примену директне методе Љапунова. Приступ је илустрован дијаграмом тока и итеративног је карактера. Симулација је спроведена компјутерским програмом ©МАТЛАБ.