

A New Approach to Correlate Heat Transfer Data for Turbulent Flow in Smooth Pipes Based on Generalization of Reynolds Number

Branislav Jaćimović

Professor
University of Belgrade
Faculty of Mechanical Engineering

Srbislav Genić

Professor
University of Belgrade
Faculty of Mechanical Engineering

This paper shows that the conventional Reynolds number has a clear physical meaning only for laminar, hydraulically developing or unsteady flow, while the generalized Reynolds number is physically meaningful for both laminar and turbulent regimes; the two Reynolds numbers coincide only in case of laminar flow.

A review of widely used heat transfer correlations for turbulent liquid flow in smooth pipes ($Re > 2000$) is presented, and new correlations are developed using the generalized Reynolds number. The analysis is based on a database of 1025 experiments (including 110 in the transitional regime), covering $Re = 2000$ – 522000 and $Pr = 0.18$ – 414 .

The proposed correlations apply over the same ranges and show improved statistical performance, exceeding commonly cited correlations by 4.7–7.8 percentage points. It is also demonstrated that the range $Re = 2000$ – 4000 represents a hybrid region where both Reynolds and Graetz numbers simultaneously affect the Nusselt number.

Keywords: pie flow, generalized Reynolds number, friction factor, heat transfer, Nusselt number

1. INTRODUCTION

Proper design of thermal systems (equipment, pipelines, etc.) is significantly dependent, among many other parameters, on using a suitable correlation for estimation of heat transfer parameters such as heat transfer coefficient or Nusselt number.

Fluid flow through pipes is commonly used in heat transfer units – process furnaces and heat exchangers such as shell and tube heat exchangers, bayonet tube heat exchangers, double pipe heat exchangers, tube–fin heat exchangers in heating or cooling applications. The fluid in such applications is forced to flow by a fan or a pump through tubes and the flow regime can be laminar or turbulent which is evaluated by the value of Reynolds number. The Reynolds number calculated as

$$Re = \frac{w_{av} \cdot d \cdot \rho}{\mu} \quad (1)$$

is commonly defined as the ratio of inertial and viscous forces [1,30,31]. In the preceding definition of the Reynolds number the following nomenclature is used:

- w_{av} (m/s) average fluid velocity in pipe;
- d (m) pipe diameter;
- μ (Pa·s) dynamic viscosity;
- ρ (kg/m³) density.

In laminar flow viscous forces in fluid prevail the inertia forces (that are proportional to the velocity

squared) and fluid flows in layers. In this case pipe friction factor is calculated from Hagen–Poiseuille equation and depends only on the Reynolds number, and heat transfer coefficient can be calculated by some of well-known correlations like Sieder and Tate [2] or Jaćimović et.al [3].

As the Reynolds number (Re) increases above the certain value, the inertia forces prevail the influence of fluid viscosity. Reynolds number plays crucial role in transition from laminar to turbulent flow and vice versa. Citation from [4]: “Reynolds numbers below 2000 to 2100 correspond to laminar or viscous flow; numbers from 2000 to 3000 – 4000 correspond to a transition region of peculiar flow, and numbers above 4000 correspond to turbulent flow.” Similar opinion on this phenomena is established in numerous literature sources (books, papers etc.) and transition between laminar and turbulent flow is in fluid mechanics called critical flow or critical regime.

Turbulent flow is characterized by the development of transverse velocity component that causes the mixing of fluid perpendicular to the flow direction. For turbulent flow the influence of the pipe wall roughness (ar , m) plays an important role, and it is taken into account by relative roughness

$$\varepsilon = ar / d \quad (2)$$

In turbulent flow, smooth pipes are defined as those having small surface irregularities when compared with the thickness of the boundary layer and rough pipes are characterized with irregularities of the walls sufficient to break up the laminar boundary layer. In fluid mechanics there are three cases of turbulent flow divided as follows [5,6]:

- flow in hydraulically smooth pipes (smooth turbulent flow) $Re \cdot Rr \cdot \sqrt{\varepsilon / 8} < 5$

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Correspondence to: Srbislav Genić, University of Belgrade, Faculty of Mechanical Engineering, Kraljice Marije 16, 11120 Belgrade 35, Serbia

E-mail: sgenic@mas.bg.ac.rs

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- flow in hydraulically rough pipes (complete turbulence, rough turbulent flow)

$$\text{Re} \cdot \text{Rr} \cdot \sqrt{\xi/8} > 70$$
- transitional turbulent flow

$$\text{Re} \cdot \text{Rr} \cdot \sqrt{\xi/8} = 5 - 70.$$

Genić and Jaćimović provided in [7] friction factor correlations for complete range of turbulent flow.

Unlike previous case, in heat transfer researches heat transfer coefficient is usually (with a very few exceptions) discussed only in case of smooth turbulent flow. There are more than a few correlations for calculation of the heat transfer coefficient in this case, published throughout the 20th century up to recent days – a comprehensive overview is given in [28].

Among others there are several well-known and often cited equations for the calculation of the heat transfer coefficient for turbulent flow of Newtonian fluids in pipes: von Karman [8], Gnielinski [9,10] and Hausens correlation [11]. It has to be noted that full statistical analysis, concerning primarily standard deviation of listed correlations is missing, both in the original papers and in the literary sources that cite them.

The goal of the research that will be presented hereby is to check the listed heat transfer correlations using the available experimental data from the open literature. Moreover, new and improved correlations for turbulent flow will also be introduced, as well as new and improved definition of Reynolds number.

2. HEAT TRANSFER CORRELATIONS

Heat transfer correlations for forced turbulent flow using the dimensional analysis technique yield the basic relationship in the following form [1]

$$\text{Nu} = f(\text{Re}, \text{Pr}, \dots) = \frac{\alpha \cdot d}{\lambda} \quad (3)$$

in which:

- Nu is Nusselt number (dimensionless parameter characterizing convective heat transfer);
- α (W/(m²·K)) is convective heat transfer coefficient;
- λ (W/(m·K)) is thermal conductivity.

The Prandtl number is defined as

$$\text{Pr} = \frac{\nu}{a} = \frac{\frac{\mu}{\rho}}{\frac{\lambda}{\rho \cdot c_p}} = \frac{c_p \cdot \mu}{\lambda} \quad (4)$$

and represents the ratio of momentum diffusivity (kinematic viscosity ν in m²/s) and thermal diffusivity and it is, for Newtonian fluids, dependent only on fluid properties [1]. In the preceding definition of the Prandtl number the following nomenclature is used:

- ν (m²/s) kinematic viscosity;
- a (m²/s) thermal diffusivity

$$a = \frac{\lambda}{\rho \cdot c_p} \quad (5)$$

- c_p (J/(kg·K)) isobaric specific heat.

As said before, there are numerous correlations for forced turbulent flow in open literature. The most widely cited equations (three in total) are discussed in this paper. The scope of their application is taken from the open literature, including both the original papers as well as the discussions and statements of other authors published in the open literature.

2.1 Von Karman correlation [8]

Von Karman correlation stands

$$\text{Nu} = \frac{\frac{f_W}{8} \cdot \text{Re} \cdot \text{Pr}}{1 + 5 \cdot \sqrt{\frac{f_W}{8}} \cdot \left[\text{Pr} - 1 + \ln \left(\frac{5 \cdot \text{Pr} + 1}{6} \right) \right]} \quad (6)$$

in which f_W denotes Weisbach friction factor for smooth pipes. Authors recommendation is to calculate f_W using equation from [12]

$$f_W = \left[0.79 \cdot \ln(\text{Re}) - 1.64 \right]^{-2} \quad (7)$$

Equation (7) is valid for $\text{Re} = 10^4 - 5 \cdot 10^6$ and $\text{Pr} = 0.5 - 10$.

2.2 Gnielinski correlation [9–10]

Gnielinski correlation for turbulent heat transfer is

$$\text{Nu} = \frac{\frac{f_W}{8} \cdot (\text{Re} - 1000) \cdot \text{Pr}}{1 + 12.7 \cdot \sqrt{\frac{f_W}{8}} \cdot (\text{Pr}^{2/3} - 1)} \cdot \left[1 + \left(\frac{d}{L} \right)^{2/3} \right] \cdot \phi_t \quad (8)$$

in which f_W is friction factor for smooth pipes calculated by (7) and ϕ_t is viscosity ratio correction factor which according to [1] takes into consideration the relative change of heat transfer due to viscosity dependence on temperature. There is a huge list of published correlations that include ϕ_t in various forms and some of them are listed in [29].

In [10] author, for liquids, defines

$$\phi_t = \left(\frac{\text{Pr}_b}{\text{Pr}_w} \right)^{0.11} \quad (9)$$

where Pr_b is Prandtl number at bulk temperature and Pr_w is Prandtl number at wall temperature, and the range of ratio $\text{Pr}_b / \text{Pr}_w$ is from 1 to 10.

Some 40 years earlier and also for liquids, Sieder and Tate [2] have defined ϕ_t for case of laminar flow in somewhat different shape

$$\phi_t = \left(\frac{\mu_b}{\mu_w} \right)^{0.14} \quad (10)$$

where μ_b and μ_w (Pa·s) are dynamic viscosities at bulk and wall temperatures respectively.

It has to be noticed that ϕ_t calculated by (9) and (10) don't differ significantly since, for liquids, only

viscosity is strongly (and nonlinearly) affected by temperature.

According to author himself equation (8) is applicable for turbulent flow in the range $Re = 2300 - 10^6$, $Pr = 0.5 - 500$ and for $L \geq d$ [10]. On the other hand, in [13] the same equation is recommended to be used in the range $Re = 2300 - 5 \cdot 10^6$. In [14] the authors do not recommend equation (8) for the critical regime ($Re = 2000 - 4000$). In their opinion, the overall accuracy of the equation is merely 10%.

However, it is important to notice that the author of equation himself in [9] limits its use to $Re = 4000 - 10^6$.

2.3 Hausen correlation [11]

In his paper [11] Hausen proposes the following correlation for Nusselt number

$$Nu = 0.0235 \cdot (Re^{0.8} - 230) \cdot (1.8 \cdot Pr^{0.3} - 0.8) \cdot \left[1 + \left(\frac{d}{L} \right)^{2/3} \right] \cdot \phi_t \quad (11)$$

which is valid in the range $Re = 2320 - 5 \cdot 10^6$ and $Pr = 0.42 - 1000$. Viscosity ratio correction factor is taken from [2] i.e., equation (10) is applied.

3. REDEFINING REYNOLDS NUMBER

Fully developed laminar steadyflow (shear flow) in straight pipe (i.e. Poiseuille flow) is important in fluid mechanics as it defines the relationship between the mean fluid velocity, the pressure drop (friction factor) and wall shear rate for both Newtonian and non-Newtonian fluids. This kind of flow is observed when the streamline paths of fluid particles are parallel with the local velocity (i.e. adjacent layers of fluid move parallel to each other with different speeds). In this case, the velocity profile is defined by the shear stress between two adjacent fluid layers.

3.1 Reynolds number for laminar flow

In Cartesian coordinate system, for a hydraulically developing steady flow or unsteady flow ($\partial w / \partial t \neq 0$), the relationship between the mean velocity and the pressure drop (friction factor) is a three-dimensional problem while for the hydraulically developed steady flow, on the other hand, it is reduced to a two-dimensional problem.

Navier-Stokes equations for steady-state **hydraulically developing flow or unsteady flow** in single direction (e.g. x -axis) of a Newtonian incompressible isotropic fluid in Cartesian coordinate system ($w_x = w(y, z)$ and $w_y = w_z = 0$) can be reduced to

$$w_x \cdot \frac{\partial w_x}{\partial x} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \cdot \left(\frac{\partial^2 w_x}{\partial x^2} \right) + g_x \quad (12)$$

in which:

- p (Pa) is pressure;
- g_x (m/s^2) is acceleration acting on the continuum.

Dimensional analysis and similarity theory will be used for further analysis of equation (12), since it

provides the means of combining the physical parameters of one phenomenon into several dimensionless groups with the aim to facilitate correlation and interpretation of experimental data [15].

The similarity of physical phenomena implies complete similarity between the flow phenomena in two systems of fluids (for example, systems a and b).

Let one class of phenomena be described by the equation having the following general form

$$F(u_1; u_2; \dots; u_n) = 0 \quad (13)$$

in which $u_1; u_2; \dots; u_n$ are physical parameters by means of which the observed class of appearances is described (velocity, time, temperature, density, concentration etc.).

For two similar systems (a and b) within one class of phenomena, the following two systems of equations can be formed

$$\begin{aligned} F(u_1^a; u_2^a; \dots; u_n^a) &= 0 \\ F(u_1^b; u_2^b; \dots; u_n^b) &= 0 \end{aligned} \quad (14)$$

The similarity of two physical parameters is defined by the coefficient of similarity

$$K_{ui} = \frac{u_i^b}{u_i^a} \quad (15)$$

for $i = 1; 2; \dots; n$.

In this case the system of equations (14) can be rewritten in the following form

$$\left. \begin{aligned} F(u_1^a; u_2^a; \dots; u_n^a) &= 0 \\ F(K_{u_1} \cdot u_1^a; K_{u_2} \cdot u_2^a; \dots; K_{u_n} \cdot u_n^a) &= 0 \end{aligned} \right\} \quad (16)$$

As it is obvious from (16), coefficients of similarity cannot be chosen randomly, but there has to be a well-established relationship between them [16].

The relationship between coefficients of similarity can be obtained by applying the theory of similarity on equation (12). This results in the following relationship

$$\frac{K_w^2}{K_L} = \frac{1}{K_\rho} \cdot \frac{K_p}{K_L} = \frac{K_\mu \cdot K_w}{K_\rho \cdot K_L^2} = K_g \quad (17)$$

Relationship between inertial forces and viscous forces, i.e. the first and the third segment of (17) is

$$\frac{K_w^2}{K_L} = \frac{K_\mu \cdot K_w}{K_\rho \cdot K_L^2} \quad (18)$$

or

$$\frac{K_w \cdot K_L \cdot K_\rho}{K_\mu} = 1 \quad (19)$$

Based on the above considerations the following (typical) definition of Reynolds number is established

$$Re = \frac{w \cdot L \cdot \rho}{\mu} \quad (20)$$

The usual textual description of the Reynolds number is that it is the ratio of inertial forces to viscous forces.

However, inertial forces exist only in case of hydraulically developing or unsteady flow (i.e. $\partial w / \partial t \neq 0$). It is obvious that in the zone of hydraulically developed steady flow in a channel there are no inertial forces (i.e. $\partial w / \partial t = 0$). The conclusion is that, in the case of hydraulically fully developed steady flow, the commonly used textual description of the Reynolds number is not appropriate. Nevertheless, despite this limitation, the above definition of the Reynolds number continues to be widely employed for flow characterization in fluid mechanics, as well as in correlations for pressure drop, heat transfer, and mass transfer.

In the following sections, a more appropriate and comprehensive definition of the generalized Reynolds number will be presented, one that overcomes the aforementioned limitations in its physical interpretation.

3.2 Generalization of Reynolds number for laminar flow

Reynolds number can also be defined by using a different analysis approach. In further text, equation (21) will be analyzed instead of equation (12)

$$w_x \cdot \frac{\partial w_x}{\partial x} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \frac{1}{\rho} \cdot \left(\frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) + g_x \quad (21)$$

in which τ_{yx} (Pa) is shear stress in y - x plane and τ_{zx} (Pa) is shear stress in z - x plane.

The above equation essentially is a more generalized form of equation (12) and is valid for any incompressible viscous fluid i.e., equation is valid for both laminar and turbulent flow.

By applying the similarity theory on (21), it can be shown that

$$\frac{K_w^2}{K_L} = \frac{1}{K_\rho} \cdot \frac{K_p}{K_L} = \frac{K_\tau}{K_\rho \cdot K_L} = K_g \quad (22)$$

In equation (22) the first and third terms must be equal and therefore

$$\frac{K_w^2 \cdot K_\rho}{K_\tau} = 1 \quad (23)$$

Based on (23), the following, somewhat different form of Reynolds number for shear flow is obtained

$$\text{Re}_{Gen} = \frac{w^2 \cdot \rho}{\tau} = 2 \cdot \frac{w^2 \cdot \rho}{\tau} \quad (24)$$

Defined in this way, Reynolds number has a completely different physical meaning: it is defined as the ratio between the fluid kinetic energy ($w^2 \cdot \rho / 2$) and viscous dissipation of energy (τ) i.e. shear stress.

Depending on system boundary conditions, an integral solution of equation (21) yields a generalized Reynolds number Re_{Gen} for the observed y - z plane cross section and flow in the direction of the x -axis

$$\text{Re}_{Gen} = C \cdot \frac{w_{av}^2 \cdot \rho}{\tau_w} \quad (25)$$

in which τ_w (Pa) is the average wall shear stress.

Determination of the constant C will be shown on the example of laminar developed flow of Newtonian incompressible fluid through pipe (i.e. Poiseuille flow). Poiseuille flow could be analyzed as one-dimensional problem (the velocity depends on radius) and wall shear rate is

$$\gamma_w = \left[-\frac{dw(r)}{dr} \right]_{r=R} = \frac{4 \cdot w_{av}}{R} = \frac{8 \cdot w_{av}}{d} \quad (26)$$

in which R (m) is pipe radius and d (m) is pipe diameter (equivalent to hydraulic diameter $d_h = d = 2 \cdot R$).

Average wall shear stress equals

$$\tau_w = \mu \cdot \gamma_w = \mu \cdot \frac{8 \cdot w_{av}}{d} \quad (27)$$

As mentioned before, commonly, the Re number for the flow of Newtonian fluid through pipe is defined as

$$\tau_w = \mu \cdot \gamma_w = \mu \cdot \frac{8 \cdot w_{av}}{d} \quad (1)$$

In further text commonly defined Reynolds number will be labelled as Re and Re_{Gen} will be used for generalized Reynolds number that covers both laminar and turbulent flow.

Based on the above, it is clear that the equation (12), which is applicable to the Newtonian fluids, is only a special case of equation (21), and so the ratio between the corresponding dimensionless coefficients of similarity (i.e. first and last terms of equations (17) and (22)) which form the corresponding dimensionless parameters in this case must be numerically identical.

Since the condition $\text{Re} = \text{Re}_{Gen}$ must be fulfilled for laminar flow, based on equations (25), (27) and (1) it can be shown that

$$\text{Re}_{Gen} = C \cdot \frac{w_{av}^2 \cdot \rho}{\tau_w} = C \cdot \frac{w_{av}^2 \cdot \rho}{\mu \cdot \frac{8 \cdot w_{av}}{d}} \quad (28)$$

$$\text{Re}_{Gen} = C \cdot \frac{w_{av} \cdot d \cdot \rho}{8 \cdot \mu}$$

From this it follows that $C = 8$, meaning that the generalized Reynolds number in this specific case equals

$$\text{Re}_{Gen} = 8 \cdot \frac{\rho \cdot w_{av}^2}{\tau_w} \quad (29)$$

3.3 Generalization of Reynolds number for turbulent flow

Similar approach can be applied in case of hydraulically fully developed turbulent steadyflow in the direction of x -axis of an incompressible viscous fluid through straight channel of an arbitrary z - y cross-section. In this case, the differential force balance equation is

$$p_L = \frac{\partial p}{\partial x} = \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \quad (30)$$

in which p_L is pressure drop (Δp) per channel length (L , m)

$$p_L = \frac{\Delta p}{L} \quad (31)$$

Integral form of equation (30) is

$$\Delta p \cdot A = \tau_w \cdot P \cdot L \quad (32)$$

in which:

- A (m^2) is cross-section area of channel;
- P (m) is wetted perimeter of channel;
- τ_w (Pa) is the average wall shear stress.

Based on equation (32) it can be shown that

$$\Delta p = \tau_w \cdot \frac{L}{A/P} \quad (33)$$

Equation (33) can be transformed into

$$\Delta p = 2 \cdot \frac{1}{\frac{\rho \cdot w_{av}^2}{\tau_w}} \cdot \frac{L}{A/P} \cdot \frac{\rho \cdot w_{av}^2}{2} \quad (34)$$

Hydraulic radius of the channel of an arbitrary cross-section is

$$R_h = \frac{A}{P} \quad (35)$$

so

$$\Delta p = 2 \cdot \frac{1}{\frac{\rho \cdot w_{av}^2}{\tau_w}} \cdot \frac{L}{R_h} \cdot \frac{\rho \cdot w_{av}^2}{2} \quad (36)$$

At this point generalized Re number can be introduced, so after insertion of (29) equation (36) becomes

$$\Delta p = \frac{16}{Re_{Gen}} \cdot \frac{L}{R_h} \cdot \frac{\rho \cdot w_{av}^2}{2} \quad (37)$$

For fluid flow in (cylindric) pipe hydraulic radius is

$$R_h = d / 4 \quad (38)$$

so (37) becomes

$$\Delta p = 4 \cdot f_F \cdot \frac{L}{d} \cdot \frac{\rho \cdot w_{av}^2}{2} \quad (39)$$

in which Fanning friction factor is

$$f_F = 16 / Re_{Gen} \quad (40)$$

On the other hand, eq. (37) can be transformed by implementing the Weisbach [17] pressure drop equation

$$\Delta p = f_W \cdot \frac{L}{d} \cdot \frac{\rho \cdot w_{av}^2}{2} \quad (41)$$

in which Weisbach friction factor is

$$f_W = 64 / Re_{Gen} \quad (42)$$

The relationship between generalized Re number and friction factors is

$$Re_{Gen} = 64 / f_W = 16 / f_F \quad (43)$$

$$\text{so } f_W = 4 \cdot f_F.$$

3.4 Linkage between generalized and ordinary Reynolds number

In opposite to laminar flow, turbulent flow is characterized by the development of transverse velocity component that causes the mixing of fluid perpendicular to the flow direction. Transitional zone between the laminar and turbulent flow (aka critical fluid flow), according to the various authors is defined as $Re = 2000 \div 3000$ in [18], or $Re = 2000 \div 4000$ in [9] and [7]. For Re numbers greater than the upper limit(s) is assumed to be turbulent.

Region of "fully developed turbulent flow" according to von Kármán [8] and many Soviet authors [12] is defined for $Re > 10000$. According to [10] and [11] the turbulent flow occurs when $Re > 2300$.

In further analysis we will assume that critical zone is in range $Re = 2000 \div 4000$.

For the new correlations developed in further text, generalized Re number was calculated by using Weisbach friction factor. Weisbach friction factor (f_W) is explained in detail in [7] and calculated the following:

- for $Re < 2000$

$$f_W = 64 / Re \quad (44)$$

- in the range $Re = 2000 - 4000$

$$f_W = 0.032 + 52 \cdot 10^{-6} \cdot (Re - 2000) \cdot (\varepsilon^{0.8} + 0.089) \quad (45)$$

- for $Re = 4000 - 35.7 \cdot 10^6$

$$f_W = \left\{ -1.8 \cdot \log \left[\frac{7.35 - 1200 \cdot \varepsilon^{1.25}}{Re} + \left(\frac{\varepsilon}{3.15} \right)^{1.15} \right] \right\}^{-2} \quad (46)$$

Having in mind that experimental data set for heat transfer coefficient taken from [19–27] refer to the hydraulically smooth pipes, on the basis of equations (44–46) the relationship between Re and Re_{Gen} is as follows:

- for $Re < 2000$

$$Re_{Gen} = Re \quad (47)$$

- in the range $Re = 2000 - 4000$

$$Re_{Gen} = \frac{64}{f_W(Re, 0)} = \frac{64}{0.032 + 4.628 \cdot 10^{-6} \cdot (Re - 2000)} \quad (48)$$

- for $Re > 4000$

$$Re_{Gen} = \frac{64}{f_W(Re, 0)} = \frac{64}{\left[-1.8 \cdot \log \left(\frac{7.35}{Re} \right) \right]^{-2}} \quad (49)$$

Functional dependence $Re_{Gen} = f(Re)$ based on equations (50), (51) and (52) is shown in Figures 1 and 2.

Figure 1 shows three characteristic regions:

- A – laminar flow region for $Re = Re_{Gen} < 2000$ in which the function $Re_{Gen}(Re)$ is monotonically increasing;
- B – critical (transitional) flow region in which the function $Re_{Gen}(Re)$ is monotonically decreasing within the range $Re = 2000 - 4000$ and Re_{Gen} varies from $Re_{Gen} = 2000$ to $Re_{Gen} = 1551$;
- C – turbulent flow region in which the function $Re_{Gen}(Re)$ is once again monotonically increasing for $Re > 4000$ ($Re_{Gen} > 1551$).

It has to be noted that for $Re = 9375$ corresponding numerical value of Re_{Gen} equals 2000, and this closely corresponds to the previously mentioned lower limit of "fully developed turbulent flow" (Figure 1).

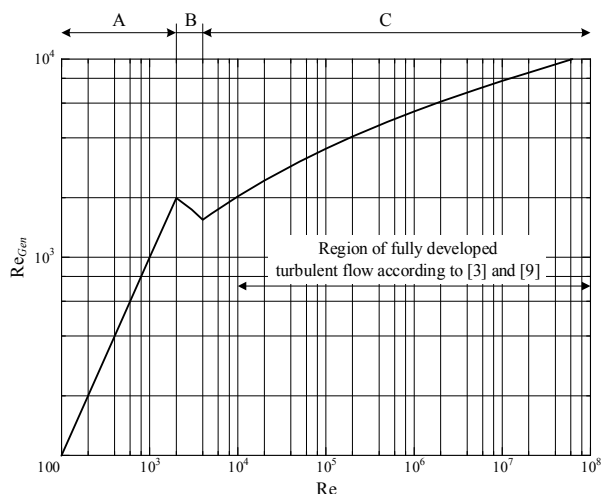


Figure 1 Generalized Reynolds number as a function of typical Reynolds number

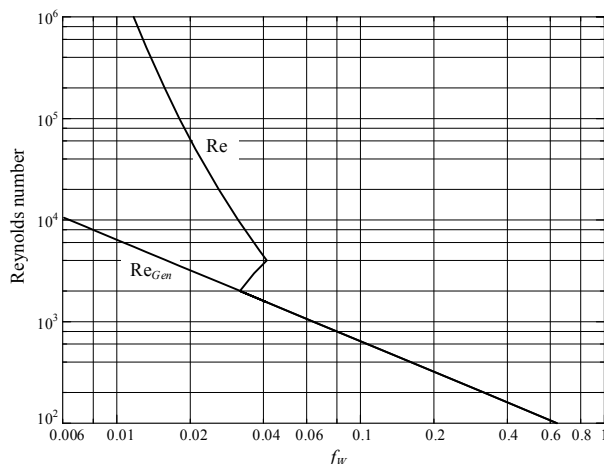


Figure 2 Weisbach friction factor as a function of Reynolds numbers for smooth pipes

3.5 General comment on physical significance of generalized Reynolds number

For practical engineering calculations (40) and (43) are accompanied by appropriate correlations in form $f_w = 4 \cdot f_F = f(Re, Rr)$, and for more than a century the above was considered as a completely closed system that fully describes the physical phenomenon of pipe flow.

In the preceding sections of this paper, it has been demonstrated that the commonly used definition of the Reynolds number (1) possesses a well-defined physical meaning only in the case of hydraulically developing or unsteady flow. For other flow regimes in pipes - namely, hydraulically fully developed laminar flow and turbulent flow - the conventional definition of the Reynolds number (1) does not retain a clear physical interpretation.

In contrast, the generalized Reynolds number (29), defined as the ratio of the fluid kinetic energy ($\rho \cdot w^2/2$) to the viscous dissipation of energy (τ_w), has been shown to be applicable across the entire range of flow regimes, encompassing both fully developed laminar and turbulent flow. Furthermore, it will be demonstrated in the subsequent sections that this generalized Reynolds number (Re_{Gen}) is not only relevant in fluid mechanics but also plays an important role in the analysis of phenomena associated with heat transfer.

4. NEW CORRELATIONS FOR HEAT TRANSFER COEFFICIENT IN PIPES FOR TURBULENT AND CRITICAL FLOW

The scope of the authors effort was to use Re_{Gen} for deriving correlations for heat transfer coefficient in pipes for turbulent and critical flow. Experimental data used in this paper refer to the measured values of the heat transfer coefficient for fluid flow through smooth tubes and cover the turbulent and critical fluid flow heat transfer for (practically) constant tube wall temperature, constant heat flux, as well as the cooling/heating cases of the fluid.

This paper considers the data from a total of $N = 1025$ experimental runs taken from [19–27], from which 110 refers to critical flow. Experiments were done using different liquids: water, petroleum oils, light and heavy fuel oil, light hydrocarbon oil, acetone, benzene, kerosene and n-butyl alcohol and gases. The range of dimensionless numbers and pipe ID covered in experiments are given in Table 1; the database is large enough to cover practically all cases of heat transfer in pipes, that are of the interest in engineering practice. It must be noticed that there are some other datasets mentioned in previously published papers, like a huge database mentioned in [28], but numerical values of the database are not available.

Table 1 Range of parameters covered in experiments

Parameter	Minimal	Maximal
Nu	5.4	2061
Re	2011	522000
Re_{Gen}	1560	4880
Pr	0.18	414
Gz	13.5	68900
μ/μ_w	0.039	7.29
L/d	37.9	213.4
d , mm	6.88	19.1

Above listed data has been used not only for establishing a new correlations, but also for the analysis of existing equations mentioned in section 2.

4.1 New correlation for turbulent flow

For data from [19–27] the following correlation is obtained in turbulent flow

$$Nu = \left[1 + 2.21 \cdot 10^{-14} \cdot (Re_{Gen} - 934)^4 \right] \cdot (0.026 \cdot Re_{Gen} - 34) \cdot (2.8 \cdot Pr^{0.3} - 1) \cdot \left[1 + \left(\frac{d}{L} \right)^{2/3} \right] \cdot \left(\frac{\mu_b}{\mu_w} \right)^{0.11} \quad (50)$$

Correlation (53) is based on $N = 915$ experimental runs in turbulent flow $Re > 4000$ and it is shown in Figure 3.

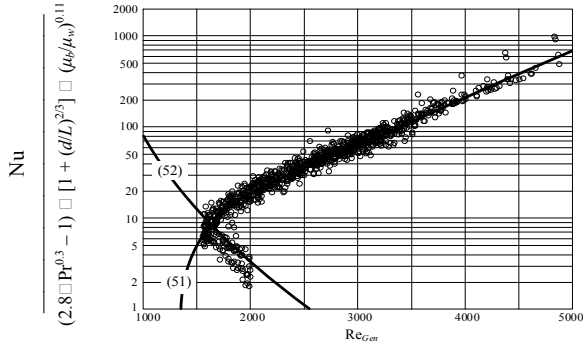


Figure 3 Nu vs. Re_{Gen} for regions B and C

Heat transfer correlations in case of turbulent flow, presented in this paper, have different definitions of the lower and upper limit of Reynolds number as shown in Table 2. Because of this fact the comparison of the results obtained by using the existing correlations and the experimental data is presented in the two distinct Tables 3 and 4, each one representing statistical parameters (defined in Appendix) for a different range of Reynolds number.

General conclusion, based on the Tables 3 and 4 is that the newly proposed correlation has superior statistical parameters in comparison to others equations for the complete range of data (i.e. for $Re > 4000$) as well as for region of “fully developed turbulent flow” (i.e. for $Re > 10000$).

Table 2. Re number limits for the heat transfer equations

Eq.	Author	Range
(6)	fon Karman [8]	$Re = 10^4 - 5 \cdot 10^6$
(8)	Gnielinski [9]	$Re = 4000 - 10^6$
(11)	Hausen [11]	$Re = 2320 - 5 \cdot 10^6$
(53)	Jaćimović and Genić	$Re = 4000 - 522 \cdot 10^3$

Table 3. Statistical parameters for range $Re = 4000 - 522000$ and $Pr = 0.18 - 340$

Eq.	Author	N	$RMRSE$ %	RE_{max}^- %	RE_{max}^+ %	CR %
(8)	Gnielinski [9]	915	21.8	-88.9	53.9	94.2
(11)	Hausen [11]		20.0	-87.8	52.9	95.9
(53)	Jaćimović and Genić		15.3	-48.7	50.0	96.6

Table 4. Statistical parameters for range $Re = 10000 - 522000$ and $Pr = 0.18 - 185$

Eq.	Author	N	$RMRSE$ %	RE_{max}^- %	RE_{max}^+ %	CR %
(6)	fon Karman [8]	728	20.1	-69.7	63.0	93.9
(8)	Gnielinski [9]		22.7	-88.9	53.9	93.4
(11)	Hausen [11]		20.1	-77.0	52.9	95.4
(53)	Jaćimović and Genić		14.9	-48.7	50.0	96.1

4.2 Heat transfer correlating in critical flow

In previous text it was clearly shown that the introduction of generalized Reynolds number leads to significant improvement of heat transfer coefficient estimation.

4.2.1 Reformulation of correlation (53) for critical flow

In order to correlate data in critical flow zone we have used the same form of correlation (53), but with a different member that involves Re_{Gen} . The following form of correlation is obtained

$$Nu = \left(2.8 \cdot 10^{14} \cdot Re_{Gen}^{-4.24} - 6.54 \cdot 10^{-13} \right) \cdot (2.8 \cdot Pr^{0.3} - 1) \cdot \left[1 + \left(\frac{d}{L} \right)^{2/3} \right] \cdot \left(\frac{\mu_b}{\mu_w} \right)^{0.11} \quad (51)$$

For data set containing 110 experimental runs statistical parameters are given in Table 5.

Table 5 Statistical parameters for correlation (54)

Eq.	Re	N	$RMRSE$ %	RE_{max}^- %	RE_{max}^+ %	CR %
(54)	2000 – 4000	110	26.9	-69.3	53.1	84.7

4.2.2 Use of laminar flow correlation in the critical flow

This section shows that the correlation referring to the heat transfer for laminar flow can be used even in the critical flow if Re_{Gen} is used. The following correlation for calculation of Nusselt number for laminar flow is taken from [3]

$$Nu = Nu_{fd} + \frac{0.01 \cdot Gz^{1.7}}{1 + 0.01 \cdot Gz^{1.3}} \cdot \left(\frac{\mu_b}{\mu_w} \right)^{0.14} \quad (52)$$

in which Gz is Graetz number

$$Gz = Re \cdot Pr \cdot d / L \quad (53)$$

For constant tube wall temperature $Nu_{fd} = 3.657$ and for constant heat flux and for all other cases $Nu_{fd} = 4.364$. Correlation (52) is based on 390 runs and it covers the region A.

In order to check whether Re_{Gen} can be directly related to heat transfer in laminar to turbulent flow in regions A and B it is necessary to transform the correlation (55) into the form

$$Nu = Nu_{fd} + \frac{0.01 \cdot Gz_{Gen}^{1.7}}{1 + 0.01 \cdot Gz_{Gen}^{1.3}} \cdot \left(\frac{\mu_b}{\mu_w} \right)^{0.14} \quad (54)$$

where generalized Graetz number is

$$Gz_{Gen} = Re_{Gen} \cdot Pr \cdot d / L \quad (55)$$

When the database for laminar flow (Region A) from [3] is combined with the database for turbulent flow within the range $Re = 2000 - 3000$ (48 runs within the range $Re_{Gen} = 2000 - 1500$, region B) a new joint database is formed having a total of 438 experimental runs. Statistical parameters for correlations (52) and (54) are given in Table 6, and according to them it can

be concluded that the generalized Reynolds number (and consequently generalized Graetz number) has a complete physical sense. In other words, although the correlation (57) is obtained primarily for the laminar flow, it can also be used in the region B ($Re = 2000 - 3000$). Correlation (57) is presented in Figure 4.

Table 6 Statistical parameters for correlation (55) taken from [3] and correlation (57)

Eq.	Re	N	RMRSE %	RE_{max}^- %	RE_{max}^+ %	CR %
(55) from [3]	<2000	390	16.2	-60.2	46.2	96.6
(57)	<3000	442	21.2	-55.4	71.8	88.7

4.2.3 New correlation for critical flow– complement to laminar heat transfer correlation

As shown by previous correlations (51) and (54) both generalized Re_{Gen} and Gz_{Gen} have an effect on the Nusselt number in critical flow. In order to incorporate this conclusion in valuable correlation we came to this one

$$Nu = \left[12.5 + 4.18 \cdot Gz_{Gen}^{0.4} \cdot \left(\frac{\mu_b}{\mu_w} \right)^{0.14} \right] \cdot \left(3 - 1.39 \cdot 10^{-3} \cdot Re_{Gen} \right) \quad (56)$$

Statistical parameters for (56) are given in Table 7, and it is obvious that correlation (56) is better than (51) and (54).

Table 7 Statistical parameters for correlation (59)

Eq.	Re	N	RMRSE %	RE_{max}^- %	RE_{max}^+ %	CR %
(59)	2000 – 4000	110	23.4	-45.2	50.1	88.3

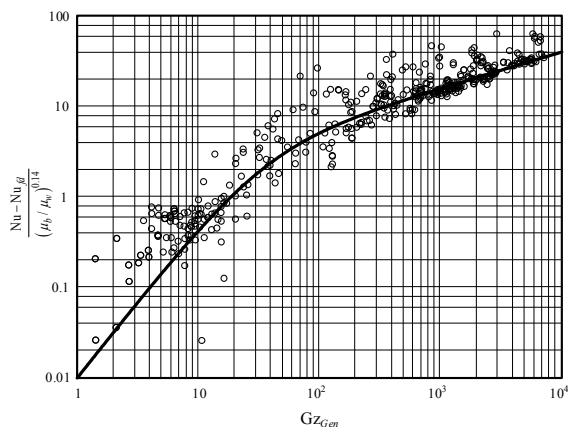


Figure 4 Correlation field Nu vs. Gz_{Gen}

4.2.4 Statistical overview of new correlations along the whole range of turbulent flow

As mentioned before, the main goal of the research presented here was to establish new correlations for the entire area of turbulent flow of Newtonian fluids and to check their quality in relation to previously published (and frequently cited) correlations. Unlike the von Karmann (6) and Gnielinski (8) correlations, the Hausen correlation (11) is the only one that covers the entire area

of critical and turbulent flow ($Re > 2320$). Table 8 shows the statistical indicators for the new correlations (53), (54) and (59) as well as for the Hausen correlation, on the basis of 1025 points in turbulent flow (Regions B + C).

Table 8 Statistical parameters for range $Re = 2000 - 522000$ and $Pr = 0.18 - 414$

Eq.	Author	N	RMRSE %	RE_{max}^- %	RE_{max}^+ %	CR %
(11)	Hausen [11]	1025	22.0	-97.5	53.3	96.2
(54) and (53)	Jaćimović and Genić		16.9	-69.3	53.1	96.8
(59) and (53)	Jaćimović and Genić		16.3	-48.7	50.1	96.8

5. CONCLUSIONS

This paper proposes redefining the commonly used Reynolds number (Re) as the generalized Reynolds number (Re_{Gen}), which holds full physical relevance for both laminar and turbulent pipe flow. It is demonstrated that the traditional Reynolds number is physically meaningful only for hydraulically developing laminar flow, where Re_{Gen} and Re are equivalent. However, in turbulent flow, the conventional Reynolds number loses its physical significance due to significant changes in the relationship between tangential stress and the kinetic energy of fluid flow. Friction factor correlations (47-49) from [7] were utilized to link Re_{Gen} and Re .

To evaluate the validity of previously published correlations from [8], [9], and [11] for estimating the heat transfer coefficient in turbulent flow through circular pipes, data from open literature sources [19–27] were compiled. This data set spans a range of $Re = 2000 - 522000$ and $Pr = 0.18 - 414$, and includes a total of 1025 experimental data points, of which 110 pertain to critical flow.

A statistical analysis for the range $Re = 4000 - 522000$ revealed that Hausen's equation is slightly more accurate than Gnielinski's, with a standard deviation ($RMRSE$) of 20.0%. In the range $Re = 10000 - 522000$, correlations of von Kármán and Hausen (both with $RMRSE = 20.1\%$) outperformed those published by Gnielinski. By introducing the generalized Reynolds number, a new correlation (53) for the heat transfer coefficient in turbulent flow was developed, which exhibits a better standard deviation than previously published correlations: $RMRSE = 15.3\%$ for $Re = 4000 - 522000$, and $RMRSE = 14.9\%$ for $Re = 10000 - 522000$.

To give full physical meaning to the generalized Reynolds number, the correlation for laminar flow from [3] was applied. For a newly compiled database encompassing laminar and critical fluid flow ($Re < 3000$), a transformed version (57) of the laminar correlation from [3] showed an $RMRSE$ of 21.2%. Additionally, a modified version of correlation (53) in the form of (54) was analyzed. It was found that while correlation (54) exhibits a different trend than (53), it is also applicable to critical flow. From this analysis, it is concluded that, in the context of heat transfer, the critical region B ($Re = 2000 - 4000$) is a hybrid area where the heat transfer

coefficient can be determined using equations for either laminar or turbulent flow. This implies that in critical flow, the Nusselt number (Nu) is simultaneously a function of both Re_{Gen} and Gz_{Gen} , leading to the development of an appropriate correlation in the form of (59) with an *RMRSE* of 23.4%.

In conclusion, by using the generalized Reynolds number, the newly derived correlations (53) and (59) for the heat transfer coefficient, expressed as $Nu = f(Re_{Gen}, Pr...)$, along with the previously published correlation (55) for laminar flow, form a comprehensive set of equations that can be effectively applied in practical engineering calculations, such as the sizing of heat exchangers and other related applications. This final recommendation can be expressed in the following form:

- for $Re < 2000$ [3]

$$Nu = Nu_{fd} + \frac{0.01 \cdot Gz^{1.7}}{1 + 0.01 \cdot Gz^{1.3}} \cdot \left(\frac{\mu}{\mu_w} \right)^{0.14} \quad (52)$$

- for $Re = 2000 - 4000$

$$Nu = \left[12.5 + 4.18 \cdot Gz_{Gen}^{0.4} \cdot \left(\frac{\mu_b}{\mu_w} \right)^{0.14} \right] \cdot \left(3 - 1.39 \cdot 10^{-3} \cdot Re_{Gen} \right) \quad (56)$$

- for $Re > 4000$

$$Nu = \left[1 + 2.21 \cdot 10^{-14} \cdot (Re_{Gen} - 934)^4 \right] \cdot \left(0.026 \cdot Re_{Gen} - 34 \right) \cdot \left(2.8 \cdot Pr^{0.3} - 1 \right) \cdot \left[1 + \left(\frac{d}{L} \right)^{2/3} \right] \cdot \left(\frac{\mu_b}{\mu_w} \right)^{0.11} \quad (50)$$

in which Nu_{fd} presents theoretically obtained value for fully developed flow: $Nu_{fd} = 3,657$ for constant tube wall temperature and $Nu_{fd} = 4,364$ for constant heat flux.

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NOMENCLATURE

a , m ² /s	thermal diffusivity
ar , m	absolute pipe roughness
A , m ²	cross-section of channel
c_p , J/(kg·K)	specific heat at constant pressure
CR	correlation ratio
d , m	pipe diameter
d_h , m	hydraulic diameter
f_w	Weisbach friction factor
f_F	Fanning friction factor
g , m/s ²	acting acceleration
K	coefficient of similarity for physical parameter
L , m	length
N	number of experimental runs
O , m	wetted perimeter of channel
p , Pa	pressure
p_L , Pa/m	pressure drop per channel length
R , m	pipe radius
RE	relative error

R_h , m	hydraulic radius
t , s	time
T , K	temperature
w , m/s	fluid velocity
α , W/(m ² ·K)	convective heat transfer coefficient
γ_w , 1/s	wall shear rate
Δp , Pa	pressure drop
ε	relative pipe roughness
ϕ_t	viscosity ratio correction factor
ξ	friction factor
λ , W/(m·K)	thermal conductivity
μ , Pa·s	dynamic viscosity
μ_b , Pa·s	dynamic viscosity at bulk temperature
μ_w , Pa·s	dynamic viscosity at wall temperature
ν , m ² /s	kinematic viscosity
ρ , kg/m ³	density
τ , Pa	shear stress
τ_w , Pa	average wall shear stress

Dimensionless numbers

Gz	Graetz number
Nu	Nusselt number
Pr	Prandtl number
Re	Reynolds number

Subscripts

b	bulk
g	acting acceleration
Gen	generalized (in physical sense)
h	hydraulic
L	laminar
max	maximal
$mean$	mean
min	minimal
T	turbulent
w	wall

Superscripts

a, b	similar appearances within one class of phenomena
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APPENDIX – STATISTICAL PARAMETERS

The comparison of the experimental (y_i) and correlated (y_i^c) parameters can be done by a certain number of statistical parameters:

- root-mean-squared relative error

$$RMRSE = \sqrt{\frac{\sum_{i=1}^N \left(\frac{y_i - y_i^c}{y_i} \right)^2}{N}}$$

- the maximal positive relative error

$$RE_{max}^+ = \max \left(\frac{y_i - y_i^c}{y_i} \right)$$

- the maximal negative relative error

$$RE_{max}^- = \max \left(\frac{y_i^c - y_i}{y_i} \right)$$

- correlation ratio

$$CR = \sqrt{1 - \frac{\sum_{i=1}^N (y_i - y_i^c)^2}{\sum_{i=1}^N (y_i - y_{av})^2}}$$

where y_{av} is the average value of y_i for complete set of N experimental data

$$y_{av} = \frac{\sum_{i=1}^N y_i}{N}$$

НОВИ ПРИСТУП КОРЕЛИСАЊУ ПОДАТАКА О ПРЕНОСУ ТОПЛОТЕ ЗА ТУРБУЛЕНТНО СТРУЈАЊЕ У ГЛАТКИМ ЦЕВИМА ЗАСНОВАН НА ГЕНЕРАЛИЗАЦИЈИ РЕЈНОЛДСОВОГ БРОЈА

Б. Јаћимовић, С. Генић

У овом чланку је показано да „уобичајено дефинисан“ Рејнолдсов број има јасно физичко значење само за ламинарно, хидраулички неразвијено или нестационарно струјање, док генерализовани Рејнолдсов број има физички смисао и за ламинарни и за турбулентни режим било оно развијено или не; ова два Рејнолдсова броја се поклапају само у случају ламинарног струјања.

Дат је преглед најчешће коришћених корелација преноса топлоте за турбулентно струјање течности у глатким цевима ($Re > 2000$), а нове корелације су изведене на основу генерализованог Рејнолдсовог броја. Анализа је заснована на бази од 1025 експерименталних мерења (од којих је 110 у прелазном режиму), у опсегу $Re = 2000-522000$ и $Pr = 0,18-414$. Предложене корелације важе у истим опсезима и показују боље статистичке резултате, надмашујући често цитиране корелације за 4,7–7,8 процентних поена. Такође је показано да опсег $Re = 2000-4000$ представља хибридну зону у којој и Рејнолдсов и Грецов број истовремено утичу на Нуселтов број.